2019 Integral Calculus Calculator test markscheme

1a. *[3 marks]*

Let $f(x) = (x-1)(x-4)$.

Find the x-intercepts of the graph of f.

Markscheme

valid approach ***(M1)***

eg $f(x) = 0$, sketch of parabola showing two x-intercepts

$x = 1,\ x = 4\ \text{(accept } (1, 0), (4, 0))$ ***A1A1 N3***

[3 marks]

1b. *[3 marks]*

The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis.

Find the volume of the solid formed.

Markscheme

attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\int_1^4 (f(x))^2 \mathrm{d}x,\ \pi \int ((x-1)(x-4))^2$

$\text{volume} = 8.1\pi\ \text{(exact)}, 25.4$ ***A2 N3***

[3 marks]

2a. *[2 marks]*

Let $f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.

Find $f'(x)$.

Markscheme

expressing f as $x^{\frac{4}{3}}$ ***(M1)***

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} \; \left(= \frac{4}{3}\sqrt[3]{x}\right)$ ***A1 N2***

[2 marks]

2b. *[4 marks]*

Find $\int f(x)\mathrm{d}x$.

Markscheme

attempt to integrate $\sqrt[3]{x^4}$ ***(M1)***

eg $\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}$

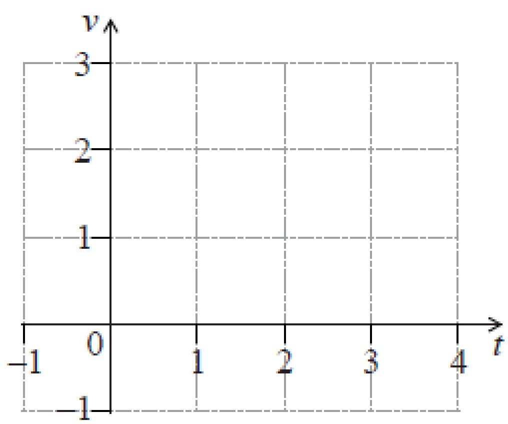
$\int f(x)\mathrm{d}x = \frac{3}{7}x^{\frac{7}{3}} - \frac{x}{2} + c$ ***A1A1A1 N4***

[4 marks]

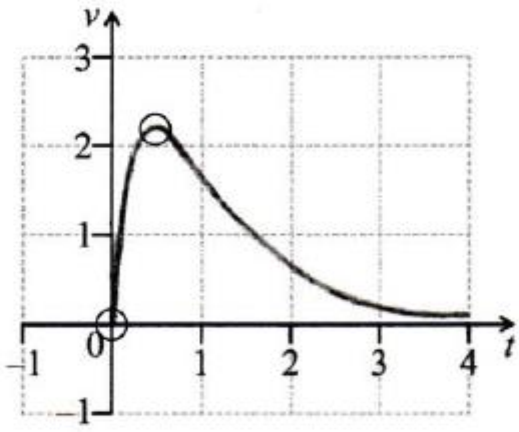
3a. *[3 marks]*

A particle moves along a straight line such that its velocity, $v\ \mathrm{ms}^{-1}$, is given by $v(t) = 10t\mathrm{e}^{-1.7t}$, for $t \geqslant 0$.

On the grid below, sketch the graph of v, for $0 \leqslant t \leqslant 4$.



Markscheme



A1A2 ***N3***

Notes: Award ***A1*** for approximately correct domain $0 \leqslant t \leqslant 4$.

The shape must be approximately correct, with maximum skewed left. **Only** if the shape is approximately correct, award ***A2*** for all the following approximately correct features, in circle of tolerance where drawn (accept seeing correct coordinates for the maximum, even if point outside circle):

Maximum point, passes through origin, asymptotic to t-axis (but must not touch the axis).

If only two of these features are correct, award ***A1***.

[3 marks]

3b. *[2 marks]*

Find the distance travelled by the particle in the first three seconds.

Markscheme

valid approach (including 0 and 3) ***(M1)***

eg $\int_0^3 10t\mathrm{e}^{-1.7t}\,\mathrm{d}t,\ \int_0^3 f(x)$, area from 0 to 3 (may be shaded in diagram)

$\text{distance} = 3.33\ (\text{m})$ ***A1*** ***N2***

[2 marks]

3c. *[3 marks]*

Find the velocity of the particle when its acceleration is zero.

