17 Using a graphic display calculator

CHAPTER OBJECTIVES:

This chapter shows you how to use your graphic display calculator (GDC) to solve the different types of problems that you will meet in your course. You should not work through the whole of the chapter – it is simply here for reference purposes. When you are working on problems in the mathematical chapters, you can refer to this chapter for extra help with your GDC if you need it.

GDC instructions on CD:
The instructions in this chapter are for the TI-Nspire model. Instructions for the same techniques using the TI-84 Plus and the Casio FX-9860GII are available on the CD.

Chapter contents

Before you start

You should know:

- Important keys on the keyboard: On, Off, menu, esc, tab, ctrl, shift, enter, del
- The home screen
- Opening new documents, adding new pages, changing settings
- Moving between pages in a document
- Panning and grabbing axes to change a window in a Graphs page
- Change window settings in a Graphs page
- Using zoom tools in a Graphs page
- Using trace in a Graphs page
- Setting the number of significant figures or decimal places

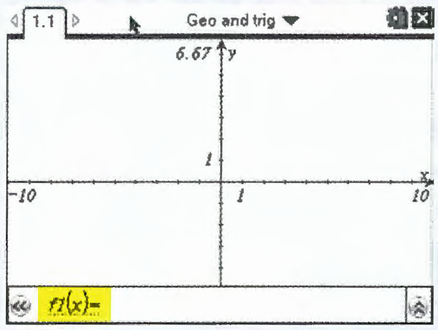
1 Functions

1.1 Graphing linear functions

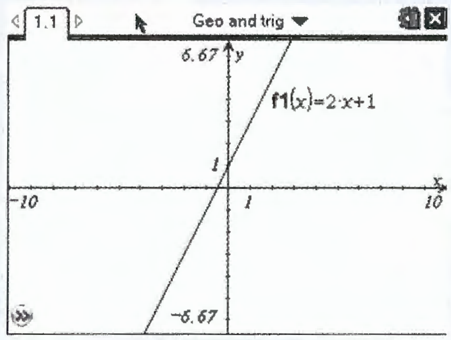
Example 1

Draw the graph of the function $y = 2x + 1$

Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function,
so the form '$f1(x)$=' is displayed.
The default axes are $-10 \le x \le 10$ and
$-6.67 \le y \le 6.67$.
Type $2x + 1$ and press enter.



The graph of $y = 2x + 1$ is now displayed and labeled on the screen.



Finding information about the graph

Your GDC can give you a lot of information about the graph of a function, such as the coordinates of points of interest and the gradient (slope).

1.2 Finding a zero

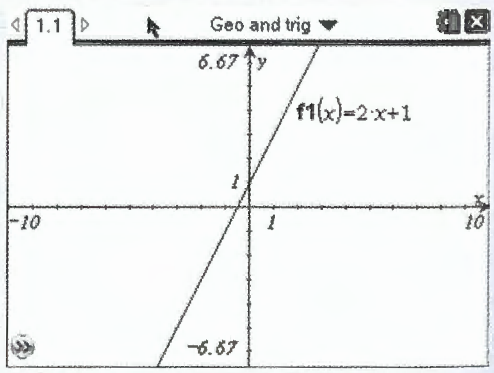
The x-intercept is known as a *zero* of the function.

At the x-intercept, $y = 0$.

Example 2

Find the zero of $y = 2x + 1$

First draw the graph of $y = 2x + 1$ (see Example 1).



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Press menu 6:Analyze Graph | 1:Zero

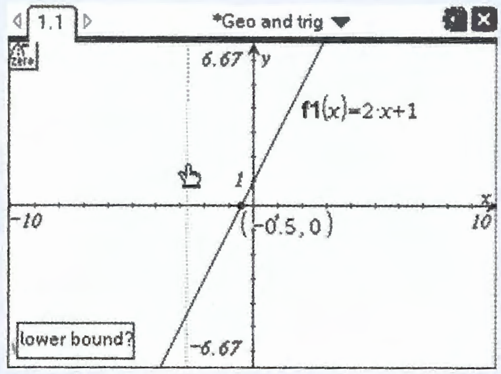
Press enter.

To find the zero you need to give the lower and upper bounds of a region that includes the zero.

The GDC shows a line and asks you to set the lower bound.

Move the line using the touchpad and choose a position to the left of the zero.

Click the touchpad.

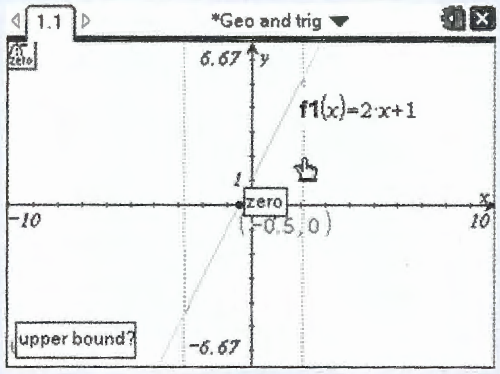


The GDC shows another line and asks you to set the upper bound.

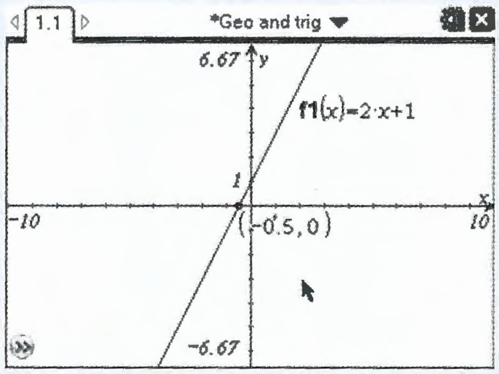
Use the touchpad to move the line so that the region between the upper and lower bounds contains the zero.

When the region contains the zero, the calculator will display the word 'zero' in a box.

Click the touchpad.



The GDC displays the zero of the function $y = 2x + 1$ at the point $(-0.5, 0)$.



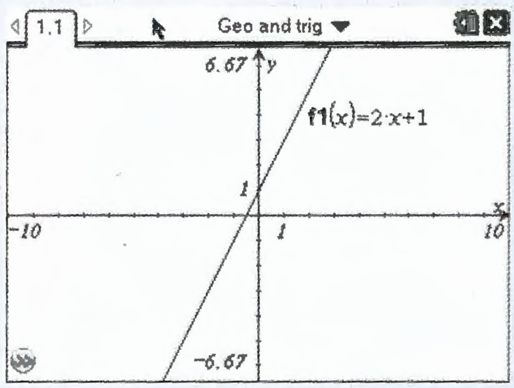
1.3 Finding the gradient (slope) of a line

The correct mathematical notation for gradient (slope) is $\frac{dy}{dx}$, and this is how the GDC denotes gradient.

Example 3

Find the gradient of $y = 2x + 1$

First draw the graph of $y = 2x + 1$ (see Example 1).



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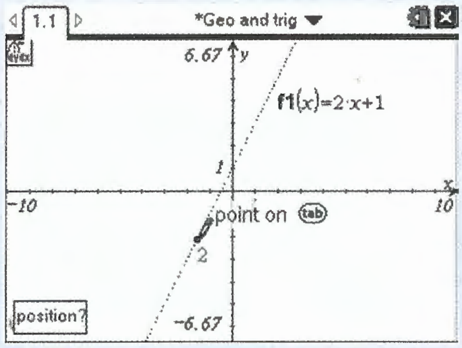
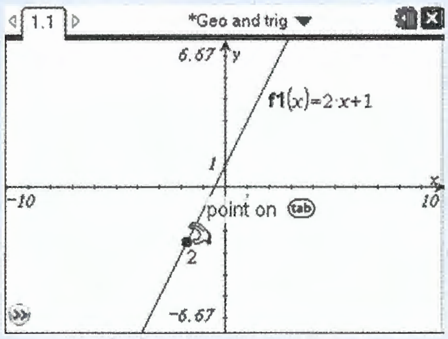
Press menu 6:Analyze Graph | 5: $\frac{dy}{dx}$

Press enter

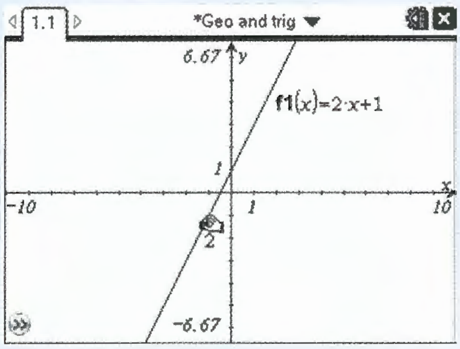
Use the touchpad to select a point on the line.
Click the touchpad.



The point you selected is now displayed together with the gradient of the line at that point.
The gradient (slope) is 2.



With the open-hand symbol showing, click the touchpad again. The hand is now grasping the point.
Move the point along the line using the touchpad.
This confirms that the gradient (slope) of $y = 2x + 1$ at every point on the line is 2.



Simultaneous equations

1.4 Solving simultaneous equations graphically

To solve simultaneous equations graphically you draw the straight lines and then find their point of intersection. The coordinates of the point of intersection give you the solutions x and y.

For solving simultaneous equations using a non-graphical method, see section 1.5.

Example 4

Use a graphical method to solve the simultaneous equations

$2x + y = 10$

$x - y = 2$

First rewrite both equations in the form '$y =$'.

$2x + y = 10$	$x - y = 2$
$y = 10 - 2x$	$-y = 2 - x$
	$y = x - 2$

The GDC will only draw the graphs of functions that are expressed explicitly, '$y =$' as a function of x. If the equations are written in a different form, you need to rearrange them before using your GDC to solve them.

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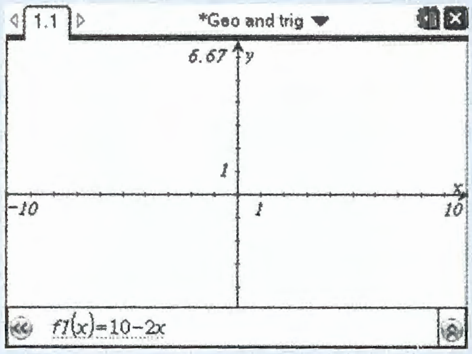
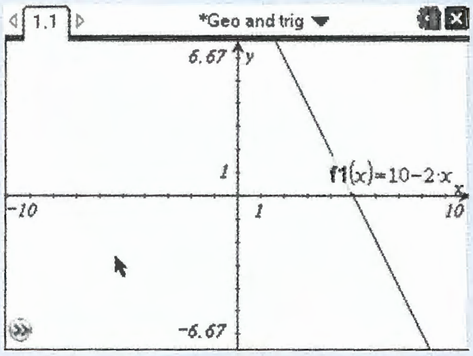
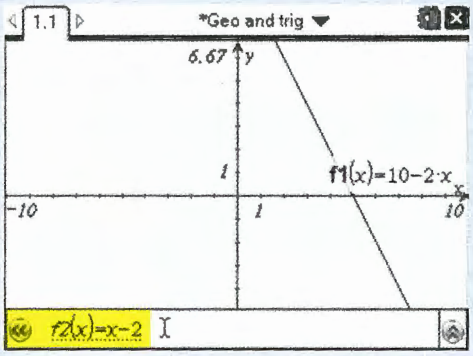
To draw the graphs $y = 10 - 2x$ and $y = x - 2$:
Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form '*f1(x)=*' is displayed.
The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.



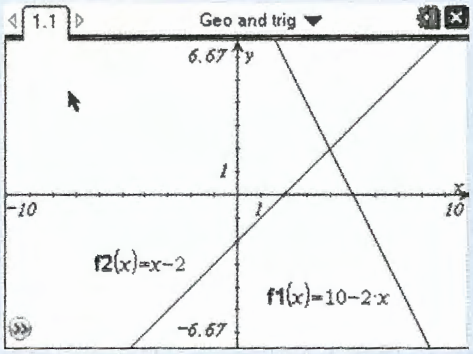
Type $10 - 2x$ and press enter.
The calculator displays the first straight-line graph:
$f1(x) = 10 - 2x$



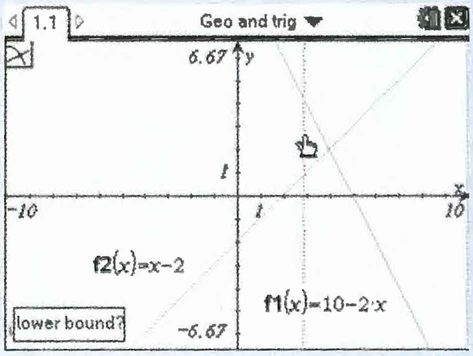
Use the touchpad to click on the arrows in the bottom left-hand corner of the screen.
This will open the entry line again. This time '$f\,2(x)=$' is displayed.
Type $x - 2$ and press enter.



The GDC now displays both straight-line graphs:
$f1(x) = 10 - 2x$
$f2(x) = x - 2$



Press menu 6:Analyze Graph | 4:Intersection Point(s)
Press enter
To find the intersection you need to give the lower and upper bounds of a region that includes the intersection.
The GDC shows a line and asks you to set the lower bound. Move the line using the touchpad and choose a position to the left of the intersection.
Click the touchpad.



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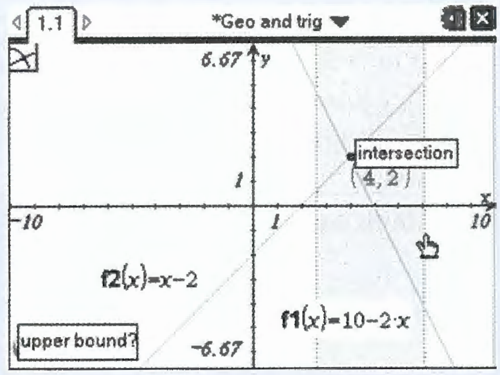
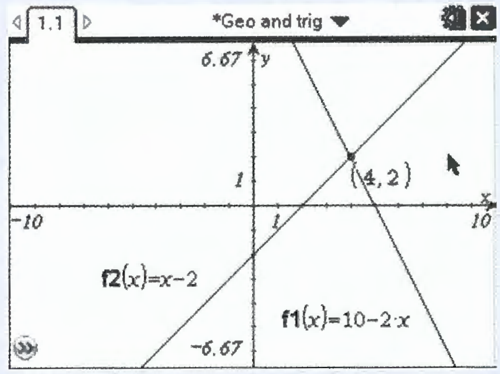
The GDC shows another line and asks you to set the upper bound.
Use the touchpad to move the line so that the region between the upper and lower bounds contains the intersection.
When the region contains the intersection, the calculator will display the word 'intersection' in a box.
Click the touchpad.



The calculator displays the intersection of the two straight lines at the point (4, 2).
The solution is $x = 4$, $y = 2$.



1.5 Solving simultaneous linear equations

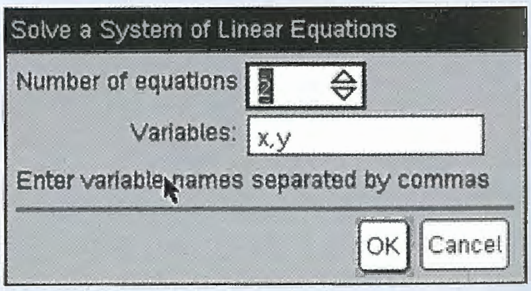
When solving simultaneous equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The GDC will do all the working for you.

You do not need the equations to be written in any particular format to use the linear equation solver, as long as they are both *linear*, that is, neither equation contains x^2 or higher order terms.

Example 5

Solve the equations:
$2x + y = 10$
$x - y = 2$

Open a new document and add a Calculator page.
Press menu 3:Algebra | 2:Solve Systems of Linear Equations…
Press enter
You will see this dialogue box, showing 2 equations and two variables, x and y.
Note: This is how you will use the linear equation solver in your examinations. In your project, you might want to solve a more complicated system with more equations and more variables.



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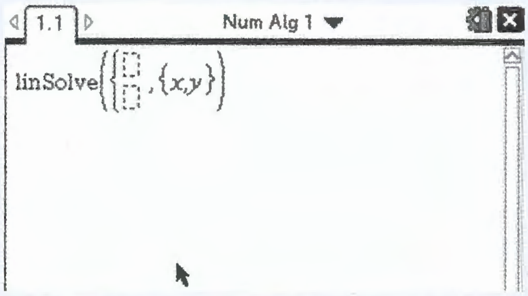
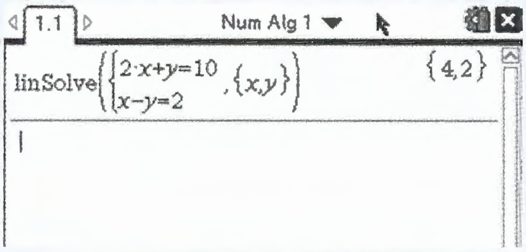
Press enter and you will see the template on the right.
Type the two equations into the template, using the arrow keys ▲▼ to move within the template.
Press enter and the GDC will solve the equations, giving the solutions in the form $\{x, y\}$.



The solutions are $x = 4$, $y = 2$.



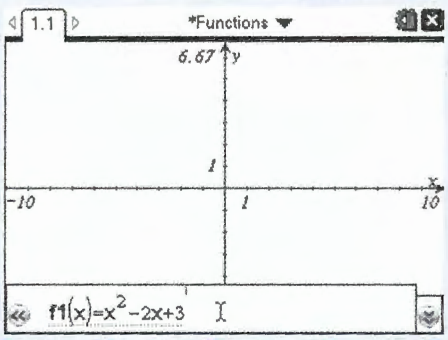
Quadratic functions

1.6 Drawing a quadratic graph

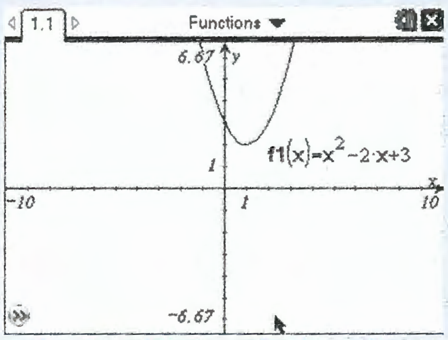
Example 6

Draw the graph of $y = x^2 - 2x + 3$ and display using suitable axes.

Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area. The default graph type is Function, so the form '$f1(x) =$' is displayed.
The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.

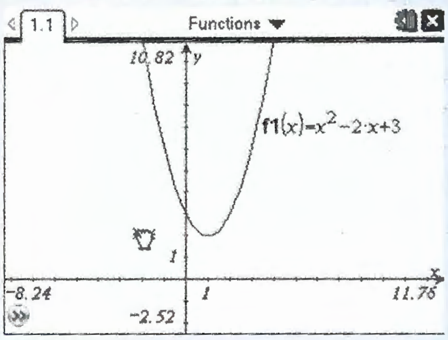


Type $x^2 - 2x + 3$ and press enter.
The calculator displays the curve with the default axes.



Pan the axes to get a better view of the curve.

For help with panning, see your GDC manual.



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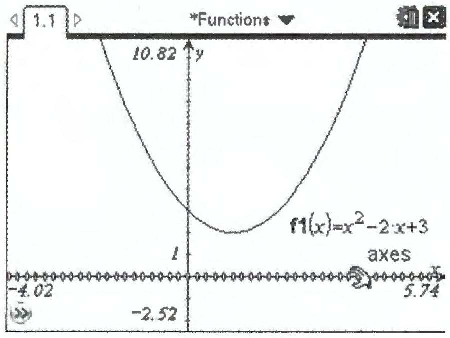
Grab the x-axis and change it to make the quadratic curve fit the screen better.

For help with changing axes, see your GDC manual.



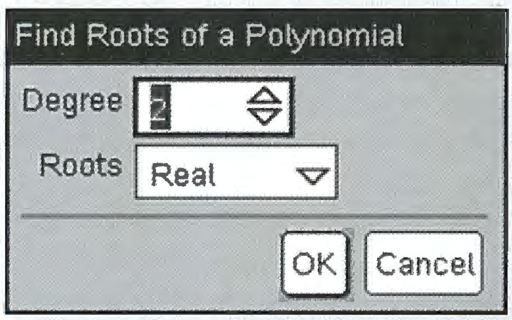
1.7 Solving quadratic equations

When solving quadratic equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The GDC will do all the working for you.

Example 7

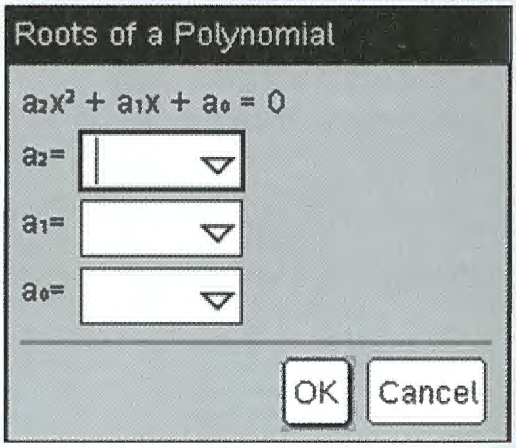
Solve $3x^2 - 4x - 2 = 0$

Press menu 3:Algebra | 3:Polynomial Tools | 1:Find Roots of a Polynomial...
Press enter
You will see this dialogue box, showing a polynomial of degree 2 (a quadratic equation) with real roots. You do not need to change anything.
Press enter



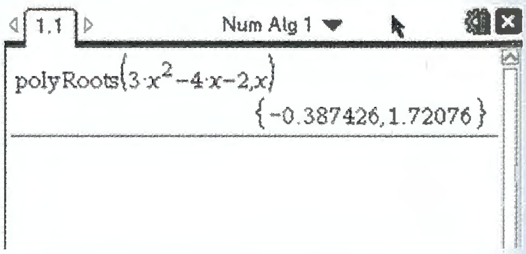
no dialogue box

Another dialogue box opens for you to enter the equation. The general form of the quadratic equation is $a_2x^2 + a_1x + a_0 = 0$, so enter the coefficients in a_2, a_1 **and** a_0.
Here, $a_2 = 3$, $a_1 = -4$ and $a_0 = -2$. Be sure to use the (−) key to enter the negative values. Use the tab key to move around the dialogue box.
Press enter and the GDC will solve the equation, giving the roots in the form $\{x, y\}$.



Type $3x^2-4x-2$, x (not $=0$)

The solutions are $x = -0.387$ or $x = 1.72$ (to 3 sf).

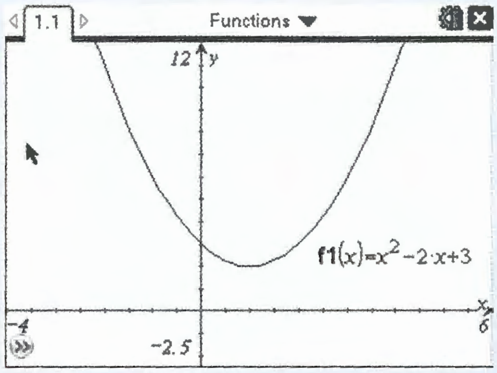


1.8 Finding a local minimum or maximum point

Example 8

Find the minimum point on the graph of $y = x^2 - 2x + 3$

First draw the graph of $y = x^2 - 2x + 3$ (see Example 6).



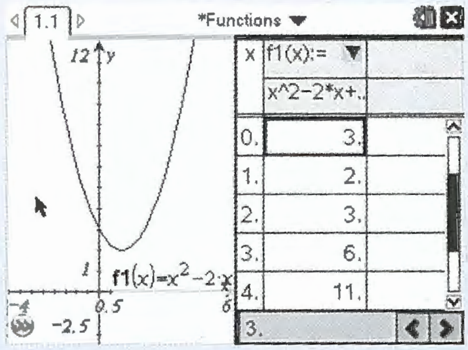
Method 1: Using a table

You can look at the graph **and** a table of the values by using a split screen.

~~Press menu 2:View | 9:Show Table~~ Menu 7 : Table : 1 : Split Screen

(or simply press ctrl T)

The minimum value shown in the table is 2 when $x = 1$.



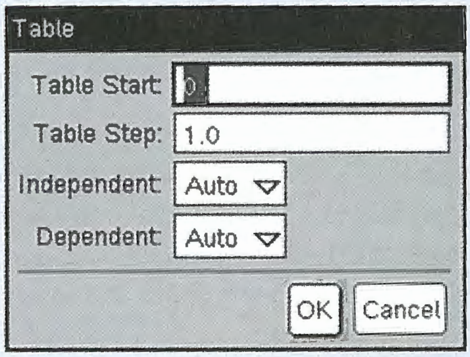
Look more closely at the values of the function around $x = 1$.

Change the settings in the table.

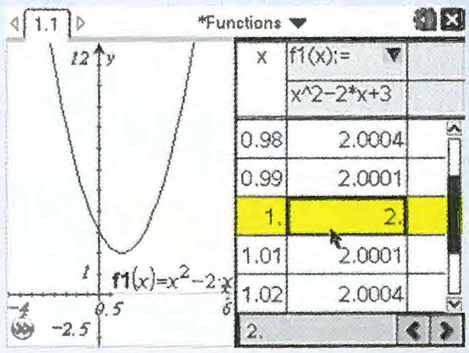
Choose any cell and press menu 5:Table | 5:Edit Table Settings…

Set Table Start to 0.98 and Table Step to 0.01.

Press enter



The table shows that the function has larger values at points around (1, 2). We can conclude that the point (1, 2) is a local minimum on the curve.



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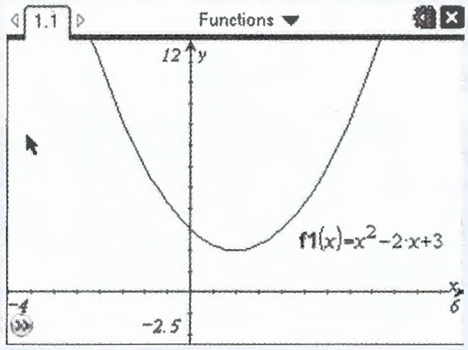
Method 2: Using the minimum function



Press menu 6:Analyze Graph | 2:Minimum

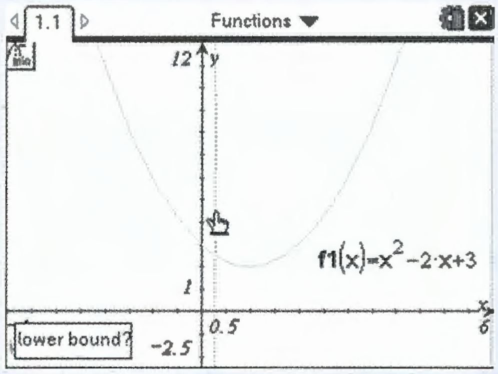
Press enter

To find the minimum you need to give the lower and upper bounds of a region that includes the minimum.

The GDC shows a line and asks you to set the lower bound.

Move the line using the touchpad and choose a position to the left of the minimum.

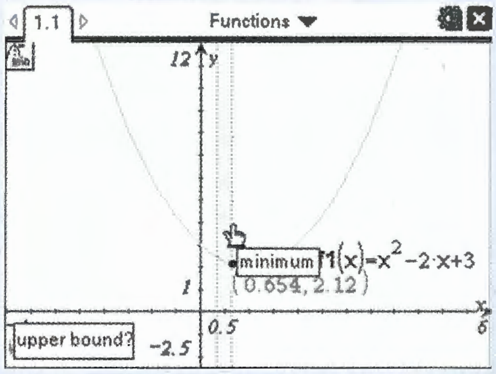
Click the touchpad.



The GDC shows another line and asks you to set the upper bound.

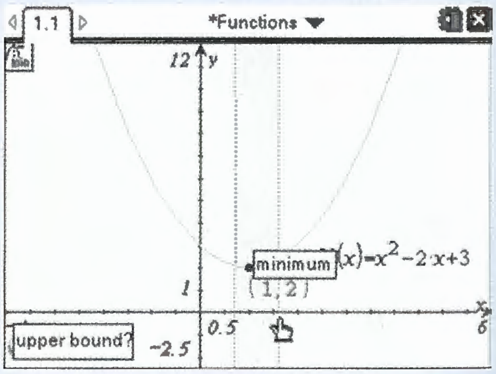
Use the touchpad to move the line so that the region between the upper and lower bounds contains the minimum.

Note: The minimum point in the region that you have defined is being shown. In this screenshot it is not the local minimum point. Make sure you move the line beyond the point you are looking for.

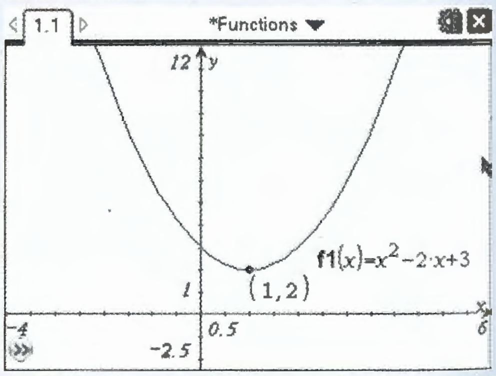


When the region contains the minimum, the GDC will display the word 'minimum' in a box and a point that lies between the lower and upper bounds. The point displayed is clearly between the upper and lower bounds.

Click the touchpad.



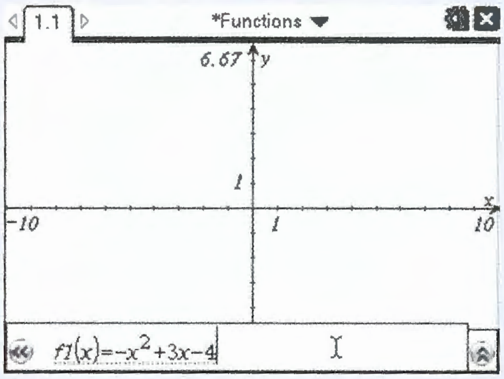
The calculator displays the minimum point on the curve at (1, 2).



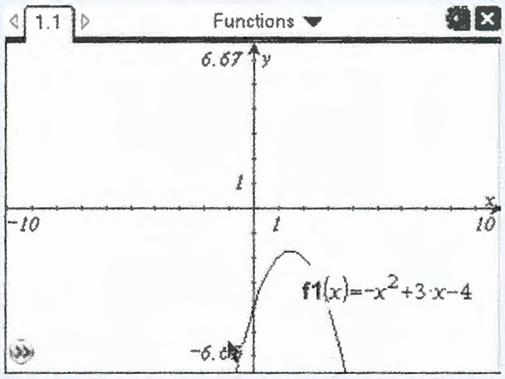
Example 9

Find the maximum point on the graph of $y = -x^2 + 3x - 4$

First draw the graph of $y = -x^2 + 3x - 4$:
Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form '*f1(x)=*' is displayed.
The default axes are $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.

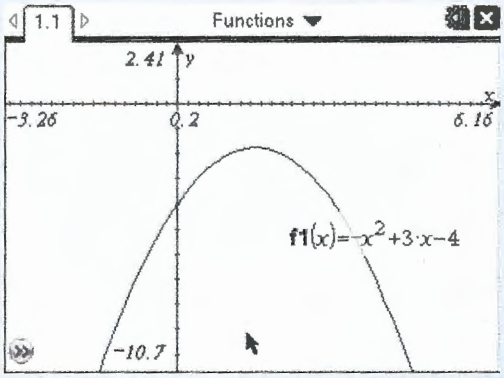


Type $-x^2 + 3x - 4$ and press enter.
The GDC displays the curve with the default axes.

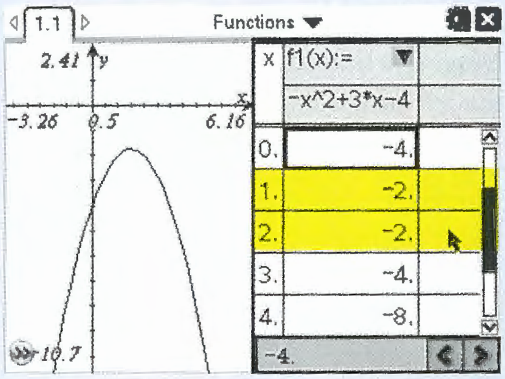


Pan the axes to get a better view of the curve.
Grab the x-axis and change it to make the quadratic curve fit the screen better.

For help with panning or changing axes, see your GDC manual.



Method 1: Using a table
You can look at the graph **and** a table of the values by using a split screen.
Press menu 2:View | 9:Show Table
(or simply press ctrl T)
The maximum value shown in the table is -2 when $x = 1$ and $x = 2$.



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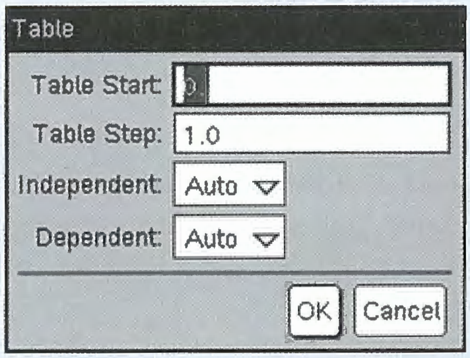
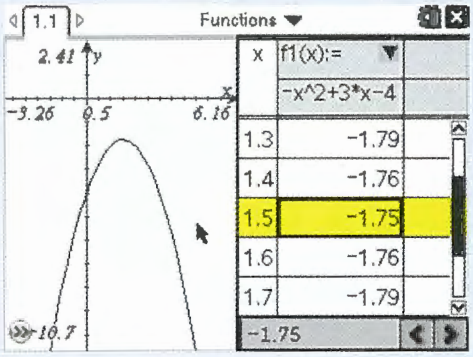
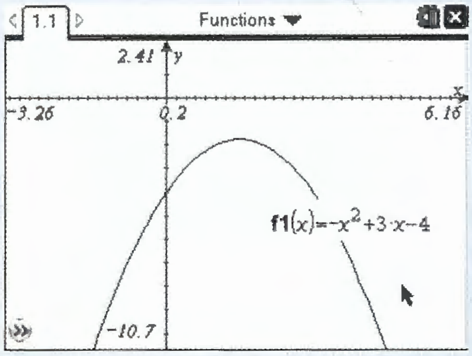
Look more closely at the values of the function between $x = 1$ and $x = 2$.
Change the settings in the table.
Choose any cell and press menu 5:Table | 5:Edit Table Settings…
Set Table Start to 1.0 and Table Step to 0.1.
Press enter



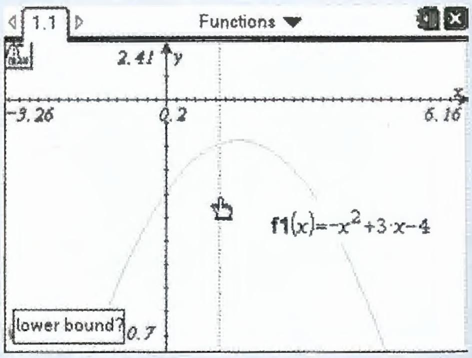
Scroll down the table and you can see that the function has its largest value at (1.5, −1.75). We can conclude that the point (1.5, −1.75) is a local maximum on the curve.



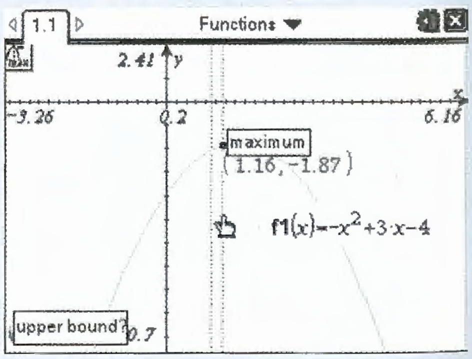
Method 2: Using the maximum function



Press menu 6:Analyze Graph | 3:Maximum
Press enter
To find the maximum you need to give the lower and upper bounds of a region that includes the maximum.
The GDC shows a line and asks you to set the lower bound.
Move the line using the touchpad and choose a position to the left of the maximum.
Click the touchpad.

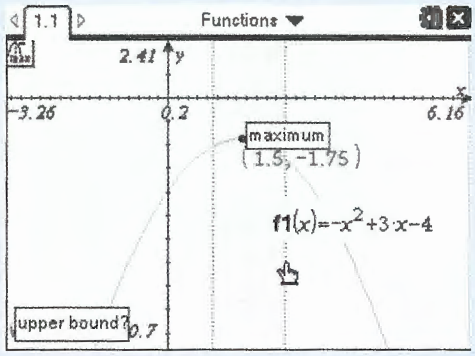


The GDC shows another line and asks you to set the upper bound.
Use the touchpad to move the line so that the region between the upper and lower bounds contains the maximum.
Note: The maximum point in the region that you have defined is being shown. In this screenshot it is not the local maximum point. Make sure you move the line beyond the point you are looking for.

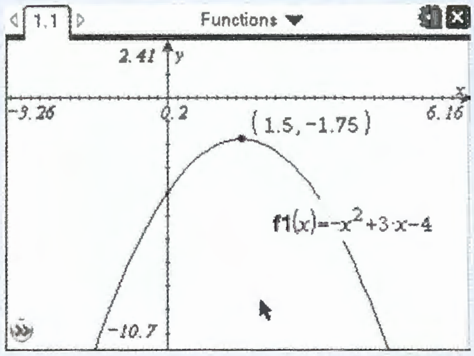


▶ Continued on next page

When the region contains the maximum, the GDC will display the word 'maximum' in a box and a point that lies between the lower and upper bounds. The point displayed is clearly between the upper and lower bounds.
Click the touchpad.



The GDC displays the maximum point on the curve at (1.5, −1.75).



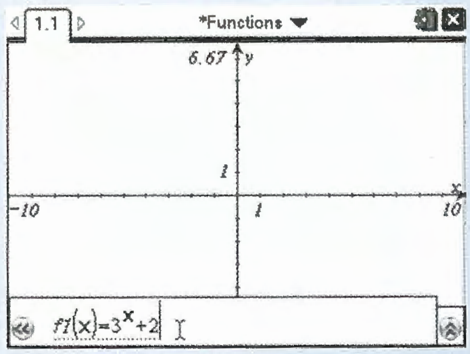
Exponential functions

1.9 Drawing an exponential graph

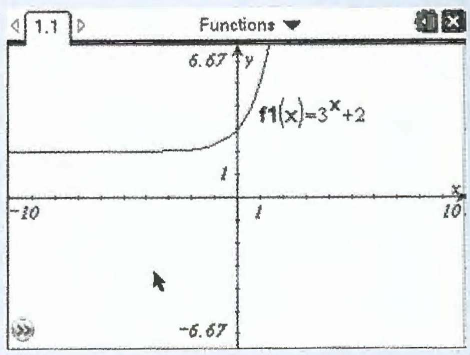
Example 10

Draw the graph of $y = 3^x + 2$

Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form '*f1(x)* =' is displayed.
The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.



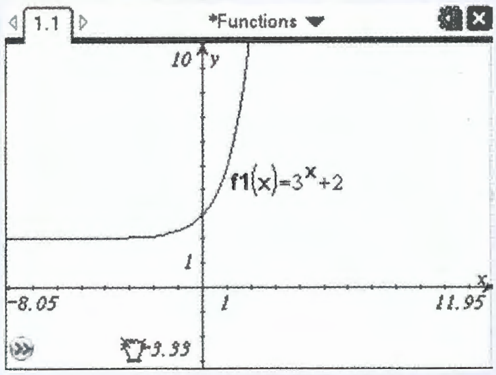
Type $y = 3^x + 2$ and press enter.
(**Note:** Type 3 ^ X ▶ to enter 3^x. The ▶ returns you to the baseline from the exponent.)
The GDC displays the curve with the default axes.



▶ Continued on next page

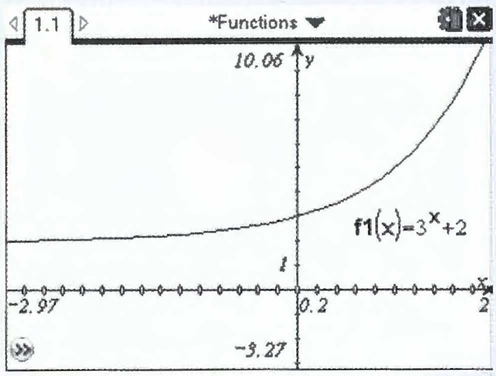
Pan the axes to get a better view of the curve.

For help with panning, see your GDC manual.



Grab the x-axis and change it to make the exponential curve fit the screen better.

For help with changing axes, see your GDC manual.

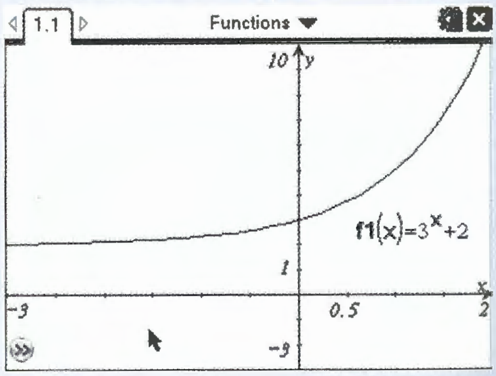


1.10 Finding a horizontal asymptote

Example 11

Find the horizontal asymptote to the graph of $y = 3^x + 2$

First draw the graph of $y = 3^x + 2$ (see Example 10).

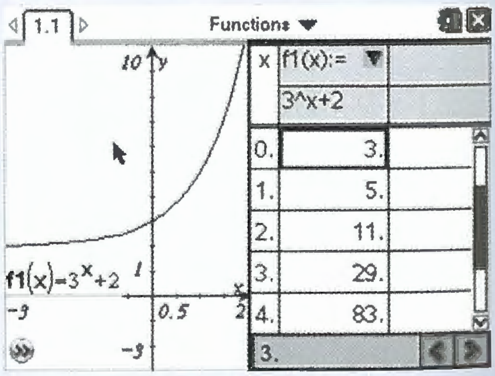


You can look at the graph **and** a table of the values by using a split screen.

~~Press menu 2:View | 9:Show Table~~ Menu 7: Table

(or simply press ctrl T) 1: Split Screen.

The values of the function are clearly decreasing as $x \to 0$.



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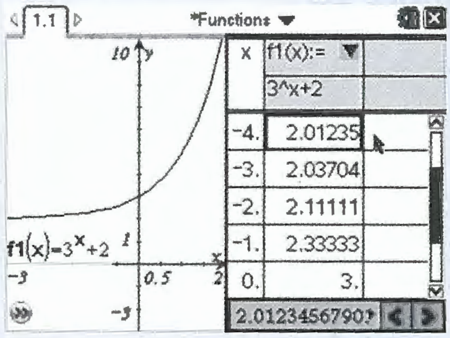
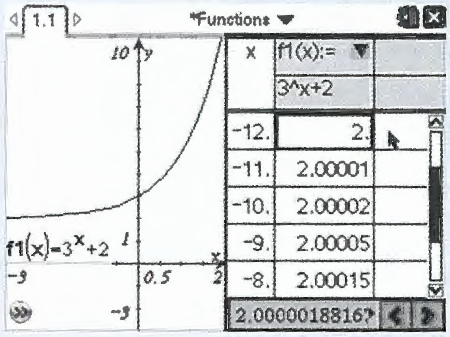
Press and hold ▲ to scroll up the table.
The table shows that as the values of x get smaller, $f1(x)$ approaches 2.



Eventually, the value of $f1(x)$ reaches 2. On closer inspection, you can see, at the bottom of the screen, that the actual value of $f1(x)$ is 2.000 001 881 6...
We can say that $f1(x) \to 2$ as $x \to -\infty$.
The line $x = 2$ is a horizontal asymptote to the curve $y = 3^x + 2$.



Logarithmic functions

1.11 Evaluating logarithms

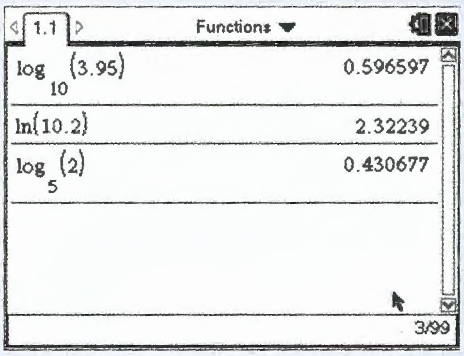
Example 12

Evaluate $\log_{10} 3.95$, $\ln 10.2$ and $\log_5 2$.

Open a new document and add a Calculator page.
Press ctrl log to open the log template.
Enter the base and the argument then press enter del

For natural logarithms it is possible to use the same method, with the base equal to e, but it is far less time consuming to press ctrl ln.

Note that the GDC will evaluate logarithms with any base without having to use the change of base formula.



1.12 Finding an inverse function

The inverse of a function can be found by interchanging the x and y values. Geometrically this can be done by reflecting points in the line $y = x$.

Example 13

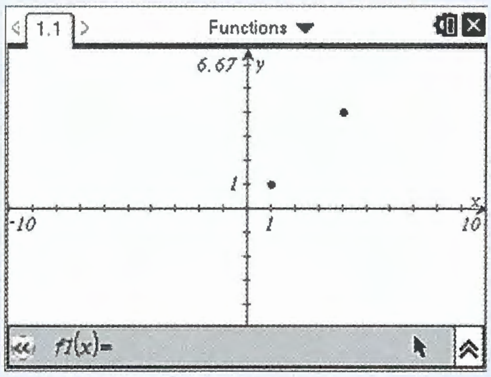
Show that the inverse of the function $y = 10^x$ is $y = \log_{10}x$ by reflecting $y = 10^x$ in the line $y = x$.

Open a new document and add a Graphs page.
First we will draw the line $y = x$. So that it can be recognised the axis of reflection, it has to be drawn and not plotted as a function. Menu 8 : Geometry

Press menu 7: Points & Lines | 1: Point

Then type (1 enter 1 enter then (4 enter 4 enter esc

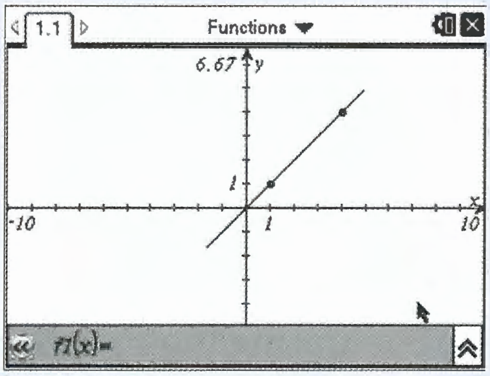
This will plot the points (1, 1) and (4, 4), which both lie on the line $y = x$



Press menu 7: Points & Lines | 4: Line

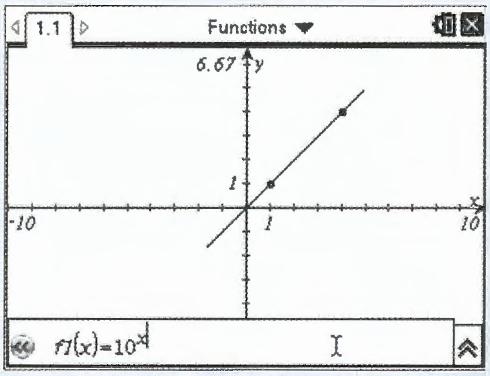
Select both the points you have plotted and draw a line through them.

Press esc to exit the drawing function.

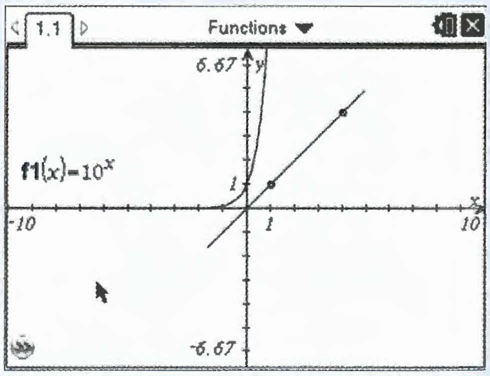


Click in the entry line at the bottom of the work area. The default graph type is Function, so the form "$f1(x)$=" is displayed.

Type 10^x and press enter.



The calculator displays the function with the default axes, $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.

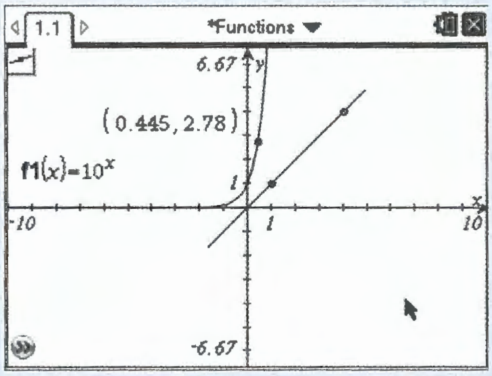


▶ Continued on next page

Press menu 7: Points & Lines | 2: Point On

Select the curve with the touchpad (you will see that it is highlighted when it is selected).

You can place a point anywhere on the curve.

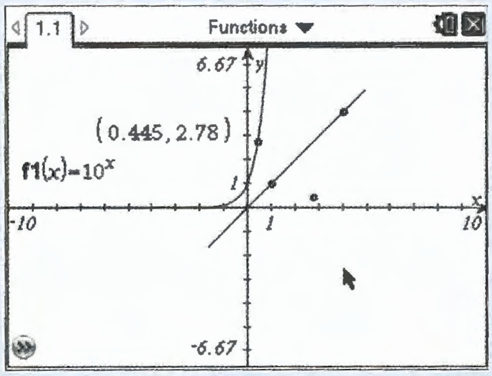


Menu 8 Geometry

Press menu 5: Transformation | 2: Reflection

Use the touch pad to select the point that you just placed on the curve and then the line $y = x$.

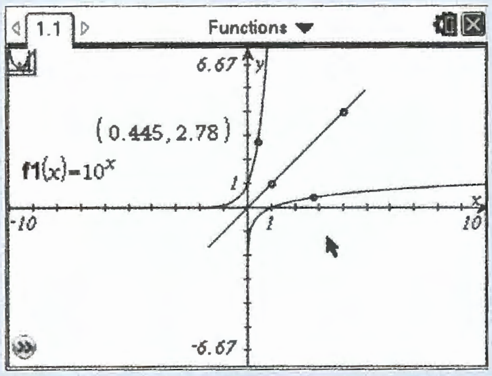
Press esc when you have finished. You should see the reflected image of the point in the line $y = x$.



Menu 8 Geometry

Press menu 4: Construction | 6: Locus

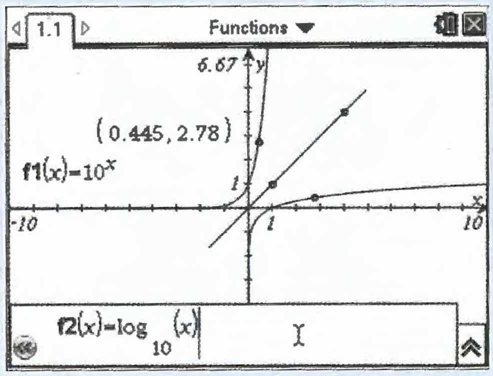
Use the touch pad to select each of the points. The calculator will display the locus of the reflection as the point moves along the curve.



Click in the entry line at the bottom of the work area. "$f2(x)$=" is displayed.

Type $\log_{10}(x)$ and press enter.

The reflected curve and the logarithmic function coincide, showing that $y = \log_{10} x$ is inverse of the function $y = 10^x$.



1.13 Drawing a logarithmic graph

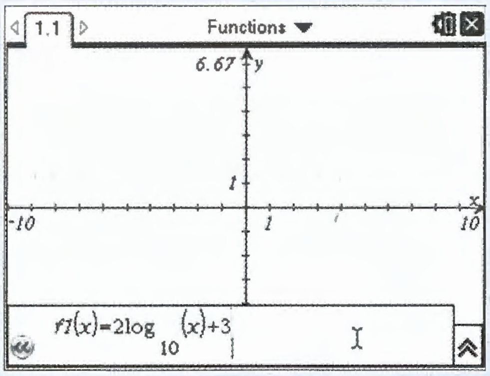
Example 14

Draw the graph of $y = 2\log_{10}x + 3$.

Open a new document and add a Graphs page.

The entry line is displayed at the bottom of the work area. The default graph type is Function, so the form "$f\,1(x)=$" is displayed.

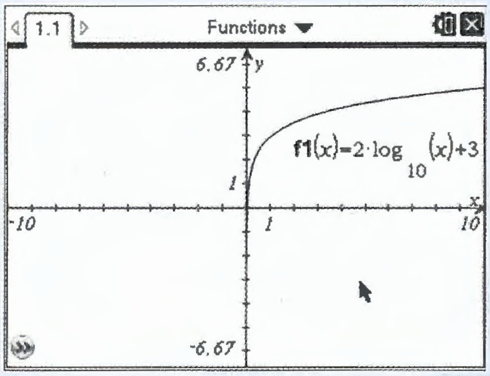
The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.



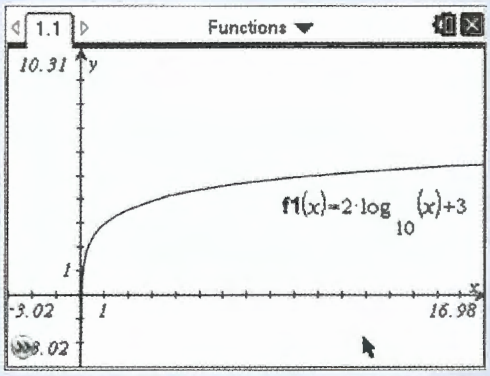
Type $2\log_{10}(x) + 3$ and press enter.

(**Note:** Type 2 ctrl log and enter 10 as the base of the logarithm. Enter x in the argument section of the template, use the ▶ to move beyond the brackets to enter +3)

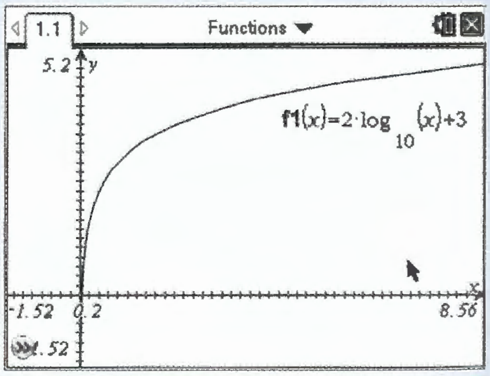
The calculator displays the curve with the default axes.



Pan the axes to get a better view of the curve.



Grab the x-axis and change it to make the logarithmic curve fit the screen better.

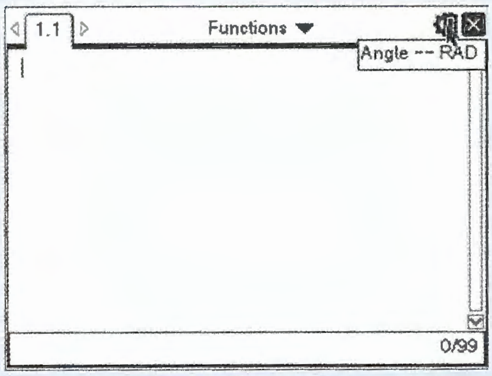
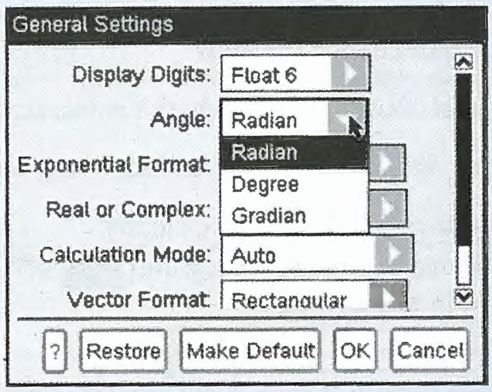
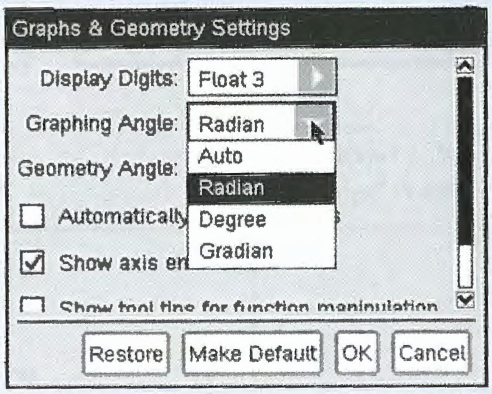


Trigonometric functions

1.14 Degrees and radians

Work in trigonometry will be carried out either in degrees or radians. It is important, therefore, to be able to check which mode the calculator is in and to be able to switch back and forth. On the TI-Nspire, there are three separate settings to make: general, graphing and geometry. The defaults for general and graphing are radians and for geometry the default is degrees. Geometry is only used for drawing plane geometrical figures. Normally the two important settings are general and graphing. General refers to the angle used in calculations and graphing is for drawing trigonometric graphs.

Example 15

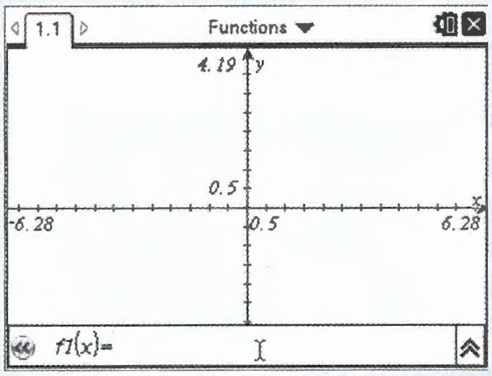
Change angle settings from radians to degrees and from degrees to radians.	
Open a new document and add a Calculator page. Move the cursor to the symbol at the top right hand side of the screen. It will display the *general* angle mode – either radians or degrees. Click in the symbol and choose 2:Settings | 1:General.	
In the dialogue box, select either degrees or radians and then click on OK.	
To change the setting for graphing, click in the symbol and choose 2:Settings | 2:Graphs & Geometry. In the dialogue box, select either degrees or radians for the Graphing Angle and then click on OK.	

1.15 Drawing trigonometric graphs

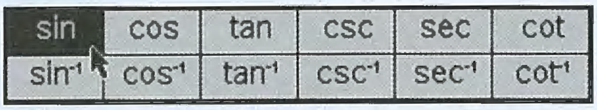
Example 16

Draw the graph of $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$.

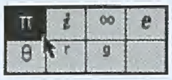
Open a new document and add a Graphs page.
Press menu 4:Window / Zoom | 8:Zoom - Trig
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form "*f1*(*x*)=" is displayed.
The default axes are $-6.28 \le x \le 6.28$ and $-4.19 \le y \le 4.19$.
These are the basic axes for graphing trigonometric graphs with x between -2π and 2π. If the calculator is in degree mode, the x-axis will be between −360 and 360.

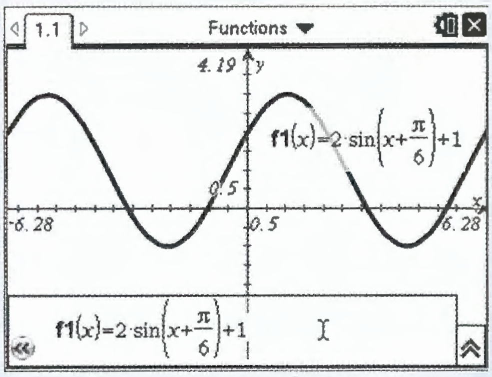


Type $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$ and press enter.
To enter sin, press and choose sin from the dialogue box.



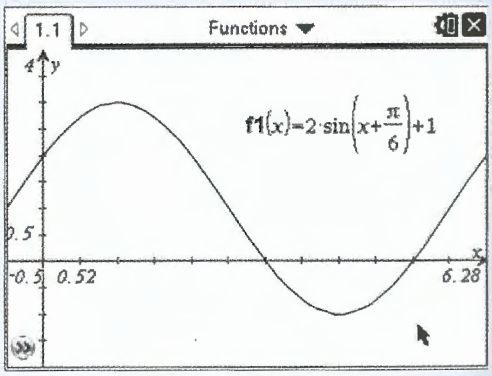
To enter π, press and choose π from the dialogue box.





Pan the axes to get a better view of the curve and grab them to change the view.

It is also useful to change the x-axis scale to a multiple of π, such as $\frac{\pi}{6}$ as this will often show the positions of intercepts and turning points more clearly.
Change the scale by pressing menu 4:Window / Zoom | 1:Window Settings
*X*Scale: pi/6
Type pi/6 in the dialogue box for *X*Scale.



More complicated functions

1.16 Solving a combined quadratic and exponential equation

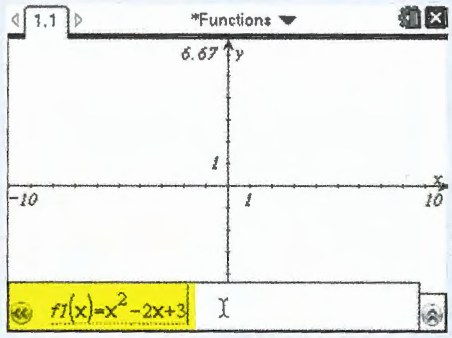
Follow the same GDC procedure when solving simultaneous equations graphically or solving a combined quadratic and exponential equation. See Examples 4 and 17.

Example 17

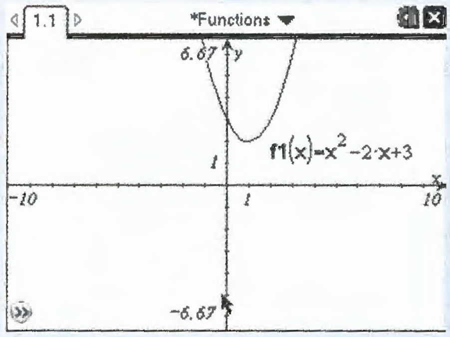
Solve the equation $x^2 - 2x + 3 = 3.2^{-x} + 4$

To solve the equation, find the point of intersection of the quadratic function $f1(x) = x^2 - 2x + 3$ with the exponential function $f2(x) = 3.2^{-x} + 4$.

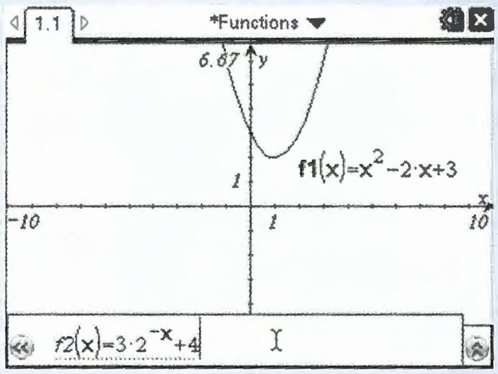
To draw the graphs $f1(x) = x^2 - 2x + 3$ and $f2(x) = 3.2^{-x} + 4$:
Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form '$f1(x)$=' is displayed.
The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.



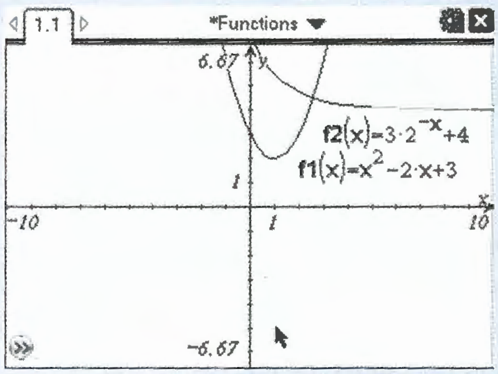
Type $x^2 - 2x + 3$ and press enter.
The GDC displays the first curve:
$f1(x) = x^2 - 2x + 3$



Use the touchpad to click on the arrows in the bottom left-hand corner of the screen.
This will open the entry line again. This time '$f2(x)$=' is displayed.
Type $3.2^{-x} + 4$ and press enter.



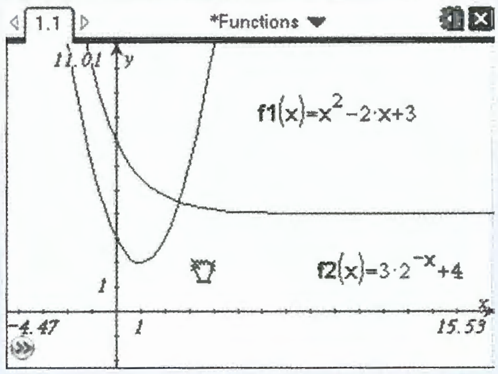
The GDC displays both curves:
$f1(x) = x^2 - 2x + 3$
$f2(x) = 3.2^{-x} + 4$



Continued on next page

Pan the axes to get a better view of the curves.

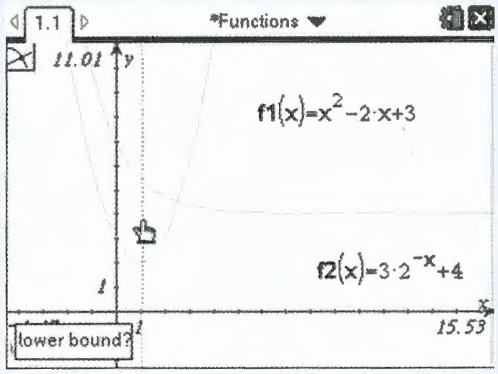
For help with panning, see your GDC manual.



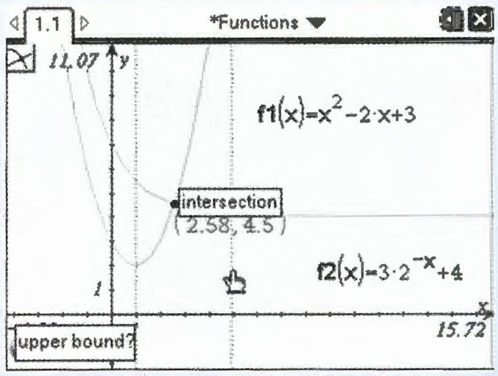
Press menu 6:Analyze Graph | 4:Intersection Point(s)

Press enter

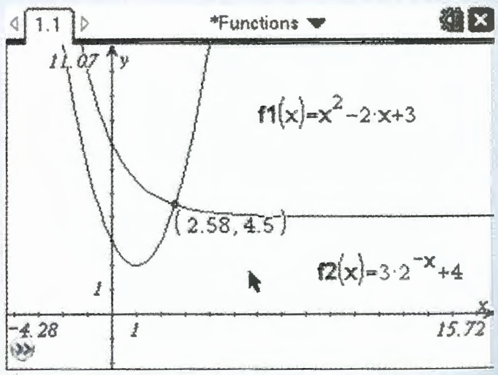
To find the intersection you need to give the lower and upper bounds of a region that includes the intersection.
The GDC shows a line and asks you to set the lower bound.
Move the line using the touchpad and choose a position to the left of the intersection.
Click the touchpad.



The GDC shows another line and asks you to set the upper bound.
Use the touchpad to move the line so that the region between the upper and lower bounds contains the intersection.
When the region contains the intersection, the calculator will display the word 'intersection' in a box.
Click the touchpad.



The GDC displays the intersection of the two curves at the point (2.58, 4.5).
The solution is $x = 2.58$.



Modeling

1.17 Using sinusoidal regression

Note: the notation $\sin^2 x$, $\cos^2 x$, $\tan^2 x$, … is a mathematical convention that has little algebraic meaning. To enter these functions on the GDC, you *should* enter $(\sin(x))^2$, etc. However, the calculator will conveniently interpret $\sin(x)^2$ and translate it as $(\sin(x))^2$.

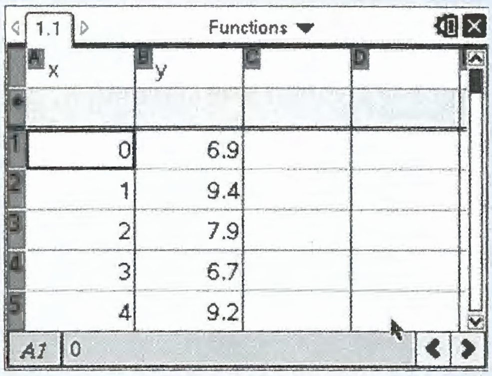
Example 18

It is known that the following data can be modeled using a sine curve.

x	0	1	2	3	4	5	6	7
y	6.9	9.4	7.9	6.7	9.2	8.3	6.5	8.9

Use sine regression to find a function to model this data.

Open a new document and add a Lists & Spreadsheet page.
Type '*x*' in the first cell and '*y*' in the cell to its right.
Type the numbers from the *x*-list in the first column and those from the *y*-list in the second.
Use the ▼ ▲ ◀ ▶ keys to navigate around the spreadsheet.

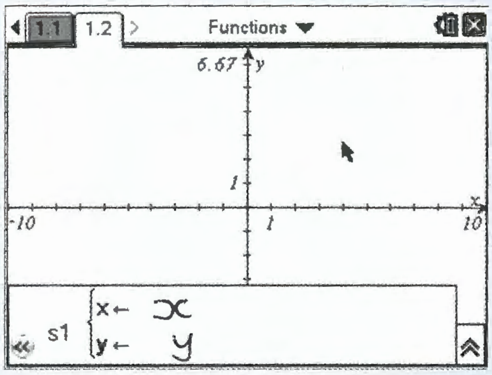


Press On and add a new graphs page to your document.

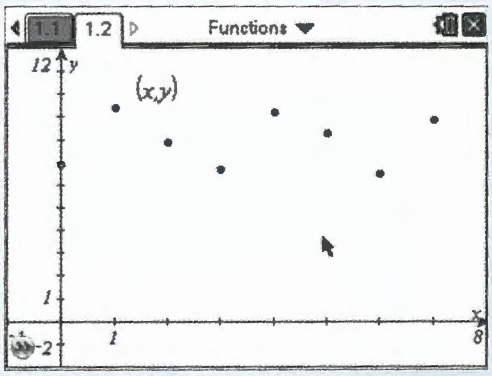
Press menu 3:Graph Type | 4:Scatter Plot

Press enter

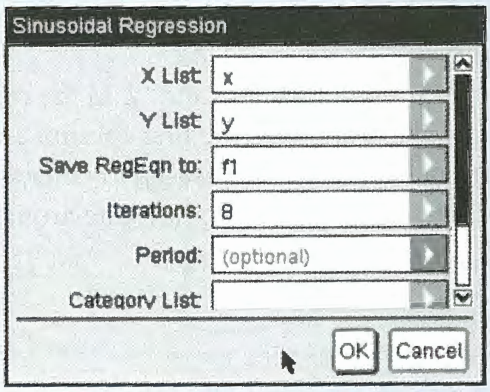
The entry line is displayed at the bottom of the work area.
Scatter plot type is displayed.
Enter the names of the lists, *x* and *y*, into the scatter plot function
Use the tab key to move from *x* to *y*.
Press enter del



Adjust your window settings to show your data and the *x*- and *y*-axes.
You now have a scatter plot of *x* against *y*.



Press ctrl ◀ to return to the Lists & Spreadsheet page.
Select an empty cell and press menu 4:Statistics | Stat Calculations | C:Sinusoidal Regression...
Press enter
From the drop down menus choose '*x*' for X List and '*y*' for Y List. You should press tab to move between the fields.
Press enter



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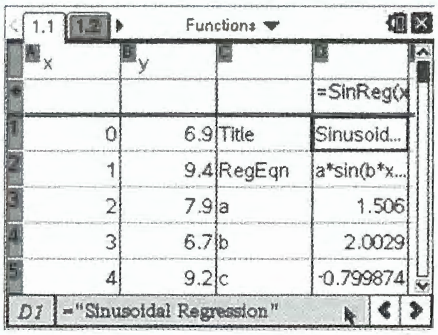
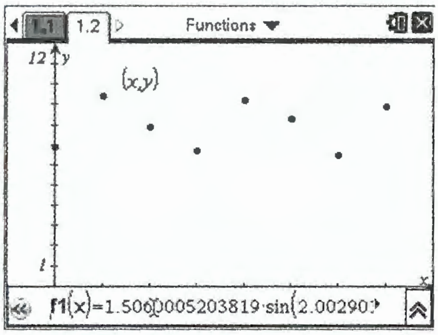
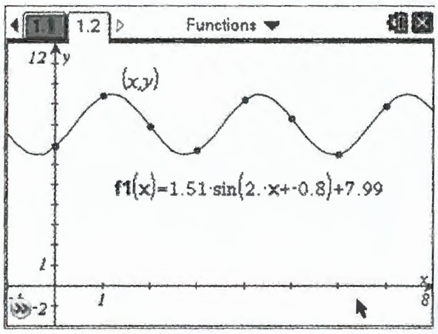
On screen, you will see the result of the sinusoidal regression in lists next to the lists for x and y.
The equation is in the form $y = a\sin(bx + c) + d$ and you will see the values of a, b, c and d displayed separately.
The equation of the sinusoidal regression line is
$y = 1.51\sin(2.00x - 0.80) + 7.99$



Press ctrl ▶ to return to the Graphs page.
Using the touchpad, click on » to open the entry line at the bottom of the work area.
You will see that the equation of the regression line has been pasted into $f1(x)$.
Press enter



The regression line is now shown on the graph.



1.18 Using transformations to model a quadratic function

You can also model a linear function by finding the equation of the least squares regression line (see section 5.15).

Example 19

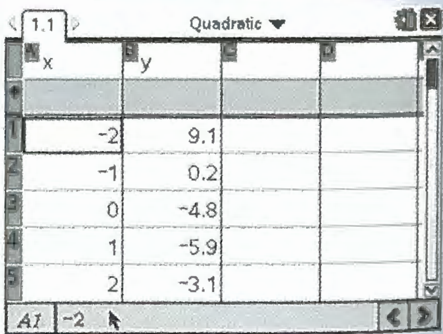
This data is approximately connected by a quadratic function.

x	−2	−1	0	1	2	3	4
y	9.1	0.2	−4.8	−5.9	−3.1	4.0	15.0

Find a function that fits the data.

Transform a basic quadratic curve to find an equation to fit some quadratic data.

Open a new document and add a Lists & Spreadsheet page.
Enter the data in two lists:
Type 'x' in the first cell and 'y' in the cell to its right.
Enter the x-values in the first column and the y-values in the second. Remember to use (-) to enter a negative number.
Use the ▼ ▲ ◀ ▶ keys to navigate around the spreadsheet.



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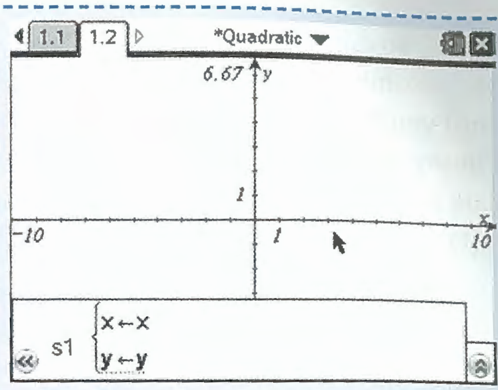
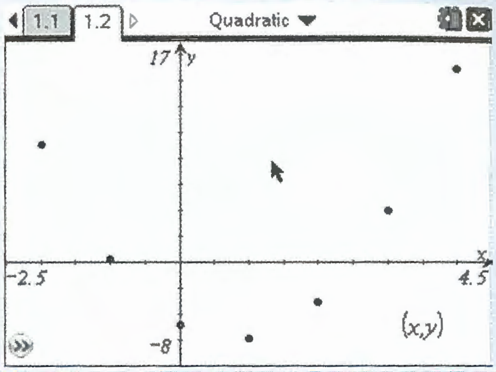
For equation: Menu :
4 : Stats
1 : Stats Calc .

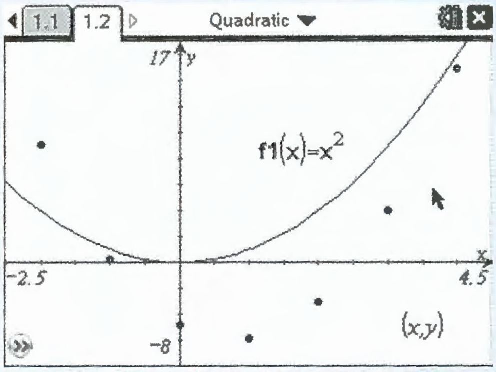
Add a Graphs page to your document.
Press menu 3:Graph Type | 4:Scatter Plot
Press enter
The entry line is displayed at the bottom of the work area.
Scatter plot type is displayed.
Enter the names of the lists, x and y, into the scatter plot function.
Use the tab key to move from x to y.
Press enter



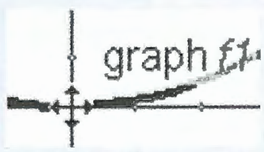
Press menu A:Zoom – Fit from the Window/Zoom menu
This is a quick way to choose an appropriate scale to show all the points.
You should recognize that the points are in the shape of a quadratic function.



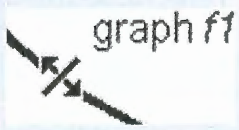
The next step is to enter a basic quadratic function, $y = x^2$, and manipulate it to fit the points.
Press menu 3:Graph Type | 1:Function
Press enter
This changes the graph type from scatter plot to function.
Type x^2 in as function $f1(x)$.
It is clear that the curve does not fit any of the points, but it is the right general shape to do so.



Use the touchpad to move the cursor so it approaches the curve. You will see one of two icons.
The first will allow you to drag the quadratic function around the screen by its vertex.

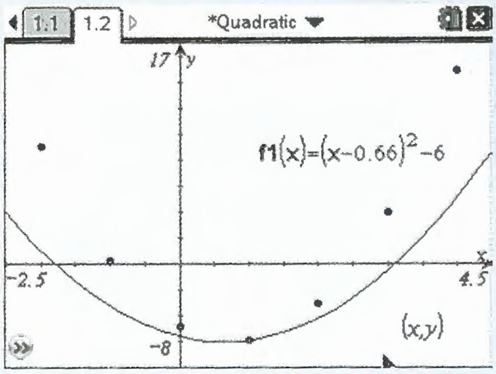


or



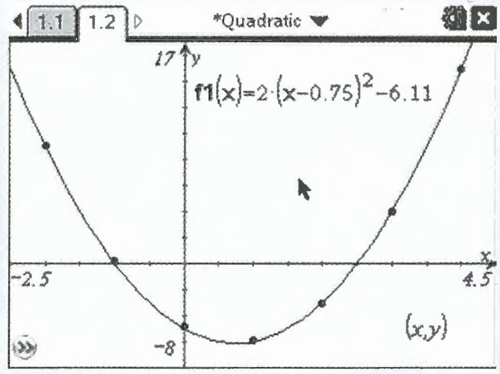
The second allows you to stretch the function either vertically or horizontally.

Use ✥ to position the vertex where you think it ought to be according to the data points.



▶ Continued on next page

Use to adjust the stretch of the curve.
Make some final fine adjustments using both the tools until you have a good fit to the data points.
The equation of the function that fits the data is:
$f1(x) = 2(x - 0.75)^2 - 6.11$



1.19 Using sliders to model an exponential function

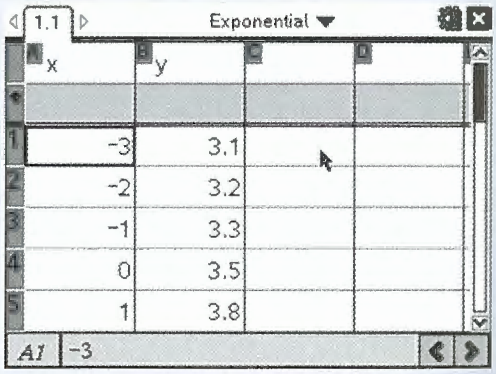
Example 20

In general, an exponential function has the form $y = ka^x + c$.
For this data, it is known that the value of a is 1.5, so $y = k(1.5)^x + c$.

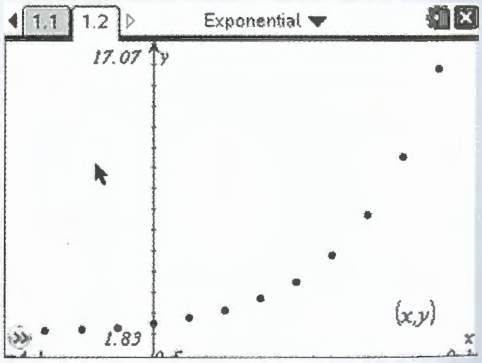
x	−3	−2	−1	0	1	2	3	4	5	6	7	8
y	3.1	3.2	3.3	3.5	3.8	4.1	4.7	5.5	6.8	8.7	11.5	15.8

Find the values of the constants k and c.

Open a new document and add a Lists & Spreadsheet page.
Enter the data in two lists:
Type 'x' in the first cell and 'y' in the cell to its right.
Enter the x-values in the first column and the y-values in the second. Remember to use (−) to enter a negative number.
Use the ▼ ▲ ◀ ▶ keys to navigate around the spreadsheet.



Add a Graphs page to your document.
Press menu 3:Graph Type | 4:Scatter Plot
Press enter
The entry line is displayed at the bottom of the work area.
Scatter plot type is displayed.
Enter the names of the lists, x and y, into the scatter plot function.
Use the tab key to move from x to y.
Press enter



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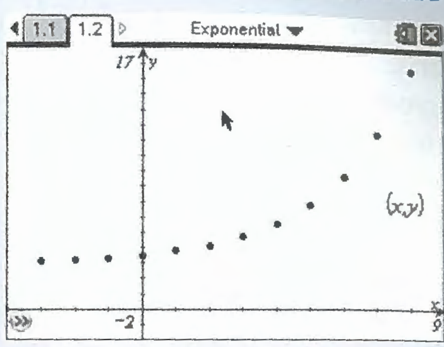
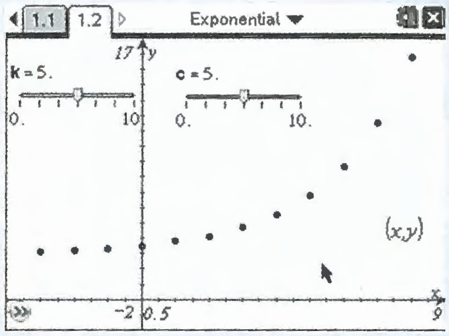
Adjust the window settings to fit the data and to display the axes clearly.

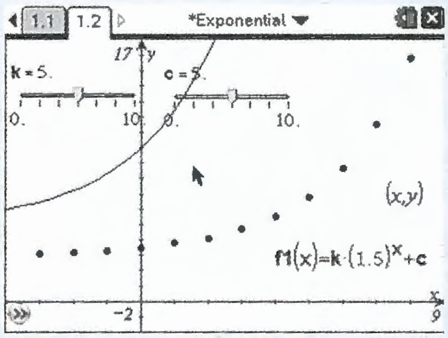


Press menu 1:Actions | 8:Insert Slider
Position the slider somewhere where it is not in the way and change the name of the constant to *k*.
Repeat and add a second slider for *c*.

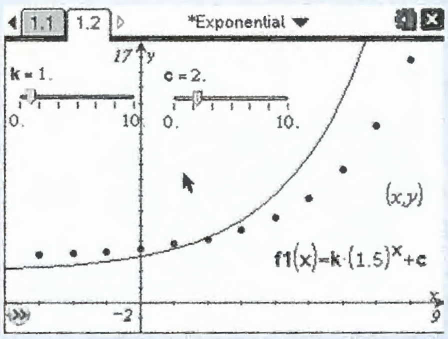
For help with sliders, see your GDC manual.



Press menu 3:Graph Type | 1:Function
Press enter
This changes the graph type from scatter plot to function.
Type $k.(1.5)^x + c$ in as function $f1(x)$.



Try adjusting the sliders.
You can get the curve closer to the points but they are not sufficiently adjustable to get a good fit.

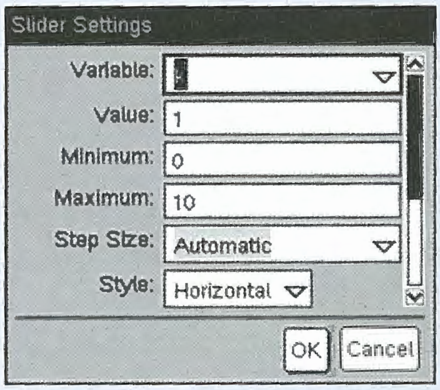


You can change the slider settings by selecting the slider, pressing ctrl menu and selecting 1:Settings.
Change the default values for *k* to:

Minimum 0
Maximum 2
Step Size 0.1

Change the default values for *c* to:

Minimum 0
Maximum 4
Step Size 0.1



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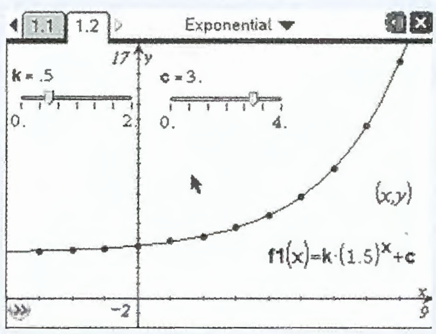
You can now adjust the sliders to get a much better fit to the curve.
The screen shows the value of k is 0.5 and c is 3.
So the best fit for the equation of the function is approximately $y = 0.5(1.5)^x + 3$.



2 Differential calculus

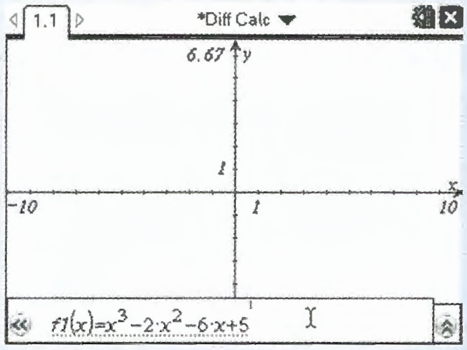
Finding gradients, tangents and maximum and minimum points

2.1 Finding the gradient at a point

Example 21

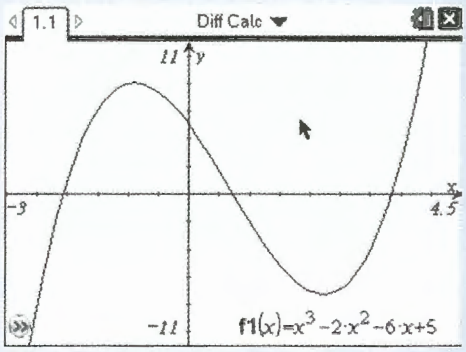
Find the gradient of the cubic function $y = x^3 - 2x^2 - 6x + 5$

Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form '$f1(x)$=' is displayed.
The default axes are $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.
Type $x^3 - 2x^2 - 6x + 5$ and press enter.
(**Note:** Type × ^ 3 ▶ to enter x^3. The ▶ returns you to the baseline from the exponent.)

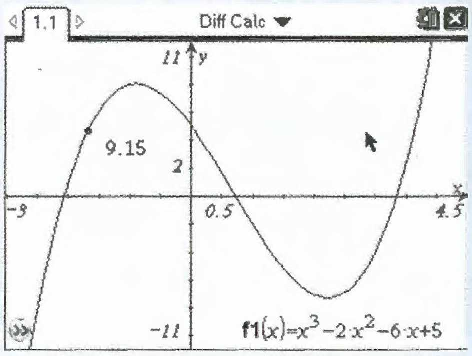


Pan the axes to get a better view of the curve and then grab the x- and y-axes to fit the curve to the window.

For help with panning and changing axes, see your GDC manual.



Press menu 6:Analyze Graph | 5: $\frac{dy}{dx}$
Press enter
Using the touchpad, move the ☝ towards the curve. As it approaches the curve, it turns to ✎ and displays the numerical value of the gradient.
Press enter to attach a point on the curve.



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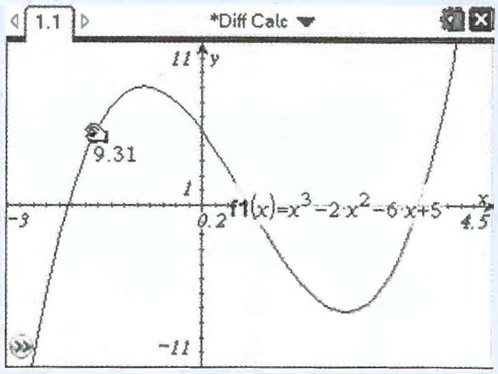
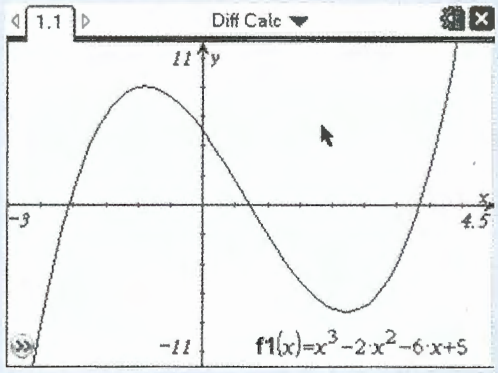
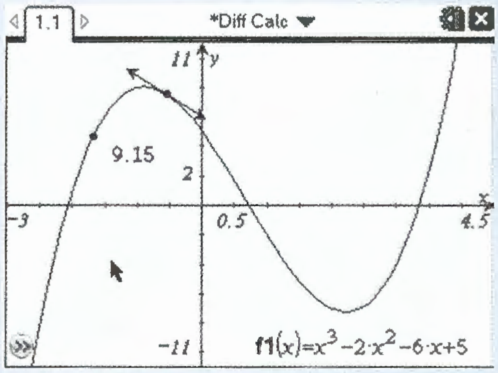
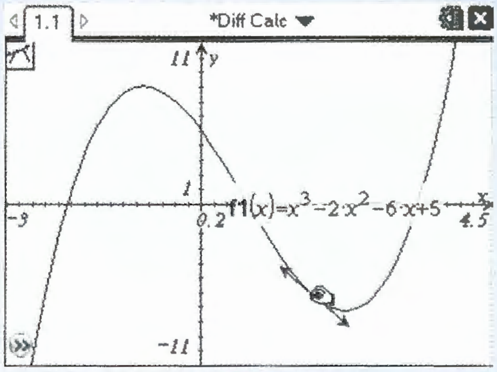
Use the touchpad to move the icon to the point. You can move the point along the curve and observe how the gradient changes as the point moves. Here, gradient at point = 9.31.	

2.2 Drawing a tangent to a curve

Example 22

Draw a tangent to the curve $y = x^3 - 2x^2 - 6x + 5$	
First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 21).	
Press menu 8:Points & Lines | 7:Tangent Press enter Using the touchpad, move the towards the curve. As it approaches the curve, it turns to . Press enter The cursor changes to and displays 'point on'. Choose a point where you want to draw a tangent and press enter.	
You can move the point that the tangent line is attached to with the touchpad.	

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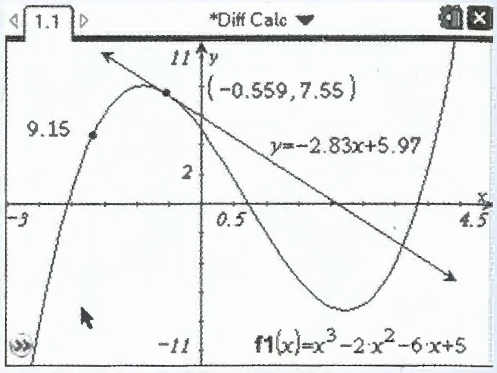
Use the touchpad to drag the arrows at each end of the tangent line to extend it.
Press ctrl menu with the tangent line selected – move to the arrow at the end and look for the word 'line'.
Choose 7:Coordinates and Equations
Click on the line to display the equation of the tangent: $y = -2.83x + 5.97$.
Click on the point to display the coordinates of the point: $(-0.559, 7.55)$.

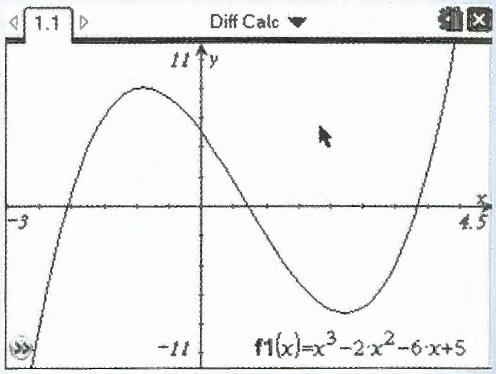


2.3 Finding maximum and minimum points

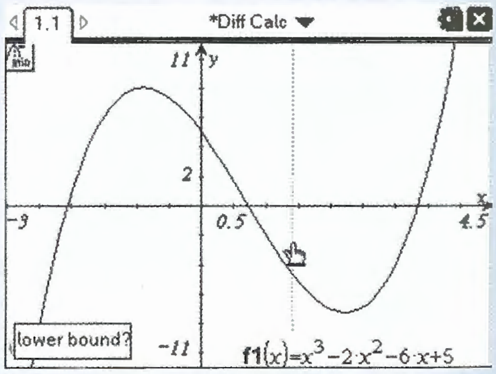
Example 23

Find the local maximum and local minimum points on the cubic curve:
$y = x^3 - 2x^2 - 6x + 5$

First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 21).



Press menu 6:Analyze Graph | 2:Minimum
Press enter
To find the minimum you need to give the lower and upper bounds of a region that includes the minimum.
The GDC shows a line and asks you to set the lower bound.
Move the line using the touchpad and choose a position to the left of the minimum.
Click the touchpad.



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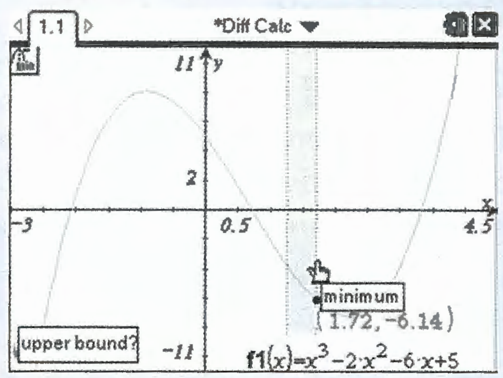
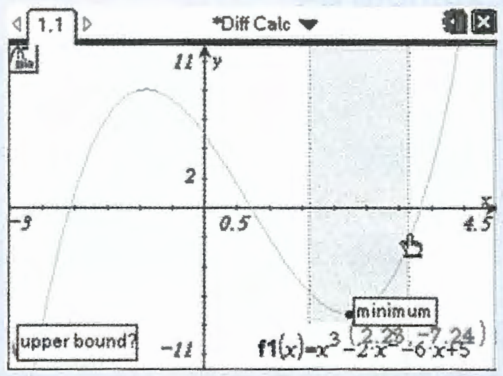
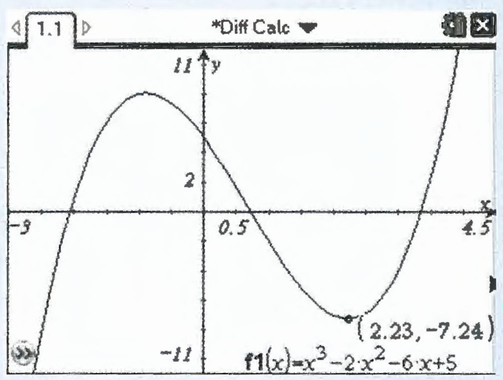
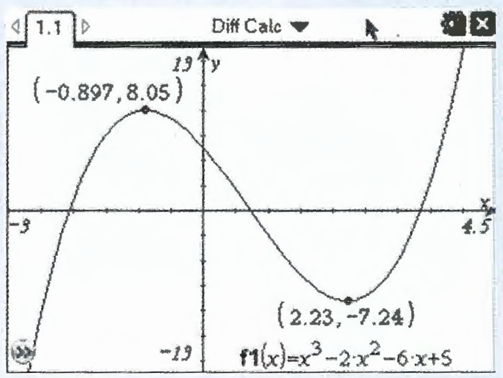
The GDC shows another line and asks you to set the upper bound. Use the touchpad to move the line so that the region between the upper and lower bounds contains the minimum. **Note:** The minimum point in the region that you have defined is being shown. In this screenshot it is not the local minimum point. Make sure you move the line beyond the point you are looking for.	
When the region contains the minimum, the GDC will display the word 'minimum' in a box and a point that lies between the lower and upper bounds. The point displayed is clearly between the upper and lower bounds. Click the touchpad.	
The GDC displays the local minimum at the point (2.23, −7.24).	
Press menu 6:Analyze Graph | 3:Maximum to find the local maximum point on the curve in exactly the same way. The maximum point is (−0.897, 8.05).	

Derivatives

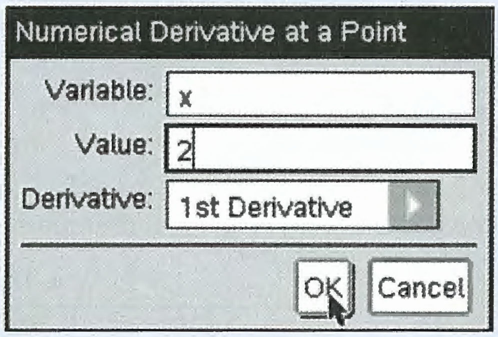
2.4 Finding a numerical derivative

Using the calculator it is possible to find the numerical value of any derivative for any value of x. The calculator will not, however, differentiate a function algebraically. This is equivalent to finding the gradient at a point graphically (see Section 2.1 example 21).

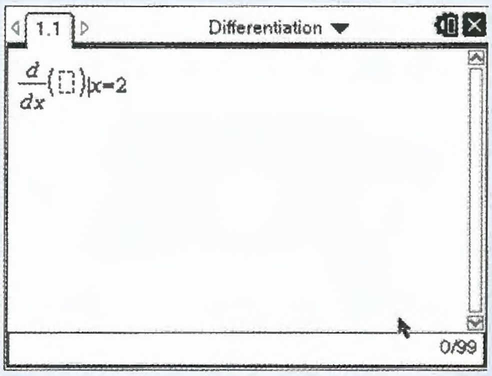
Example 24

If $y = \frac{x+3}{x}$, evaluate $\frac{dy}{dx}\Big|_{x=2}$

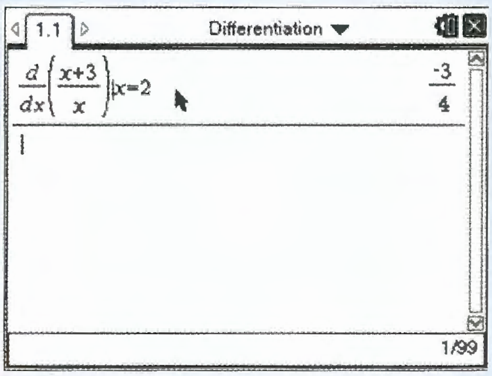
Open a new document and add a Calculator page.
Press menu 4:Calculus | 1:Numerical Derivative at a Point...
Leave the variable as x and the Derivative as 1st Derivative.
Change the Value to the value of x at which you wish to evaluate the derivative, in this case $x = 2$.



Enter the function in the template.
Press enter



The calculator shows that the value of the first derivative of $y = \left(\frac{x+3}{x}\right)$ is $-\frac{3}{4}$ when $x = 2$.

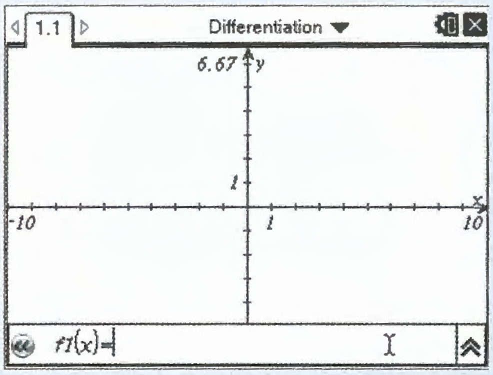
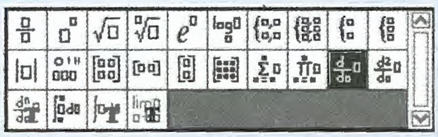
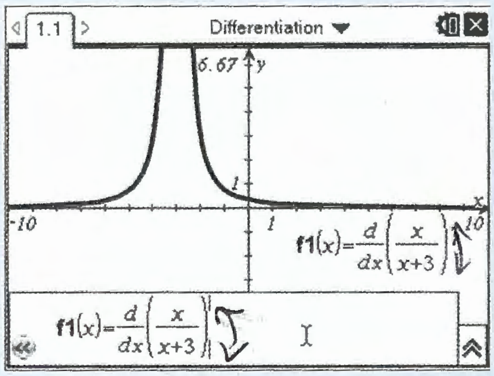


2.5 Graphing a numerical derivative

Although the calculator can only evaluate a numerical derivative at a point, it will graph the gradient function for all values of x.

Example 25

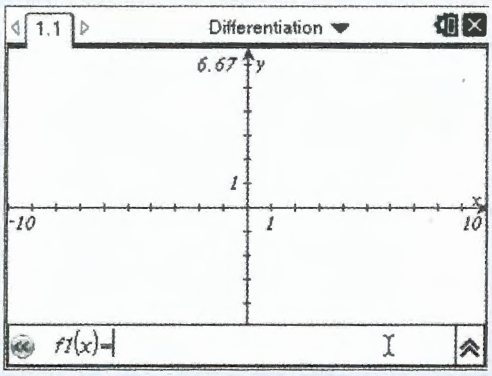
If $y = \dfrac{x+3}{x}$, draw the graph of $\dfrac{dy}{dx}$.

Open a new document and add a Graph page. The entry line is displayed at the bottom of the work area. The default graph type is Function, so the form "$f1(x)$=" is displayed. The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.	
Press the templates button marked (templates button) and choose the numerical derivative.	
In the template enter x and the function $\dfrac{x+3}{x}$. or f1(x) Press enter	
The calculator displays the graph of the numerical derivative function of $y = \dfrac{x+3}{x}$.	

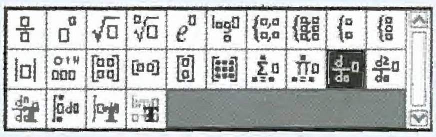
Example 26

Find the values of x on the curve $y = \frac{x^3}{3} + x^2 - 5x + 1$ where the gradient is 3.

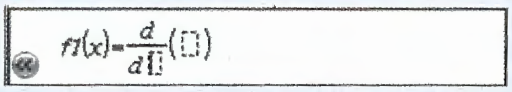
Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form "$f1(x) =$" is displayed.
The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.



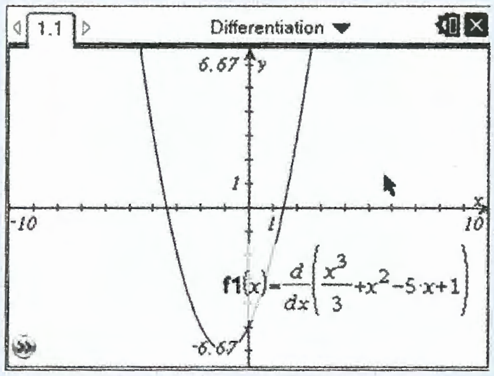
Press the templates button marked [templates button] and choose the numerical derivative.



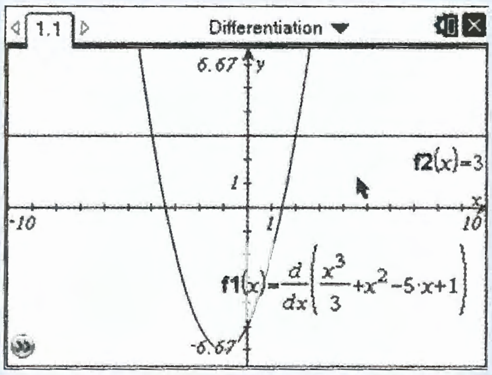
In the template enter x and the function $\frac{x^3}{3} + x^2 - 5x + 1$.
Press [enter].



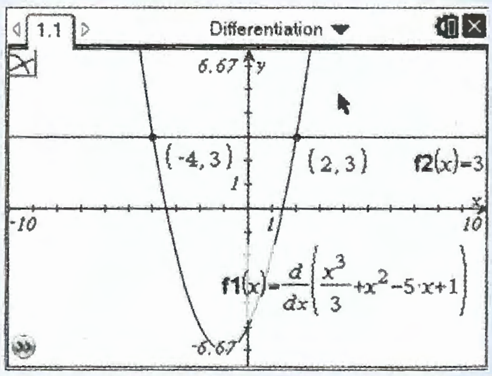
The calculator displays the graph of the numerical derivative function of $y = \frac{x^3}{3} + x^2 - 5x + 1$.



Using the touchpad, click on [button] to open the entry line at the bottom of the work area.
Enter the function $f2(x) = 3$
Press [enter]
The calculator now displays the curve and the line $y = 3$.



Press [menu] 7:Points & Lines | 3:Intersection Point(s)
Using the touchpad, select graph $f1$ and graph $f2$.
The calculator displays the coordinates of the intersection points of the gradient function and the line $y = 3$.
The curve has gradient 3 when $x = -4$ and $x = 2$



2.6 Using the second derivative

The calculator can find first and second derivatives. The second derivative can be used to determine whether a point is a maximum or minimum point.

Example 27

Find the stationary points on the curve $f(x) = x^4 - 4x^3$ and determine their nature.

$f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

At stationary points

$f'(x) = 0$

$4x^3 - 12x^2 = 0$

$4x^2 - (x - 3) = 0$

Therefore $x = 0$ or $x = 3$

Use the calculator to find the coordinates of the points and to determine their nature.

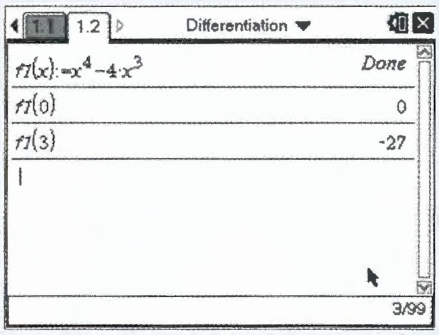
Open a new document and add a Calculator page.

Define the function $f1(x)$

Type F 1 (X) ctrl := and type the function.

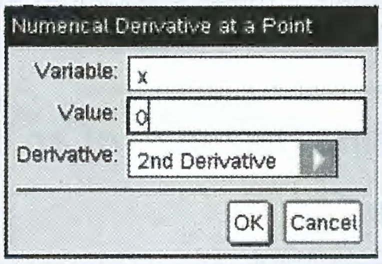
Evaluate the function when $x = 0$ and $x = 3$

The stationary points are at (0, 0) and (3, –27)



Press menu 4:Calculus | 1:Numerical Derivative at a Point...

Leave the variable as x and choose 2nd Derivative. Change the Value to the value of x at which you wish to evaluate the derivative, in this case $x = 0$ (and $x = 3$).

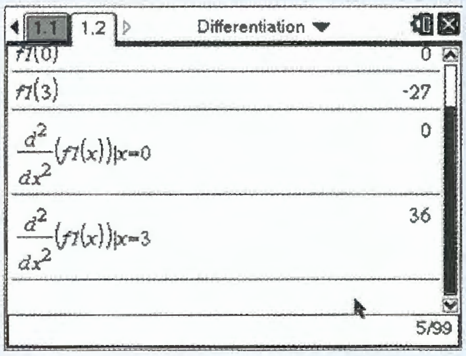


Enter $f1(x)$ in the template as the function.

Repeat for the second derivative when $x = 3$

(Note: you can cut and past the expression and change the 0 to 3)

In this case we are not certain what the nature of the stationary point is at (0, 0) but the point (3, –27) is a minimum because $f''(x) > 0$



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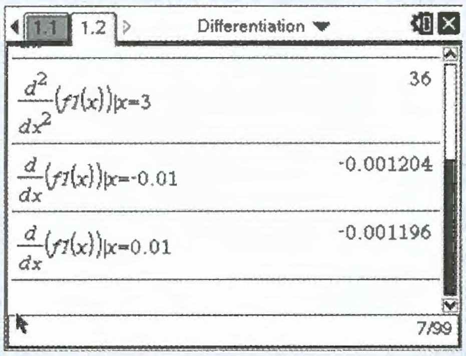
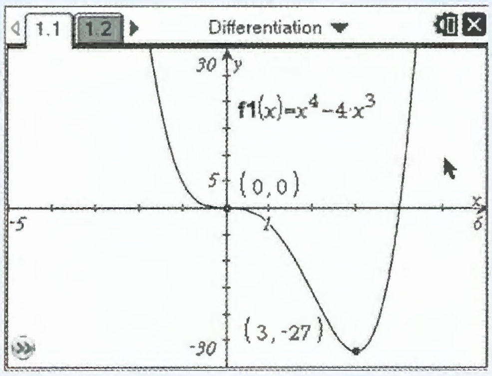
Evaluate $f'(x)$ either side of $x = 0$.

In this case using $x = -0.01$ and $x = 0.01$

The gradient is negative either side of the stationary point. Hence (0, 0) is a negative point of inflexion.



The graph on the right illustrates the curve, the minimum at (3, −27) and the point of inflexion at (0, 0).



3 Integral calculus

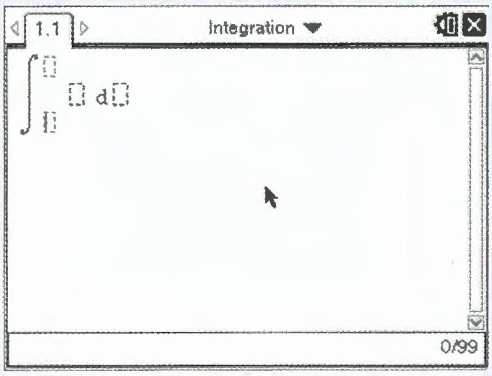
The calculator can find the values of definite integrals either on a calculator page or graphically. The calculator method is quicker, but the graphical method is clearer and shows discontinuities, negative areas and other anomalies that can arise.

3.1 Finding the value of an indefinite integral

Example 28

Evaluate $\int \left(x - \frac{3}{\sqrt{x}} \right) dx$

Open a new document and add a Calculator page.
Press menu 4:Calculus | 1:Numerical Integral...
Enter the upper and lower limits, the function and x in the template.
Use the ▼▲◀▶ keys to navigate around the template.
In this example you will also use templates to enter the rational function and the square root.



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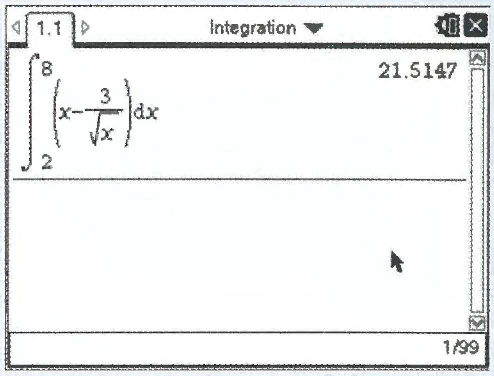
The value of the integral is 21.5 (to 3 sf)



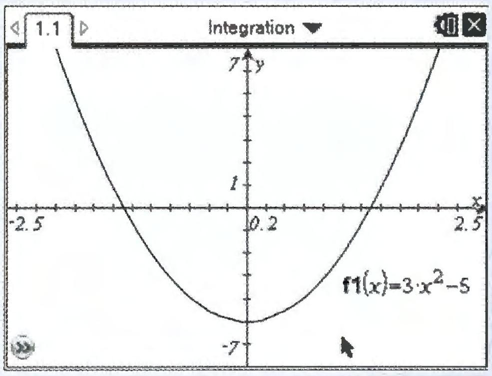
3.2 Finding the area under a curve

Example 29

Find the area bounded by the curve $y = 3x^2 - 5$, the x-axis and the lines $x = -1$ and $x = 1$.

Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form "$f1(x)$=" is displayed.

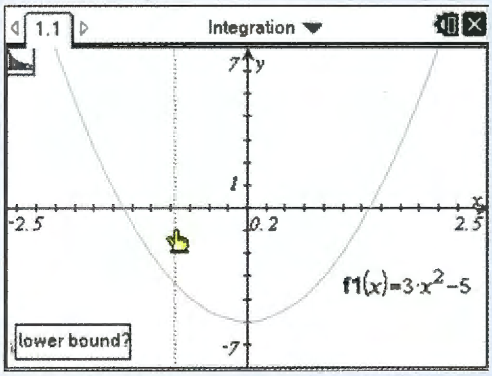
The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.
Type the function $3x^2 - 5$
Press enter



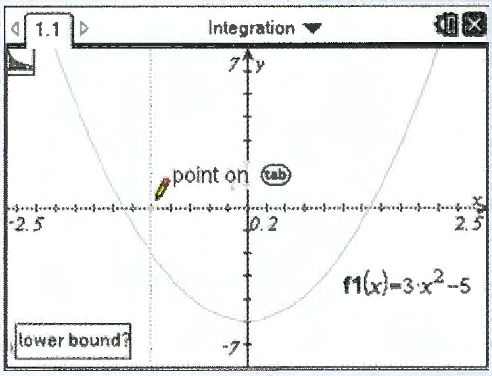
Press menu 6:Analyze Graph | 6:Integral

The calculator prompts you to enter the lower limit for the integral. There are several ways to do this.

You can click manually. This is not very accurate, however, and you will need to add the coordinates of the point you entered and edit them to obtain an accurate figure.



You can use the points on the axis.
Here the scale was set to 0.2, so the point (–1, 0) can be selected as shown.



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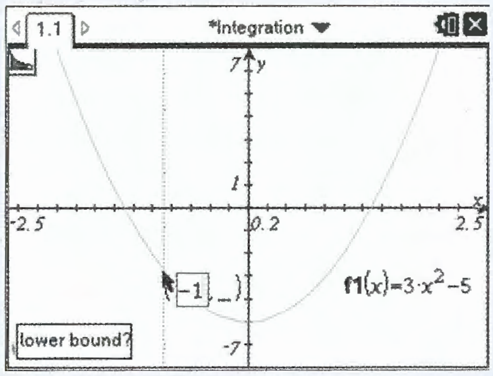
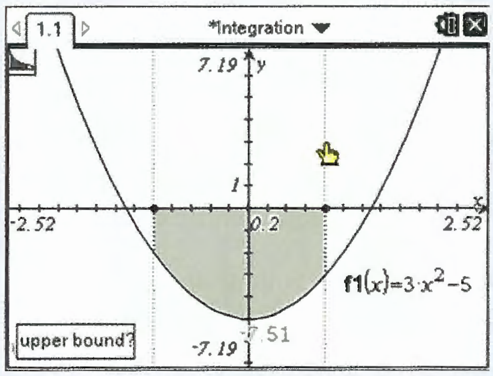
You can enter the point with the keyboard.
Enter a left bracket (and then type (−) 1 and press enter
There is no need to complete the coordinates.



Repeat for the upper limit.

The calculator displays a changing value for the area.

Using one of the methods above, select a point where the value of x is 1.

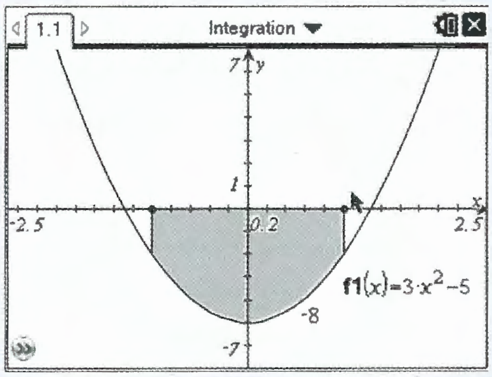


Repeat for the upper limit.

The area found is shaded and the value of the integral (−8) is shown on the screen.

Note: since the area lies below the x-axis in this case, the integral is negative.

The required area is 8.



4 Vectors

4.1 Calculating a scalar product

Example 30

Evaluate the scalar products:

a $\begin{pmatrix}1\\3\end{pmatrix}\cdot\begin{pmatrix}-3\\4\end{pmatrix}$

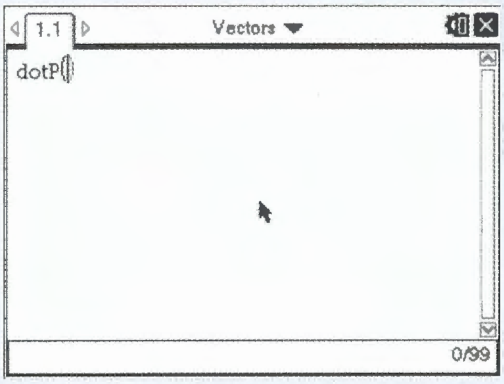
b $\begin{pmatrix}1\\-1\\4\end{pmatrix}\cdot\begin{pmatrix}3\\2\\-1\end{pmatrix}$

▶ Continued on next page

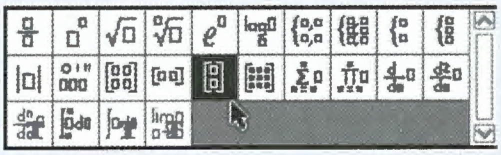
a Open a new document and add a Calculator page.

Press b 7: Matrix & Vector | C: Vector | 3: Dot Product

(or type DOTP()).



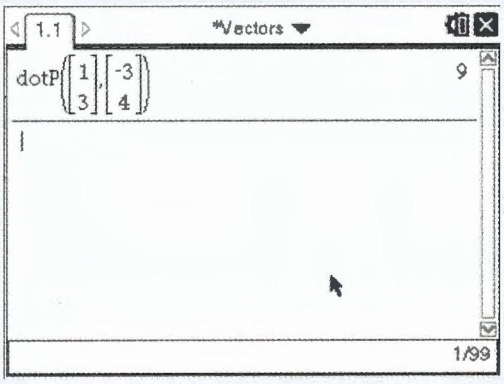
Press t and choose the 2 × 1 column vector template.



Enter the vector type , and enter the second vector.

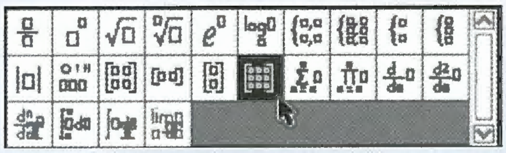
Press

$$\begin{pmatrix}1\\3\end{pmatrix}\cdot\begin{pmatrix}-3\\4\end{pmatrix}=9$$

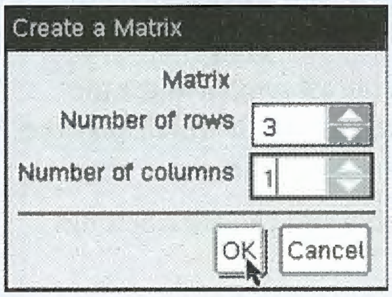


b Press b 7: Matrix & Vector | C: Vector | 3: Dot Product

Press t and choose the matrix template



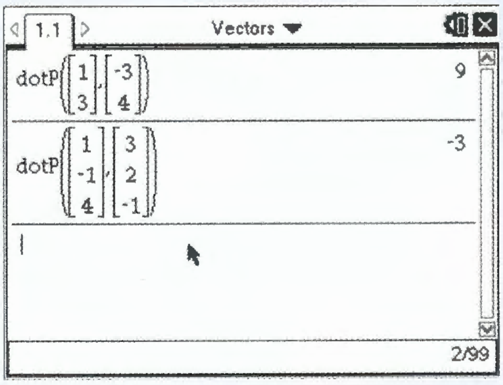
Choose 3 rows and 1 column and then click on OK.



Enter the vector type, and enter the second vector.

Press

$$\begin{pmatrix}1\\-1\\4\end{pmatrix}\cdot\begin{pmatrix}3\\2\\-1\end{pmatrix}=-3$$



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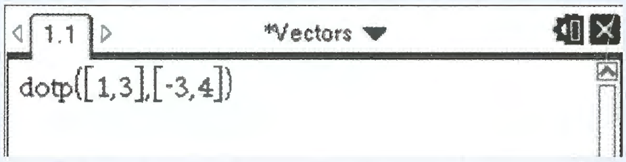
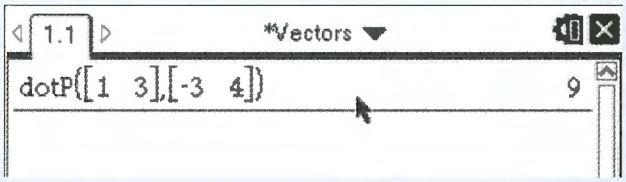
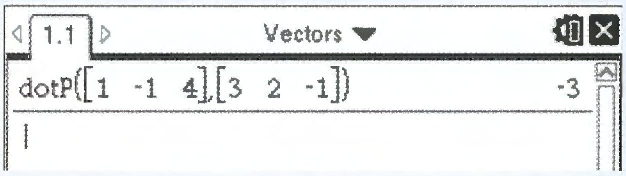
You can also enter vectors as rows by typing them in directly instead of using the templates. Separate the values in the vector with commas. When you press enter, the GDC changes the entry line and calculates the result.





This method can be quicker, especially with 3 × 1 vectors.



4.2 Calculating the angle between two vectors

The angle θ between two vectors **a** and **b**, can be calculated using the formula

$$\theta = \arccos\left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)$$

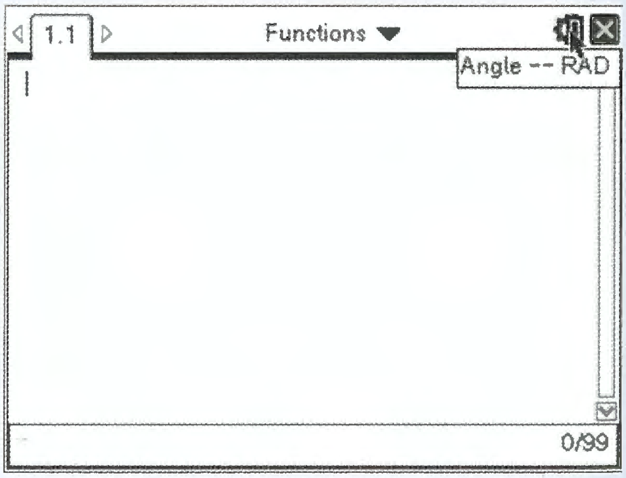
Example 31

Calculate the angle between $2\mathbf{i} + 3\mathbf{j}$ and $3\mathbf{i} - \mathbf{j}$

Open a new document and add a Calculator page.

Move the cursor to the ⚙ symbol at the top right-hand side of the screen. It will display the general angle mode – either radians or degrees.

Click in the ⚙ symbol and choose 2:Settings | 1:General.



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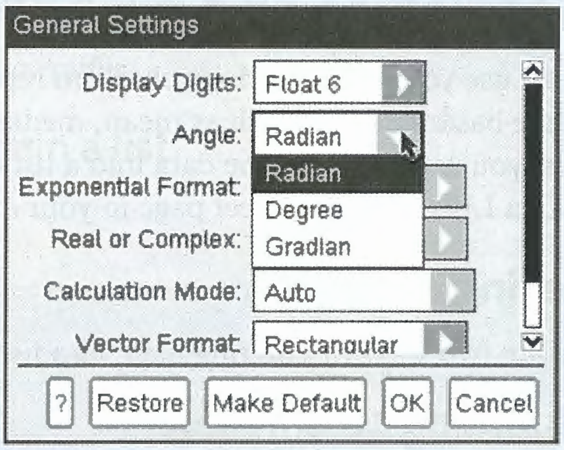
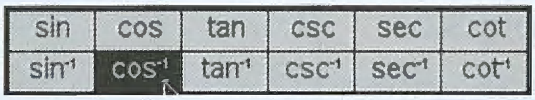
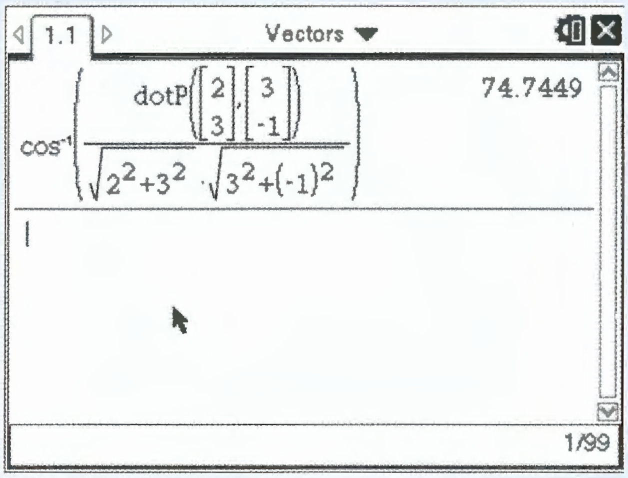
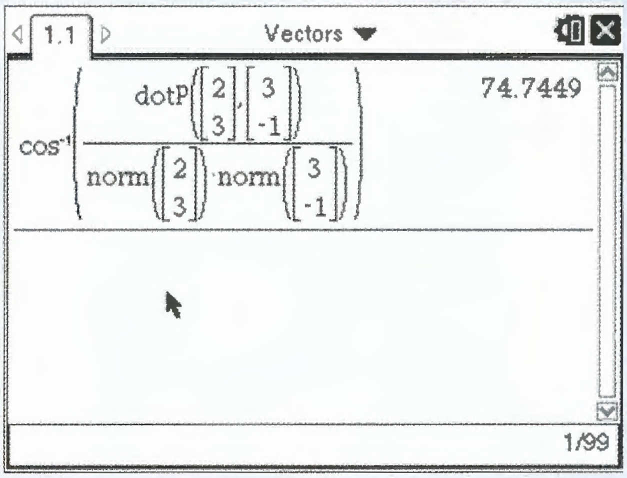
In the dialogue box, select either degrees or radians (according to the units you need your answer in) and then click on OK.	
Press μ and choose $\cos^{-1}$ from the menu.	
Enter the values in the formula as shown, using the fraction template and the 2 × 1 column vector template. To calculate the magnitudes of the vectors use the formula $\lvert a\mathbf{i} + b\mathbf{j}\rvert = \sqrt{a^2 + b^2}$	
As an alternative to using the formula for the magnitude of a vector, you can use the norm function. Press b 7: Matrix & Vector | 7: Norms | 1: Norm or simply typing norm(Instead of retyping the vectors, you can use /C and /V to cut and paste.	

5 Statistics and probability

You can use your GDC to draw charts to represent data and to calculate basic statistics such as mean, median, etc. Before you can do this, you need to enter the data into a list or spreadsheet. This is done in a Lists & Spreadsheet page in your document.

Entering data

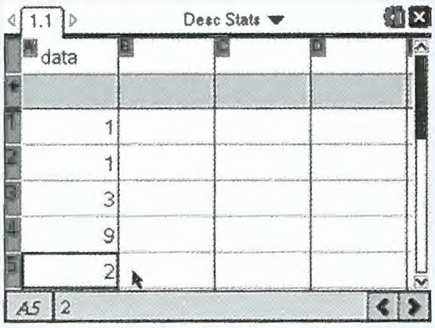
There are two ways of entering data: as a list or as a frequency table.

5.1 Entering lists of data

Example 32

Enter the data in the list
1, 1, 3, 9, 2

Open a new document and add a Lists & Spreadsheet page.
Type 'data' in the first cell.
Type the numbers from the list in the first column.
Press enter or ▼ after each number to move down to the next cell.
Note: The word 'data' is a label that will be used later when you want to create a chart or do some calculations with this data. You can use any letter or name to label the list.



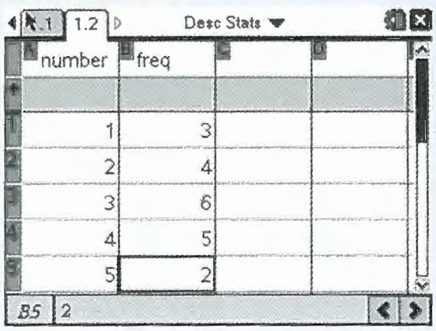
5.2 Entering data from a frequency table

Example 33

Enter the data in a table

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Add a new Lists & Spreadsheet page to your document.
To label the columns, type 'number' in the first cell and 'freq' in the cell to its right.
Enter the numbers in the first column and the frequencies in the second.
Use the ▼ ▲ ◀ ▶ keys to navigate around the spreadsheet.



Drawing charts

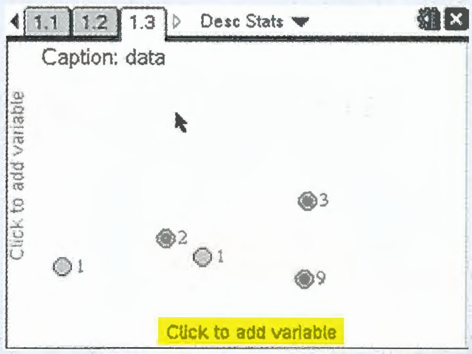
You can draw charts from a list or from a frequency table.

5.3 Drawing a frequency histogram from a list

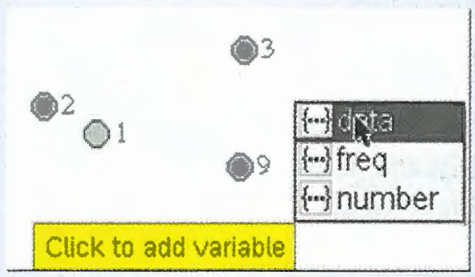
Example 34

Draw a frequency histogram for this data:
1, 1, 3, 9, 2

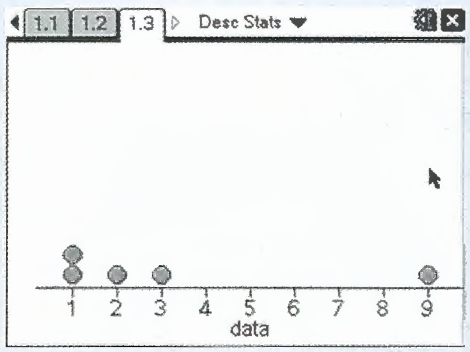
Enter the data in a list called 'data' (see Example 32).
Add a new Data & Statistics page to your document.
Note: You do not need to worry about what this screen shows.



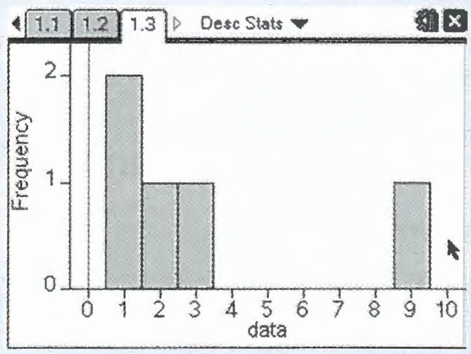
Click at the bottom of the screen where it says 'Click to add variable', choose 'data' from the list and press enter.



The first chart you will see is a dot plot of your data.



Press menu 1:Plot Type | 3:Histogram
Press enter
You should now see a frequency histogram for the data in the list.



5.4 Drawing a frequency histogram from a frequency table

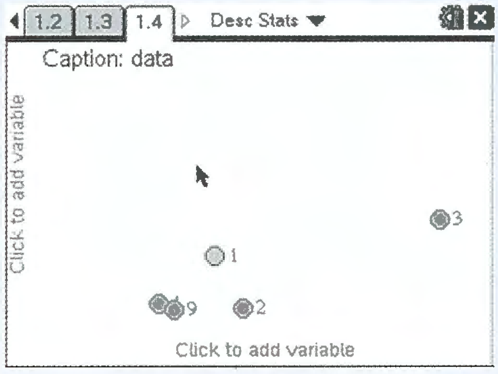
Example 35

Draw a frequency histogram for this data:

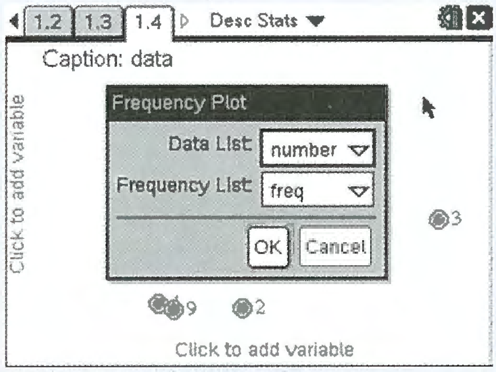
Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in lists called 'number' and 'freq' (see Example 33).
Add a new Data & Statistics page to your document.
Note: You do not need to worry about what this screen shows.

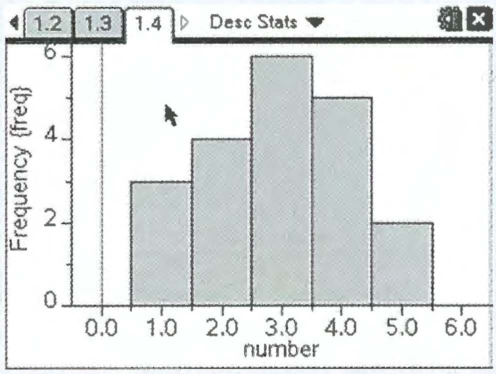
Menu 2 Plot Properties 7: Remove X Variable
A: Remove Y Variable



Press menu 2:Plot Properties | 5:Add *X* Variable with Frequency
Press enter
You will see this dialogue box.
From the drop-down menus, choose 'number' for the Data List and 'freq' for the Frequency List.
Press enter



You should now see a frequency histogram for the data in the table.

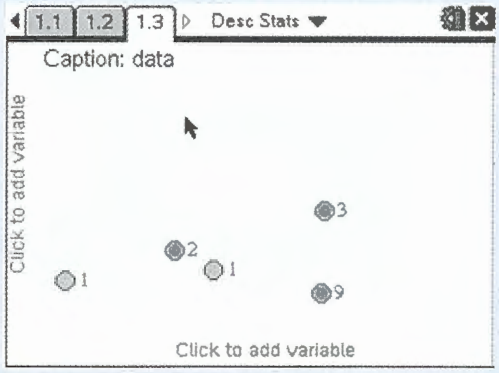
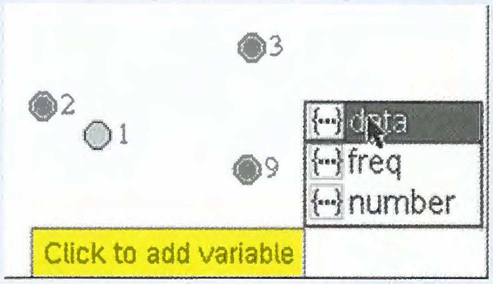
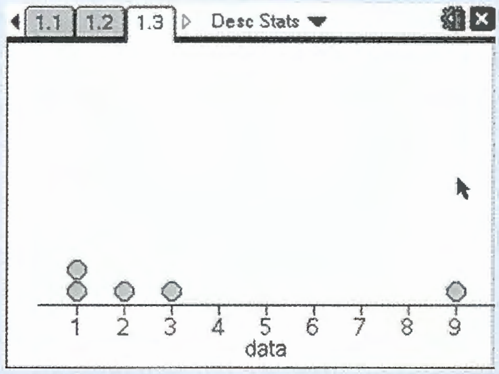
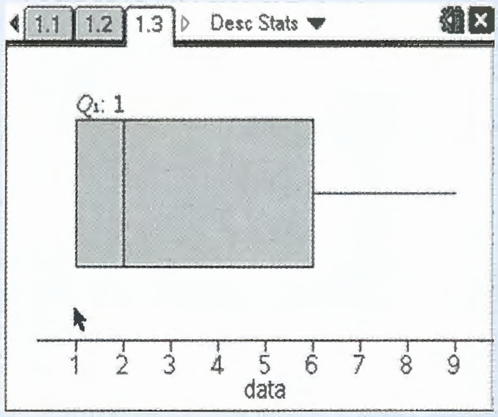
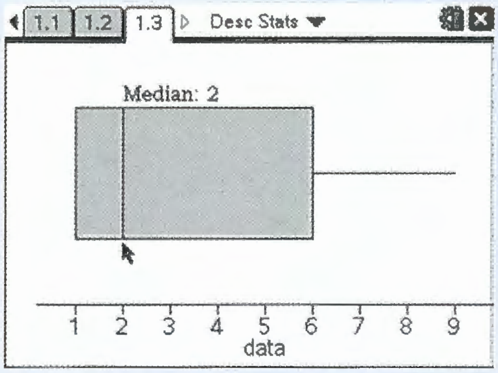


5.5 Drawing a box and whisker diagram from a list

Example 36

Draw a box and whisker diagram for this data:
1, 1, 3, 9, 2

▶ Continued on next page

Enter the data in a list called 'data' (see Example 32). Add a new Data & Statistics page to your document. **Note:** You do not need to worry about what this screen shows.	
Click at the bottom of the screen where it says 'Click to add variable', choose 'data' from the list and press enter.	
The first chart you will see is a dot plot of your data.	
Press menu 1:Plot Type | 2:Box Plot Press enter You should now see a box plot (box and whisker diagram) for the data in the list.	
Move the cursor over the plot and you will see the quartiles, Q_1 and Q_3, the median, and the maximum and minimum values.	

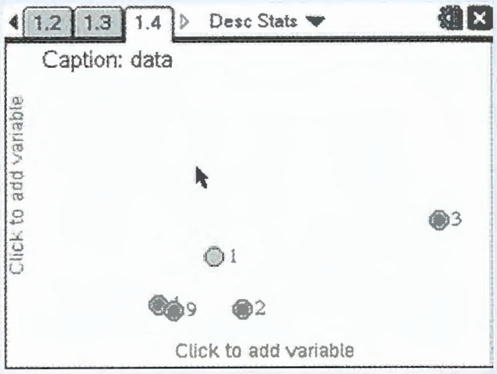
5.6 Drawing a box and whisker diagram from a frequency table

Example 37

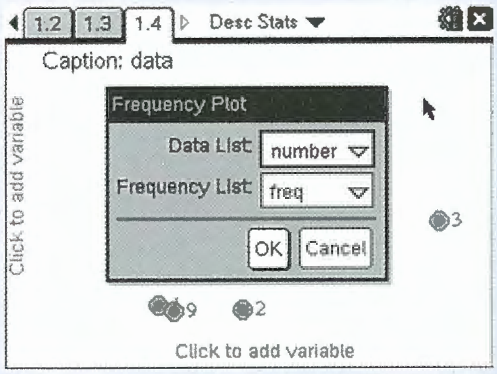
Draw a box and whisker diagram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

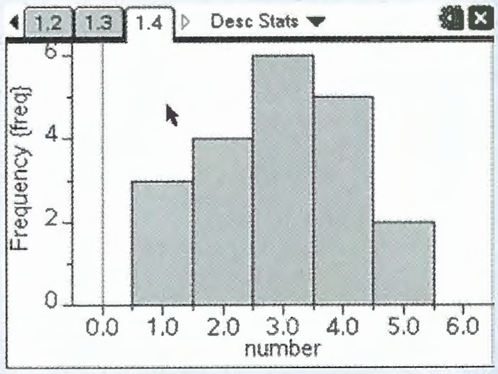
Enter the data in lists called 'number' and 'freq' (see Example 33).
Add a new Data & Statistics page to your document.
Note: You do not need to worry about what this screen shows.



