**EXPONENTIAL and LOGARITHMS TEST**

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<th>Multiple Choice</th>
<th>Short Answer</th>
<th>Applications</th>
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<td></td>
<td>8</td>
<td>77</td>
<td>13</td>
<td>98 %</td>
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Time allowed: 80 minutes (Section A: 60 mins  Section B and C: 20 mins)

**SECTION A:** Short Answer: Show all necessary working out.

1. Evaluate and simplify the following, expressing answers as a positive index where necessary.
   a) \((5x + y)^0 + 7x^0 - 6\)
   
   \[2 \text{ marks}\]

   b) \(\frac{(3a^3)^2}{6a^{-1}}\)
   
   \[2 \text{ marks}\]

   c) \(\frac{8a^6}{6a^3} + \frac{4(a^2)^4}{(3a)^3}\)
   
   \[2 \text{ marks}\]

   d) \((x - y^{-1})(x^{-1} + y)\)
   
   \[3 \text{ marks}\]

   e) \(\frac{4^{n+1} \times 2^{n-2}}{2^{3n} \times 8^{-n-3}}\)
   
   \[3 \text{ marks}\]

2. Evaluate the following:
   a) \(16^{\frac{3}{4}}\)
   
   \[2 \text{ marks}\]

   b) \(\left(\frac{1}{16}\right)^{\frac{1}{2}} \times (0.5)^{-3}\)
   
   \[3 \text{ marks}\]
3. Solve for $x$.
   (Leave answers in index notation where applicable.)
   a) $3^x = \frac{1}{3}$
   b) $3x^\frac{2}{3} + 5 = 80$
   c) $4^{x+4} = 2^{x^2}$
   d) $\sqrt[3]{x^{-1}} = \frac{1}{2}$
   e) $\left(\frac{10}{9}\right)^x = (0.9)^{\frac{3}{2}}$

4. Express each of the following in logarithm form.
   a) $49 = 7^2$
   b) $12^0 = 1$

6. Express each of the following in index form.
   a) $\log_{10} 100 = 2$
   b) $\log_a n = m$

7. Find the exact values of:
   a) $\log_2 2^9$
   b) $10^{\log_{10} 7.3}$
   c) $\log_{10} \frac{1}{1000}$
   d) $2\log_{10} 3 + \log_{10} 16 - 2\log_{10} \left(\frac{6}{5}\right)$
8. Solve and evaluate the following equations for $x$

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<tr>
<td>a)</td>
<td>$\log_{10} x = 3$</td>
</tr>
<tr>
<td>b)</td>
<td>$\log_4 x = 0.5$</td>
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<tr>
<td>c)</td>
<td>$\log_x 32 = 2.5$</td>
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9. Simplify the following:

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<td>a)</td>
<td>$\log_2 16 + \log_2 8$</td>
</tr>
<tr>
<td>b)</td>
<td>$\frac{3}{2} \log_a a - \log_a \sqrt{a}$</td>
</tr>
<tr>
<td>c)</td>
<td>$2 \log_{10} 3 + \log_{10} 16 - 2 \log_{10} \left( \frac{6}{5} \right)$</td>
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10. If $\log_a x = 4$ and $\log_a y = 5$, find the exact value of:

$$\log_a x^2 y$$

11. Show algebraically that the two graphs

\[ y = \log_3 (x - 1) \quad \text{and} \quad y = 1 - \log_4 (x + 1) \]

Intersects at one point only. State the coordinates of this point of intersection.
12. State the maximal domain, range and the x-intercept of the following equation.

\[ f(x) = \log_2(x - 3) \]

13. Sketch \( f(x) = 1 - 2^x \), on the grid below, stating the

a) equation of the asymptote.

b) domain

d) range

c) coordinates of the y-intercept

[1+1+1+1+2 (graph)=6 marks]

End of Section A.
Hand this in to your teacher and collect Sections B and C.
Section B: Multiple Choice. CAS allowed.

1. The expression \((-a)^2(-a^2)(-a)^3(a)^2\) equals
   A. \(a^9\)
   B. \(-a^9\)
   C. \(a^8\)
   D. \(-a^8\)
   E. \(a^{24}\)

2. The expression \(36(w^2y^3)^2 \div 15(w^2y^3)^3\) equals
   A. \(6w \div 15\)
   B. \(12w \div 5\)
   C. \(12 \div 5w\)
   D. \(5w \div 12\)
   E. \(5 \div 12w\)

3. The equation of the asymptote of \(y = 5 \times 2^{x-1} + 4\) is
   A. \(x = 4\)
   B. \(y = 0\)
   C. \(y = 4\)
   D. \(x = 0\)
   E. \(y = 5\)

4. The solution of the equation \(4 \times 2^{5x} = 64\) is
   A. \(x = \frac{1}{5}\)
   B. \(x = \frac{4}{5}\)
   C. \(x = \left(\frac{1}{5}\right) \log_2 60\)
   D. \(x = \left(\frac{1}{2}\right) \log_2 16\)
   E. \(x = \left(\frac{1}{5}\right) 2^5\)

5. If \(\log_a(x^2 - x + a) = 1\) where \(a > 0\) and \(a\) is a constant, then all the possible solutions for \(x\) are
   A. \(x = -1\)
   B. \(x = 0\)
   C. \(x = 1\)
   D. \(x = 0\) or \(x = 1\)
   E. \(x = 0\) or \(x = -1\) or \(x = 1\)

6. You are told there is an error in the statement \(676537 33\) sps\(rqp\). To make the statement correct, what should the left hand side be?
   A. \((3p^7q^3r^5s^6)^0\)
   B. \((3p^7)^0 q^3r^5s^6\)
   C. \(3p^7(q^3r^5s^6)^0\)
   D. \(3p^7(q^3r^5s^6)^0 \times s^6\)
   E. unchanged.

7. If \(\log_b a = 5\), which of the following statements is always true?
   A. \(a = 5^b\)
   B. \(a = b^5\)
   C. \(b = a^5\)
   D. \(b = a^5\)
   E. \(a = 5\)
8. Which of the following graphs could be the graph of the function \( f(x) = 2^{ax} + b \), where \( a \) and \( b \) are negative?

A

B

C

D

E

Section C: CAS allowed. [Total = 13 marks]

1. Solve the following equation, correct to 3 decimal places.

\[ 3^{2x-1} = 28 \]

[1 mark]

2. Tim removes a cake from a hot oven and places it on a bench to cool. The temperature \( T \), in degrees Celsius of the cake \( t \) minutes after it is removed from the oven is given by

\[ T = 20 + 160 \times 2^{-kt}, \text{ where } t \geq 0 \text{ and } k \text{ is a constant.} \]

a) What is the temperature of the cake initially?

[1 mark]

The temperature of the cake 10 minutes after it has been removed from the oven is 100°C.

b) Show algebraically that \( 0.1 \leq k \leq 0.1 \)

[2 marks]

c) Sketch the graph of \( T = 20 + 160 \times 2^{-0.1t}, t \geq 0 \) on the set of axes below. Indicate clearly any endpoints and asymptotes on your graph.

[3 marks]
d) What is the temperature of the cake 20 minutes after it has been removed from the oven?

[1 mark]

e) How long after the cake has been removed from the oven, is its temperature 25°C?

[1 mark]

Tim realises that the best time to apply the pink topping to the cake, is when its temperature is between 60°C and 40°C.

f) For how long is the cake at a temperature that is best to apply the topping?

[2 marks]

g) Find the average rate of change of temperature of the cake between $t = 20$ and $t = 30$.

[1 mark]

h) Over time, what does the temperature of the cake approach?

[1 mark]

End of Section B and C: