2019 Integral Calculus non calc test markscheme

1a.

appropriate approach ***(M1)***

eg $2\int f(x),\ 2(8)$

$\int_1^6 2f(x)\mathrm{d}x = 16$ ***A1*** ***N2***

[2 marks]

1b.

appropriate approach ***(M1)***

eg $\int f(x) + \int 2,\ 8 + \int 2$

$\int 2\mathrm{d}x = 2x$ (seen anywhere) ***(A1)***

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $2(6) - 2(1),\ 8 + 12 - 2$

$\int_1^6 \left(f(x) + 2\right)\mathrm{d}x = 18$ ***A1*** ***N3***

[4 marks]

2.

(a) **METHOD 1**

choosing quotient rule *(M1)*

eg $\frac{vu'-uv'}{v^2}$

$(\ln x)'=\frac{1}{x}$, seen in rule *(A1)*

correct substitution into the quotient rule *(A1)*

eg $\frac{x\times\frac{1}{x}-\ln x\times 1}{x^2}$

$g'(x)=\frac{1-\ln x}{x^2}$ **A1** **N4**

METHOD 2

choosing product rule *(M1)*

eg $uv'+vu'$

one correct derivative, seen in rule *(A1)*

eg $(\ln x)'=\frac{1}{x}$, $-x^{-2}$

correct substitution into the product rule *(A1)*

eg $\ln x\left(-x^{-2}\right)+x^{-1}\left(\frac{1}{x}\right)$, $\frac{1}{x^2}-\frac{\ln x}{x^2}$

$g'(x)=\frac{1-\ln x}{x^2}$ **A1** **N4**

[4 marks]

(b) attempt to use substitution or inspection *(M1)*

eg $u=\ln x$ so $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x}$, $\int u\,du$

$\int g(x)\mathrm{d}x=\frac{(\ln x)^2}{2}+C$ (accept absence of $+C$) **A2** **N3**

[3 marks]

Total [7 marks]

3.

evidence of anti-differentiation ***(M1)***

eg $\int (6e^{2t} + t)$

$s = 3e^{2t} + \frac{t^2}{2} + C$ ***A2A1***

Note: Award ***A2*** for $3e^{2t}$, ***A1*** for $\frac{t^2}{2}$.

attempt to substitute $(0, 10)$ into **their** integrated expression (even if C is missing) ***(M1)***

correct working ***(A1)***

eg $10 = 3 + C$, $C = 7$

$s = 3e^{2t} + \frac{t^2}{2} + 7$ ***A1*** ***N6***

Note: Exception to the ***FT*** rule. If working shown, allow full ***FT*** on incorrect integration which must involve a power of e.

[7 marks]

4.

correct integration, $2 \times \frac{1}{2}\ln(2x+5)$ ***A1A1***

Note: Award ***A1*** for $2 \times \frac{1}{2}(=1)$ and ***A1*** for $\ln(2x+5)$.

evidence of substituting limits into integrated function and subtracting ***(M1)***

e.g. $\ln(2 \times 5 + 5) - \ln(2 \times 0 + 5)$

correct substitution ***A1***

e.g. $\ln 15 - \ln 5$

correct working ***(A1)***

e.g. $\ln \frac{15}{5}, \ln 3$

$k = 3$ ***A1*** ***N3***

[6 marks]

(b) attempt to use substitution or inspection ***(M1)***

eg $u = \ln x$ so $\frac{du}{dx} = \frac{1}{x}$, $\int u\, du$

$\int g(x)dx = \frac{(\ln x)^2}{2} + C$ (accept absence of $+C$) ***A2*** ***N3***

[3 marks]

Total [7 marks]

5. (a) attempt to find quarter circle area ***(M1)***

eg $\frac{1}{4}(4\pi)$, $\frac{\pi r^2}{4}$, $\int_0^2 \sqrt{4 - x^2 \mathrm{d}x}$

area of region $= \pi$ ***(A1)***

$\int_0^2 f(x)\mathrm{d}x = -\pi$ ***A2 N3***

[4 marks]

(b) attempted set up with both regions ***(M1)***

eg $\text{shaded area} - \text{quarter circle}$, $3\pi - \pi$, $3\pi - \int_0^2 f = \int_2^6 f$

$\int_2^6 f(x)\mathrm{d}x = 2\pi$ ***A2 N2***

[3 marks]

Total [7 marks]

6a.

correct integration ***A1A1***

e.g. $\frac{x^2}{2} - 4x$, $\left[\frac{x^2}{2} - 4x\right]_4^{10}$, $\frac{(x-4)^2}{2}$

Notes: In the first 2 examples, award ***A1*** for each correct term.

In the third example, award ***A1*** for $\frac{1}{2}$ and ***A1*** for $(x-4)^2$.

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

e.g. $\left(\frac{10^2}{2} - 4(10)\right) - \left(\frac{4^2}{2} - 4(4)\right)$, $10 - (-8)$, $\frac{1}{2}(6^2 - 0)$

$\int_4^{10} (x-4)\mathrm{d}x = 18$ ***A1 N2***

6b.

attempt to substitute either limits or the function into volume formula ***(M1)***

e.g. $\pi \int_4^{10} f^2 \mathrm{d}x,\ \int_a^b \left(\sqrt{x-4}\right)^2,\ \pi \int_4^{10} \sqrt{x-4}$

Note: Do not penalise for missing π or d*x*.

correct substitution (accept absence of d*x* and π) ***(A1)***

e.g. $\pi \int_4^{10} \left(\sqrt{x-4}\right)^2,\ \pi \int_4^{10} (x-4)\mathrm{d}x,\ \int_4^{10} (x-4)\mathrm{d}x$

volume = 18π ***A1 N2***

[3 marks]