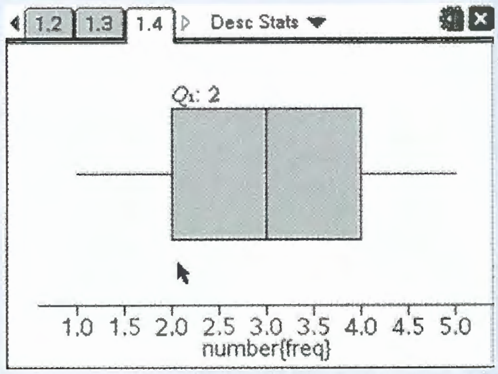
Press menu 2:Plot Properties | 5:Add *X* Variable with Frequency
Press enter
You will see this dialogue box.
From the drop-down menus, choose 'number' for the Data List and 'freq' for the Frequency List.
Press enter



You should now see a frequency histogram.

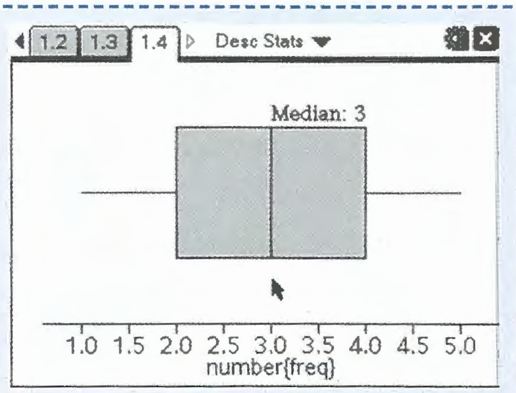


Press menu 1:Plot Type | 2:Box Plot
Press enter
You should now see a box plot (box and whisker diagram) for the data in the table.



Continued on next page

Move the cursor over the plot and you will see the quartiles, Q_1 and Q_3, the median, and the maximum and minimum values.



Calculating statistics

You can calculate statistics such as mean, median, etc. from a list, or from a frequency table.

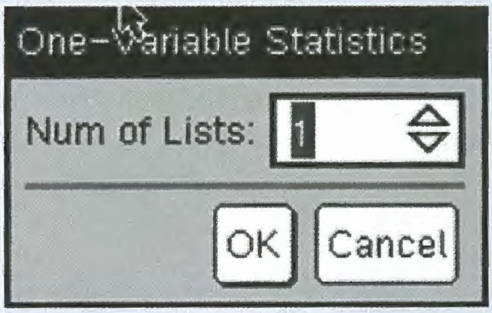
> Mean, median, range, quartiles, standard deviation, etc. are called **summary statistics**.

5.7 Calculating statistics from a list

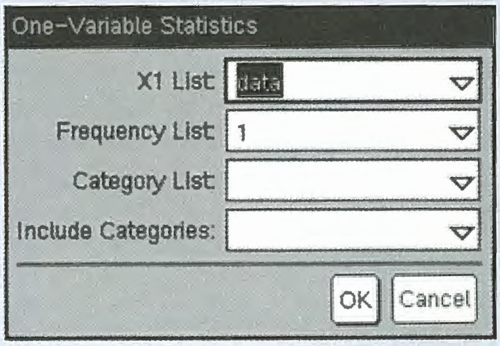
Example 38

Calculate the summary statistics for this data: 1, 1, 3, 9, 2

Enter the data in a list called 'data' (see Example 32).
Add a new Calculator page to your document.
Press menu 6:Statistics | 1:Stat Calculations | 1:One-Var Statistics…
Press enter
This opens a dialogue box.
Leave the number of lists as 1 and press enter.

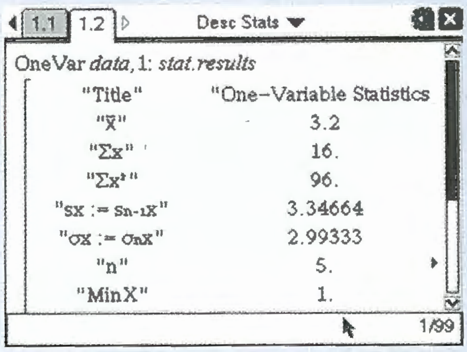


This opens another dialogue box.
Choose 'data' from the drop-down menu for $X1$ List and leave the Frequency List as 1.
Press enter



The information shown will not fit on a single screen.
You can scroll up and down to see it all.
The statistics calculated for the data are:

mean	$\bar{x}$
sum	Σx
sum of squares	Σx^2
sample standard deviation	s_x
population standard deviation	σ_x



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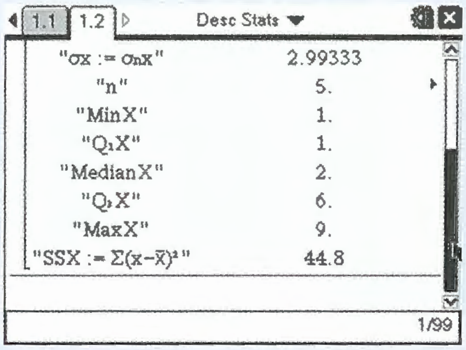
number	n
minimum value	Min$X$
lower quartile	Q_1X
median	Median$X$
upper quartile	Q_3X
maximum value	Max$X$
sum of squared deviations from the mean	SSX

Note: You should always use the population standard deviation (σ_x) in this course.



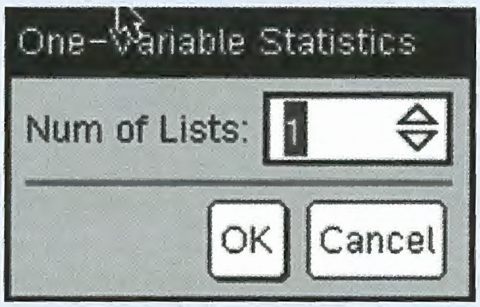
5.8 Calculating statistics from a frequency table

Example 39

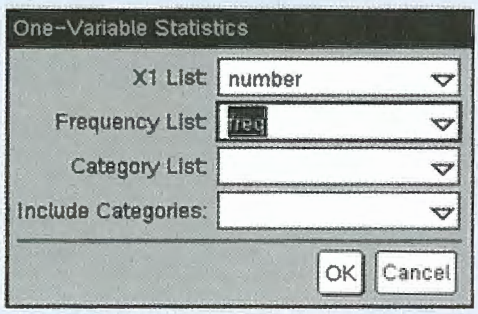
Calculate the summary statistics for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in lists called 'number' and 'freq' (see Example 33).
Add a new Calculator page to your document.
Press menu 6:Statistics | 1:Stat Calculations | 1:One-Var Statistics…
Press enter
This opens a dialogue box.
Leave the number of lists as 1 and press enter.

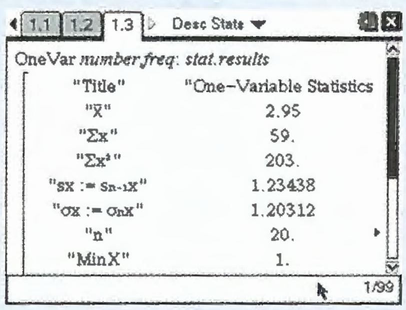


This opens another dialogue box.
From the drop-down menus, choose 'number' for X1 List and 'freq' for the Frequency List.
Press enter



The information shown will not fit on a single screen.
You can scroll up and down to see it all.
The statistics calculated for the data are:

mean	$\bar{x}$
sum	$\sum x$
sum of squares	$\sum x^2$
sample standard deviation	s_x



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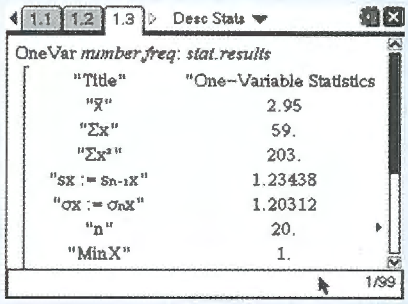
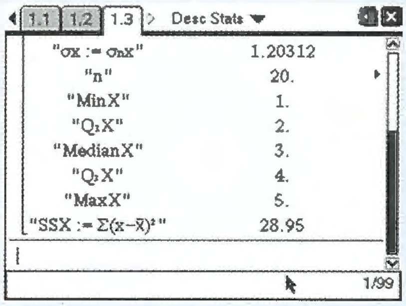
The information shown will not fit on a single screen.
You can scroll up and down to see it all.
The statistics calculated for the data are:

population standard deviation	σ_x
number	n
minimum value	Min$X$
lower quartile	Q_1X
median	Median$X$
upper quartile	Q_3X
maximum value	Max$X$
sum of squared deviations from the mean	SSX

Note: You should always use the population standard deviation (σ_x) in this course.





5.9 Calculating the interquartile range

The interquartile range is the difference between the upper and lower quartiles ($Q_3 - Q_1$).

Example 40

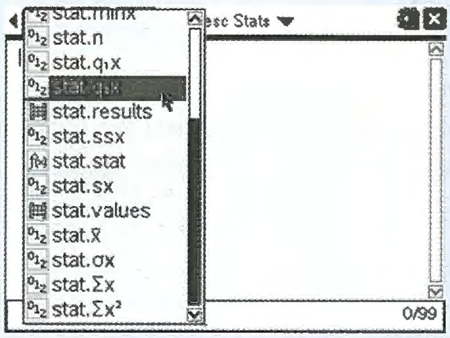
Calculate the interquartile range for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

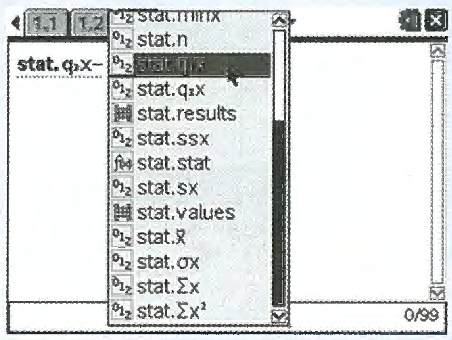
First calculate the summary statistics for this data (see Example 38).

The values of the summary statistics are stored after One-Variable Statistics have been calculated and remain stored until the next time they are calculated.

Add a new Calculator page to your document.
Press var
A dialogue box will appear with the names of the statistical variables.
Scroll down to stat.q_3x using the touchpad, or the ▼ ▲ keys, and then press enter.



Type (-) and press var again.
Scroll down to stat.q_1x using the touchpad, or the ▼ ▲ keys, and then press enter.



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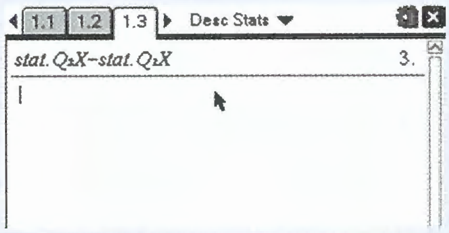
Press `enter` again.
The calculator now displays the result:
Interquartile range = $Q_3 - Q_1 = 3$



5.10 Using statistics

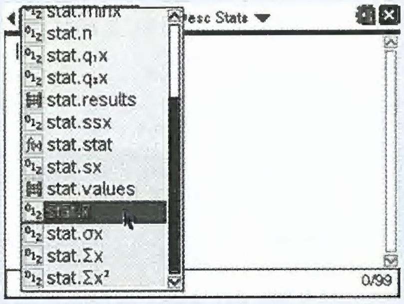
Example 41

Calculate $\bar{x} + \sigma_x$ for this data:

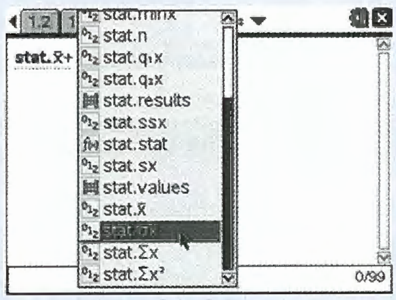
Number	1	2	3	4	5
Frequency	3	4	6	5	2

> The calculator stores the values you calculate in One-Variable Statistics so that you can access them in other calculations. The values are stored until you do another One-Variable Statistics calculation.

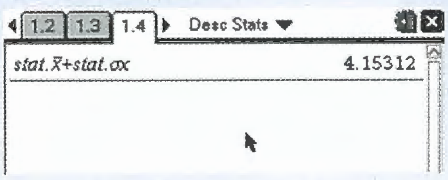
First calculate the summary statistics for this data (see Example 38).
Add a new Calculator page to your document.
Press `var`
A dialogue box will appear with the names of the statistical variables.
Scroll down to stat.$\bar{x}$ using the touchpad, or the ▼ ▲ keys, and then press `enter`.



Type `+` and press `var` again.
Scroll down to stat.σx using the touchpad, or the ▼ ▲ keys, and then press `enter`.



Press `enter` again.
The calculator now displays the result:
$\bar{x} + \sigma_x = 4.15$ (to 3 sf)



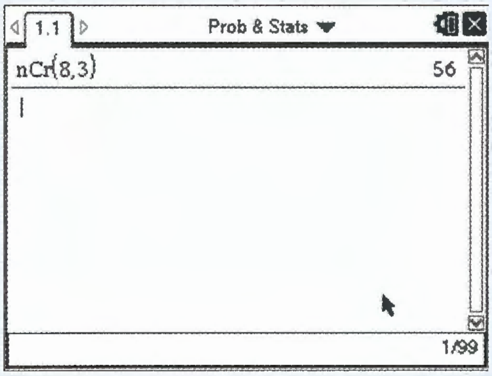
Calculating binomial probabilities

5.11 The use of nCr

Example 42

Find the value of $\binom{8}{3}$ (or ${}_8C_3$)

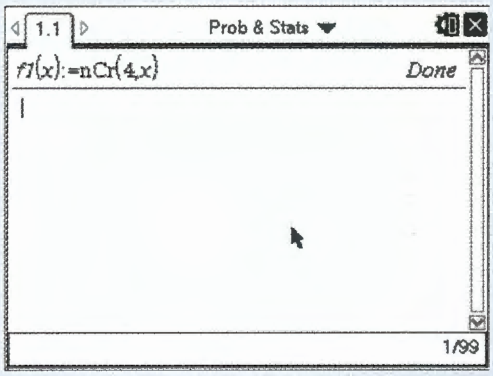
Open a new document and add a Calculator page.
Press menu 5:Probability | 3:Combinations
Alternatively you can just type N C R (.
There is no need to worry about upper or lower case, the calculator recognises the key sequence and translates it accordingly.
Type 8,3
Press enter



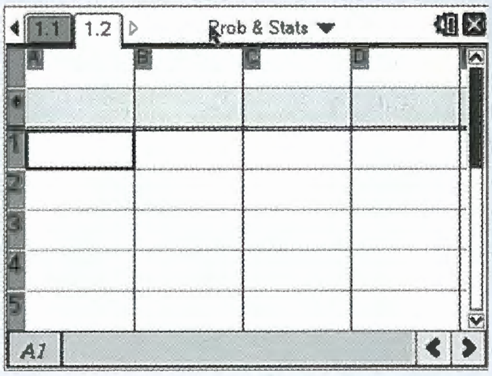
Example 43

List the values of $\binom{4}{r}$ for $r = 0, 1, 2, 3, 4$

Open a new document and add a Calculator page.
Type F 1 (X) ctrl :=
Press menu 5:Probability | 3:Combinations
Alternatively you can just type N C R (.
There is no need to worry about upper or lower case, the calculator recognises the key sequence and translates it accordingly.
Type 4, x
Press enter



Press On and add a new Lists and Spreadsheet page to your document.



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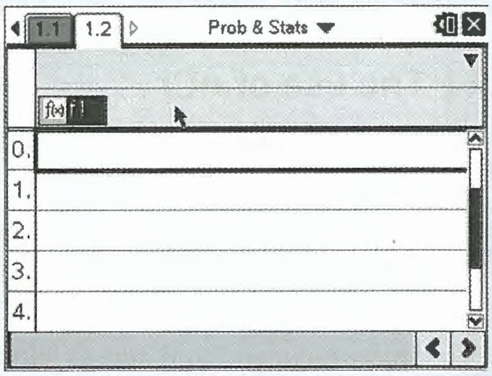
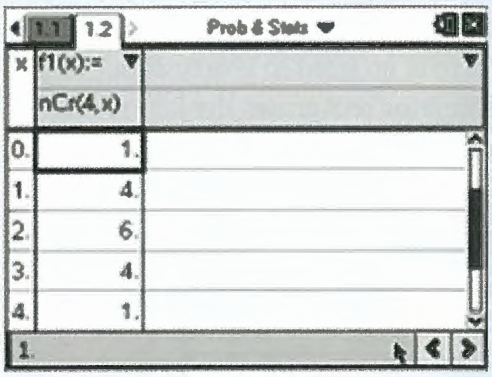
Press ctrl T to switch from spreadsheet view to table view.
Press enter to display the function $f1(x)$



The table shows that

$$\binom{4}{0}=1, \binom{4}{0}=1, \binom{4}{1}=4, \binom{4}{2}=6, \binom{4}{3}=4 \text{ and } \binom{4}{4}=1$$



5.12 Calculating binomial probabilities

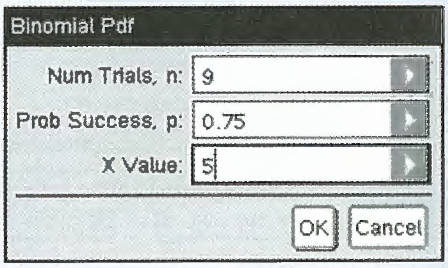
Example 44

X is a discrete random variable and $X \sim B(9, 0.75)$
Calculate $P(X = 5)$

$$P(x=5)=\binom{9}{5}0.75^5\ 0.25^4$$

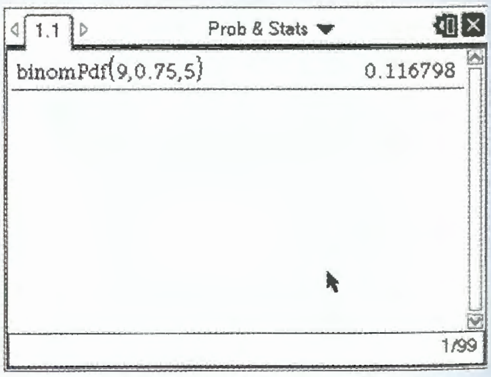
The calculator can find this value directly

Open a new document and add a Calculator page.
Press menu 5:Probability | 3:Probability | 5:Distributions | D:Binomial Pdf...
Enter the number of trials, probability of success and the X value.
Click on OK



The calculator shows that
$P(X = 5) = 0.117$ (to 3 sf)

You can also type the function straight in without using the dialogue box.



Example 45

X is a discrete random variable and $X \sim B(7, 0.3)$

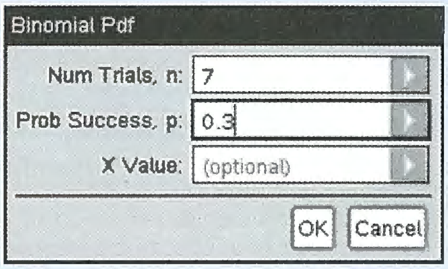
Calculate the probabilities that X takes the values {0, 1, 2, 3, 4, 5, 6, 7}

Open a new document and add a Calculator page.

Press menu 5:Probability | 3:Probability | 5:Distributions | D:Binomial Pdf...

Enter the number of trials, probability of success and leave the X value blank.

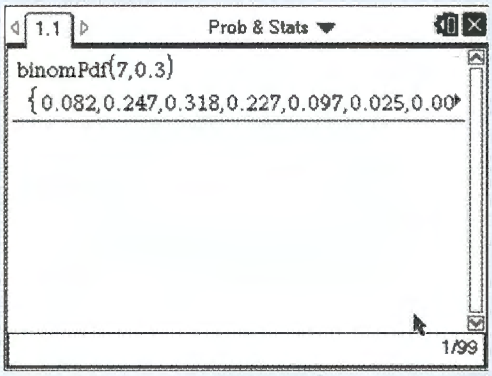
Click on OK



The calculator displays each of the probabilities.

To see the remaining values scroll the screen to the right.

The list can also be transferred to a Lists & Spreadsheet page.

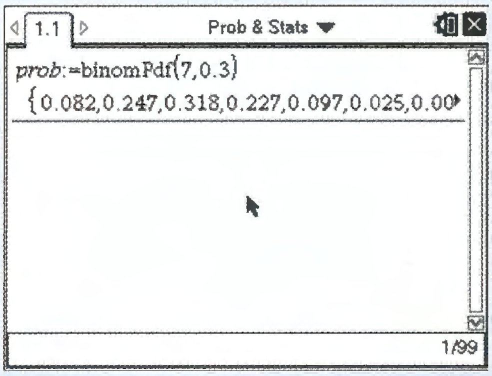


To store the list in a variable named "prob" type:

prob:=binomPdf(7,0.3)

or use the dialogue box as you did before.

Use ctrl := to enter :=

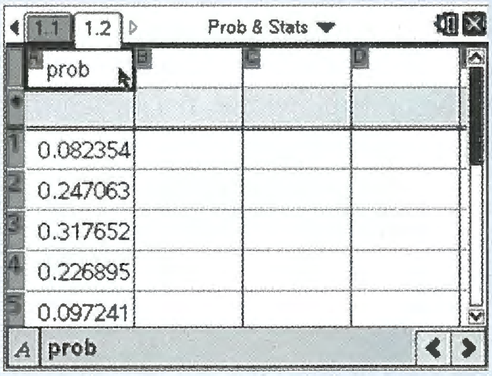


Press On and add a new Lists & Spreadsheet page

At the top of the first column type prob

Press enter

The binomial probabilities are now displayed in the first column.



Example 46

X is a discrete random variable and $X \sim B(20, 0.45)$

Calculate

a the probability that X is less than or equal to 10

b the probability that X lies between 5 and 15 inclusive

c the probability that X is greater than 11

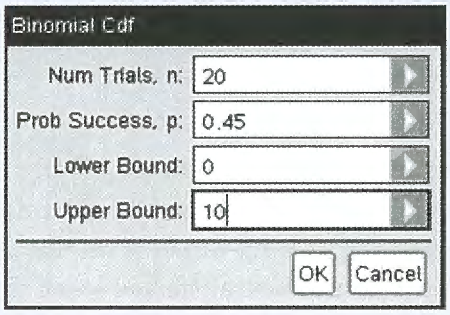
Open a new document and add a Calculator page.

Press menu 5:Probability | 3:Probability | 5:Distributions | E:Binomial Cdf

Enter the number of trials and the probability of success

The lower bound in this case is 0 and the upper bound is 10.

Click on OK

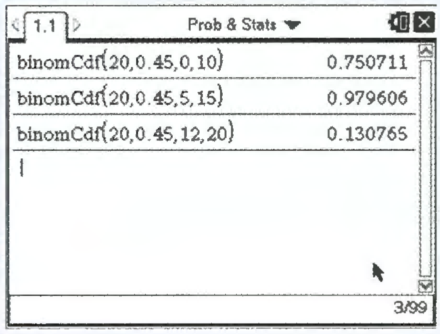


a $P(X \le 10) = 0.751$ (to 3 sf)

b $P(5 \le X \le 15) = 0.980$ (to 3 sf)

c $P(X > 11) = 0.131$ (to 3 sf)

Note: the lower bound is 12 here.



Calculating normal probabilities

5.13 Calculating normal probabilities from *X*-values

Example 47

A random variable X is normally distributed with a mean of 195 and a standard deviation of 20, or $X \sim N(195, 20^2)$. Calculate

a the probability that X is less than 190

b the probability that X is greater than 194

c the probability that X lies between 187 and 196.

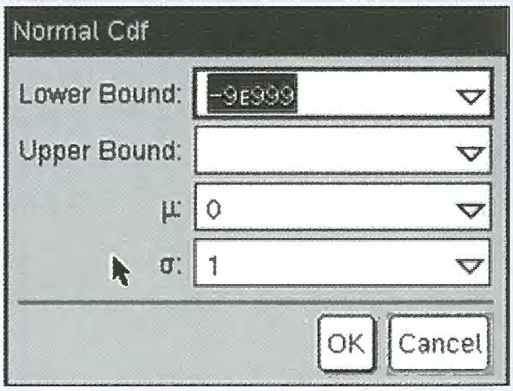
Open a new document and add a Calculator page.

Press menu 5:Probability | 5:Distributions | 2:Normal Cdf

Press enter

You need to enter the values Lower Bound, Upper Bound, μ and σ in the dialogue box.

For the Lower Bound, enter -9×10^{999} as -9E999. This is the smallest number that can be entered in the GDC, so it is used in place of $-\infty$. To enter the E, you need to press the key marked EE.



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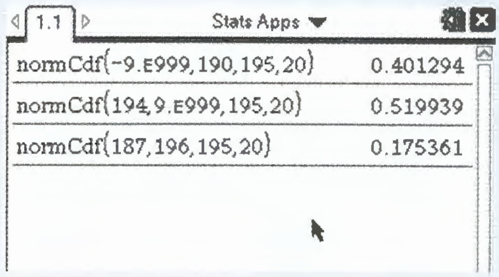
a $P(X < 190)$
Leave the Lower Bound as $-9\text{E}999$.
Change the Upper Bound to 190.
Change μ to 195 and σ to 20.
$P(X < 190) = 0.401$ (to 3 sf)

b $P(X > 194)$
Change the Lower Bound to 194.
For the Upper Bound, enter 9×10^{999} as 9E999. This is the largest number that can be entered in the GDC, so it is used instead of $+\infty$. Leave μ as 195 and σ as 20.
$P(X > 194) = 0.520$ (to 3 sf)

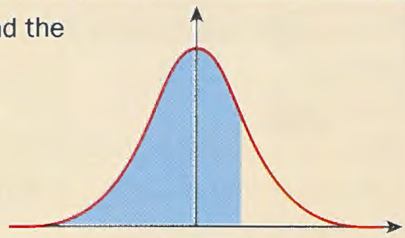
c $P(187 < X < 196)$
Change the Lower Bound to 187 and the Upper Bound to 196; leave μ as 195 and σ as 20.
$P(187 < X < 196) = 0.175$ (to 3 sf)



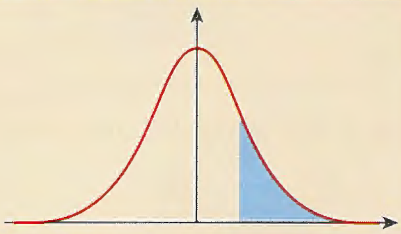
It can be quicker to type the function directly into the calculator, without using the menus and the wizard, but there are a lot of parameters to remember for the function normCdf.

5.14 Calculating *X*-values from normal probabilities

When using the inverse normal function (invNorm), make sure that you find the probability on the correct side of the normal curve. The areas are always the lower tail, that is, they are of the form P(X < x) (see Example 48).



If you are given the upper tail, P(X > x), you must first subtract the probability from 1 to before you can use invNorm (see Example 49).



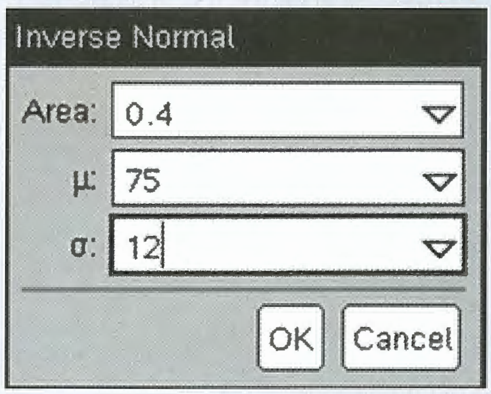
Example 48

A random variable X is normally distributed with a mean of 75 and a standard deviation of 12, or $X \sim N(75, 12^2)$.
If $P(X < x) = 0.4$, find the value of x.

You are given a *lower*-tail probability, so you can find P(X < x) directly.

Open a new document and add a Calculator page.
Press menu 5:Probability | 5:Distributions | 3:Inverse Normal...
Press enter
Enter the probability (area = 0.4), mean ($\mu = 75$) and standard deviation ($\sigma = 12$) in the dialogue box.

It can be quicker to type the function directly into the calculator, without using the menus and the wizard, but there are a lot of parameters to remember for the function invNorm.



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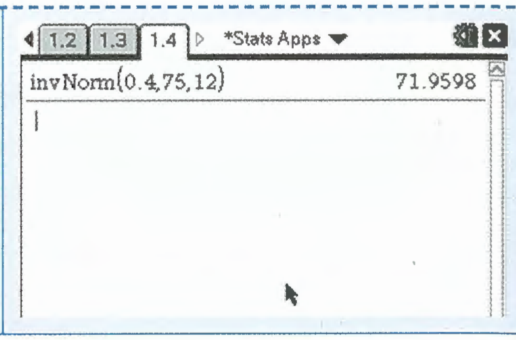
So, if $P(X < x) = 0.4$ then $x = 72.0$ (to 3 sf).

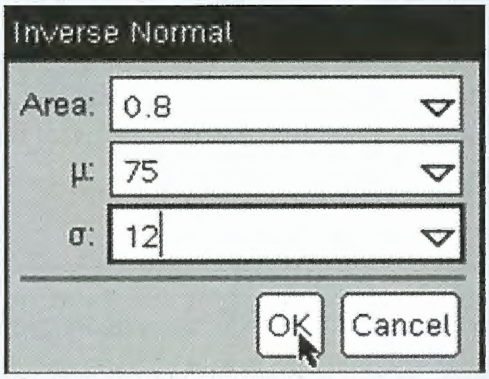


Example 49

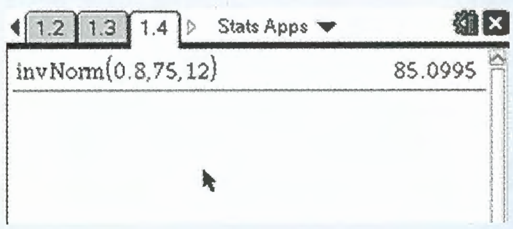
A random variable X is normally distributed with a mean of 75 and a standard deviation of 12, or $X \sim N(75, 12^2)$.
If $P(X > x) = 0.2$, find the value of x.

You are given an *upper*-tail probability, so you must first find $P(X < x) = 1 - 0.2 = 0.8$. You can now use the invNorm function as before.

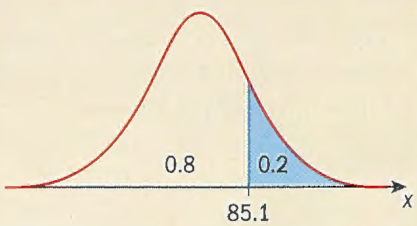
Open a new document and add a Calculator page.
Press menu 5:Probability | 5:Distributions | 3:Inverse Normal...
Press enter
Enter the probability (area = 0.8), mean ($\mu = 75$) and standard deviation ($\sigma = 12$) in the dialogue box.



So, if $P(X > x) = 0.2$ then $x = 85.1$ (to 3 sf).



This sketch of a normal distribution curve shows the value of x and the probabilities for Example 49.



Scatter diagrams, linear regression and the correlation coefficient

5.15 Scatter diagrams using a Data & Statistics page

Using a Data & Statistics page is a quick way to draw scatter graphs and find the equation of a regression line.

For Pearson's product–moment correlation coefficient, see section 5.16, Scatter diagrams using a Graphs page.

Example 50

This data is approximately connected by a linear function.

x	1.0	2.1	2.4	3.7	5.0
y	4.0	5.6	9.8	10.6	14.7

Find the equation of the least squares regression line for y on x.

Use the equation to predict the value of y when $x = 3.0$.

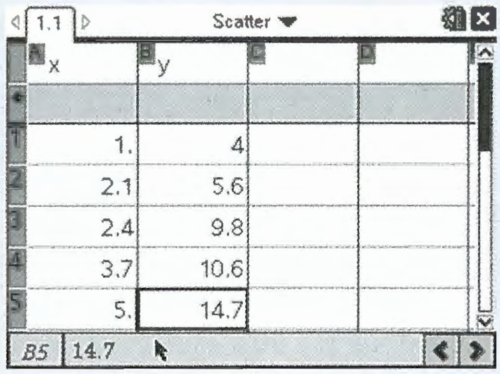
Open a new document and add a Lists & Spreadsheet page.

Enter the data in two lists:

Type 'x' in the first cell and 'y' in the cell to its right.

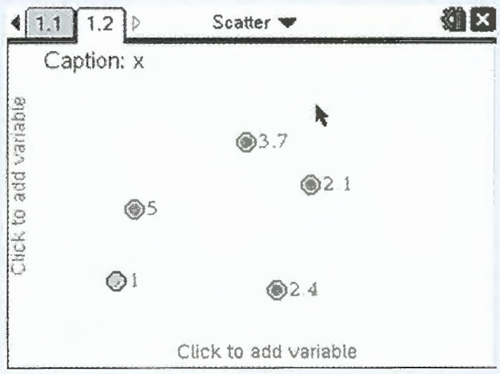
Enter the x-values in the first column and the y-values in the second.

Use the ▼ ▲ ◀ ▶ keys to navigate around the spreadsheet.

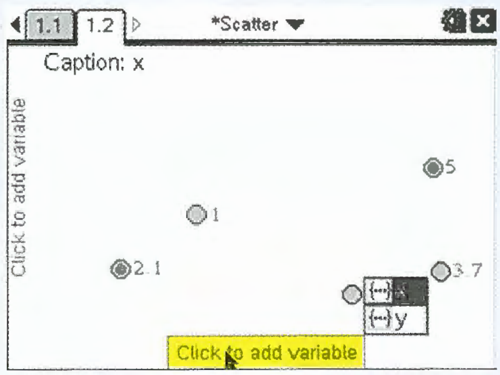


Press [⌂ On] and add a new Data & Statistics page.

Note: You do not need to worry about what this screen shows.



Click at the bottom of the screen where it says 'Click to add variable', choose 'x' from the list and press [enter].



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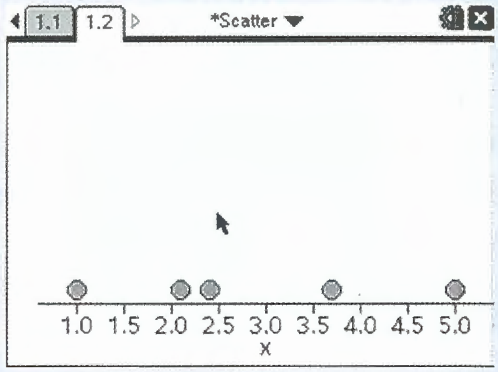
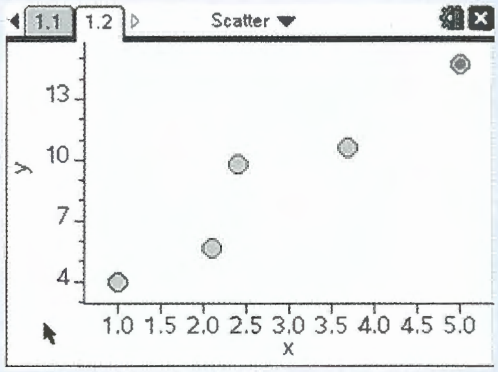
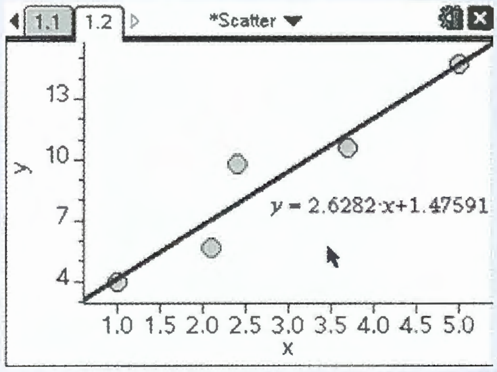
You now have a dot plot of the x-values.
Move the ↖ near to the side of the screen on the left.
The message 'Click to add variable' reappears. Click on the message, choose 'y' from the list and press enter.



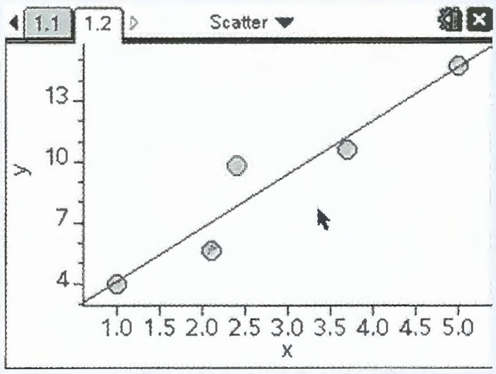
You now have a scatter graph of y against x.



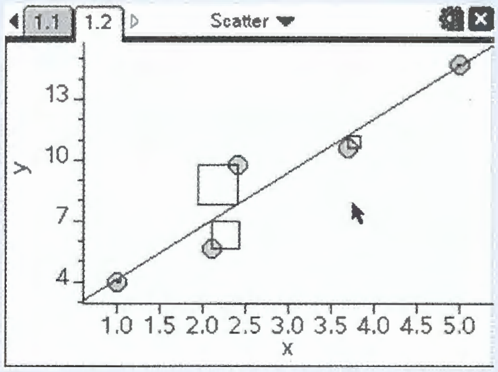
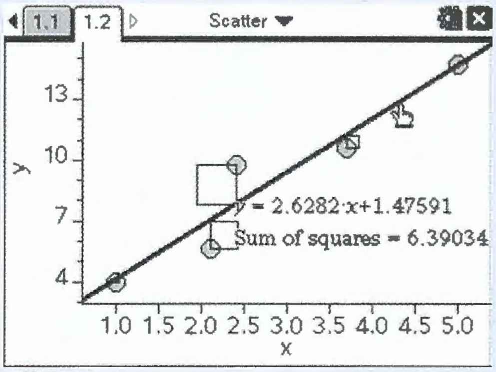
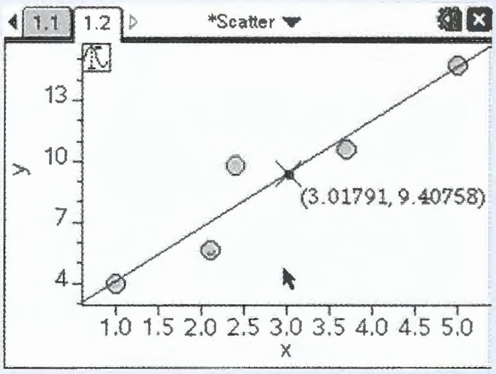
Press menu 4:Analyze | 6:Regression | 1:Show Linear($mx + b$)
Press enter
You will see the least squares regression line for y on x and its equation:
$y = 2.6282x + 1.47591$



If you click the ↖ away from the line, it will no longer be selected and the equation disappears.



▶ Continued on next page

Press menu 4:Analyze | 7:Residuals | 1:Show Residual Squares Press enter The squares on the screen represent the squared deviations of the y-values of the data from the regression line.	
Move the ↖ towards the regression line. When it becomes a ☝, click the touchpad. You now see the equation of the least squares regression line for y on x and the sum of squares. The sum of squares is related to Pearson's product–moment correlation coefficient. Press menu 4:Analyze | 7:Residuals | 1:Hide Residual Squares Press enter	
Press menu 4:Analyze | A:Graph Trace Press enter Use the ▶ ◀ keys to move the trace along the line. It is not possible to move the trace point to an exact value, so get as close to $x = 3$ as you can. From the graph, $y \approx 9.4$ when $x = 3.0$.	

5.16 Scatter diagrams using a Graphs page

Using a Graphs page takes a little longer than the Data & Statistics page, but you will get more detailed information about the data such as Pearson's product–moment correlation coefficient.

Example 51

This data is approximately connected by a linear function.

x	1.0	2.1	2.4	3.7	5.0
y	4.0	5.6	9.8	10.6	14.7

a Find the equation of the least squares regression line for y on x.
b Find Pearson's product–moment correlation coefficient.
c Predict the value of y when $x = 3.0$.

This is the same data as in Example 50.

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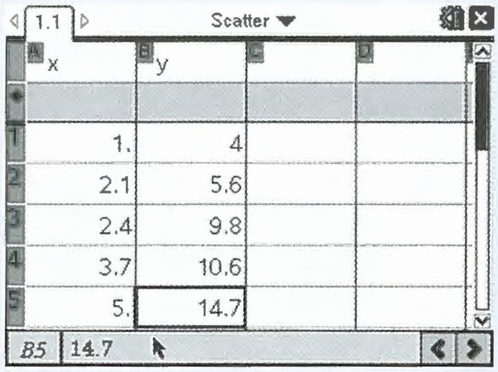
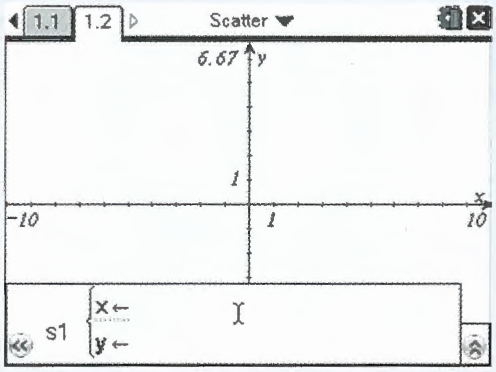
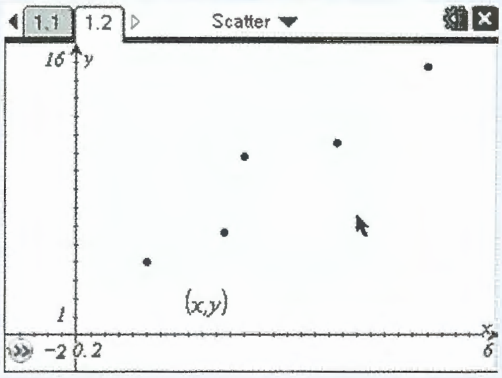
Open a new document and add a Lists & Spreadsheet page.
Enter the data in two lists:
Type 'x' in the first cell and 'y' in the cell to its right.
Enter the x-values in the first column and the y-values in the second.
Use the ▼ ▲ ◀ ▶ keys to navigate around the spreadsheet.



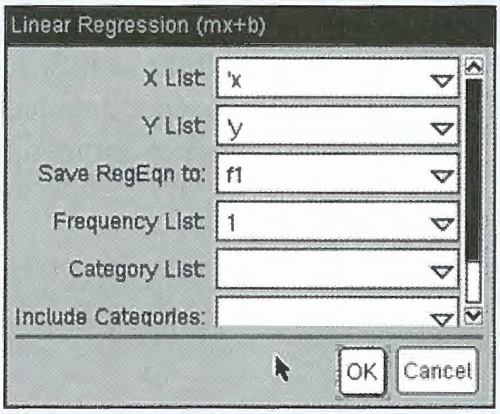
Press On and add a new Graphs page to your document.
Press menu 3:Graph Type | 4:Scatter Plot
Press enter
The entry line is displayed at the bottom of the work area.
Scatter plot type is displayed.
Enter the names of the lists, x and y, into the scatter plot function.
Use the tab key to move from x to y.
Press enter



Adjust your window settings to show the data and the x- and y-axes.
You now have a scatter plot of x against y.



Press ctrl ◀ to return to the Lists & Spreadsheet page.
Press menu 4:Statistics | 1:Stat Calculations | 3:Linear Regression ($mx + b$)
Press enter
From the drop-down menus, choose 'x' for X List and 'y' for Y List. You should press tab to move between the fields.
Press enter



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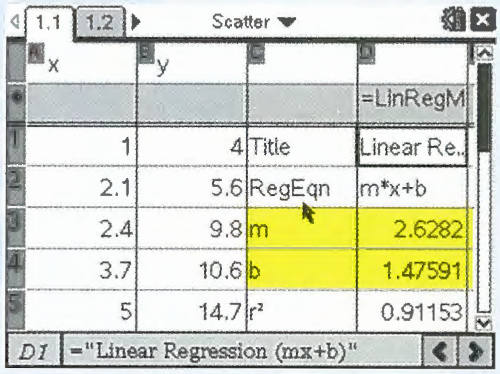
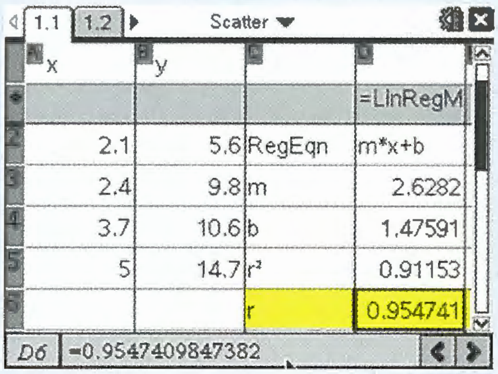
On the screen, you will see the result of the linear regression in lists next to the lists for x and y.
The values of m (2.6282) and b (1.475 91) are shown separately.

a The equation of the least squares regression line for y on x is $y = 2.6282x + 1.475\,91$.

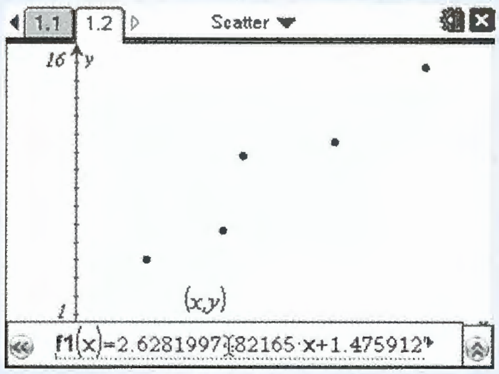


Scroll down the table to see the value of Pearson's product–moment correlation coefficient, given by r.

b Pearson's product–moment correlation coefficient, $r = 0.954\,741$.



Press ctrl ▶ to return to the Graphs page.
Using the touchpad, click on « to open the entry line at the bottom of the work area.
You will see that the equation of the regression line has been pasted into $f1(x)$.



Press enter.
The regression line is now shown on the graph.
Use the trace function menu 5:Trace | 1:Graph Trace to find the point where x is 3.0.
Using the ▶◀ keys, move the trace point close, then edit the x-coordinate and change it to exactly 3.0.

c When $x = 3.0$, $y = 9.36$.