Markscheme

recognizing acceleration is derivative of velocity ***(R1)***

eg $a = \frac{\mathrm{d}v}{\mathrm{d}t}$, attempt to find $\frac{\mathrm{d}v}{\mathrm{d}t}$, reference to maximum on the graph of v

valid approach to find v when $a = 0$ (may be seen on graph) ***(M1)***

eg $\frac{\mathrm{d}v}{\mathrm{d}t} = 0,\ 10\mathrm{e}^{-1.7t} - 17t\mathrm{e}^{-1.7t} = 0,\ t = 0.588$

$\text{velocity} = 2.16\ (\text{ms}^{-1})$ ***A1 N3***

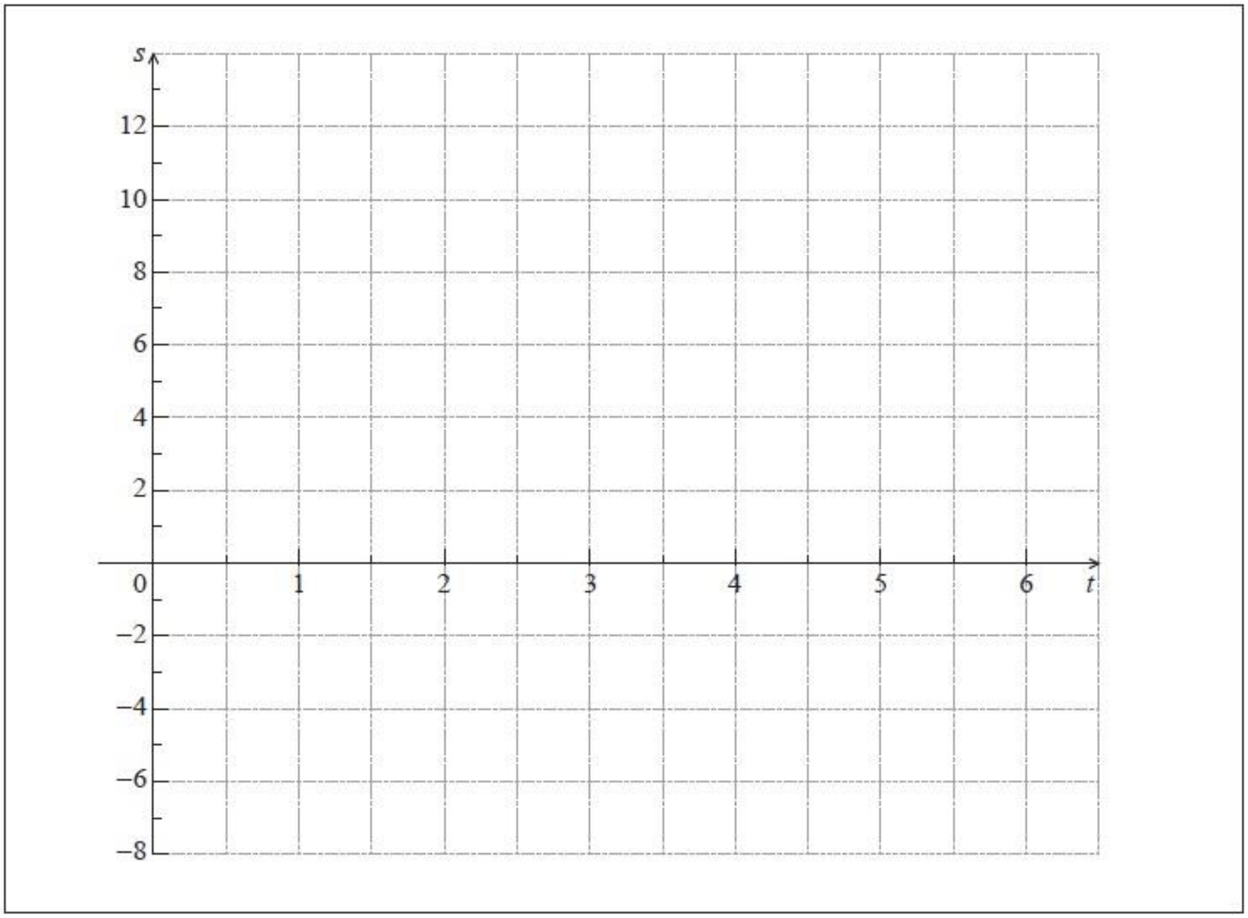
Note: Award ***R1M1A0*** for $(0.588, 216)$ if velocity is not identified as final answer

[3 marks]

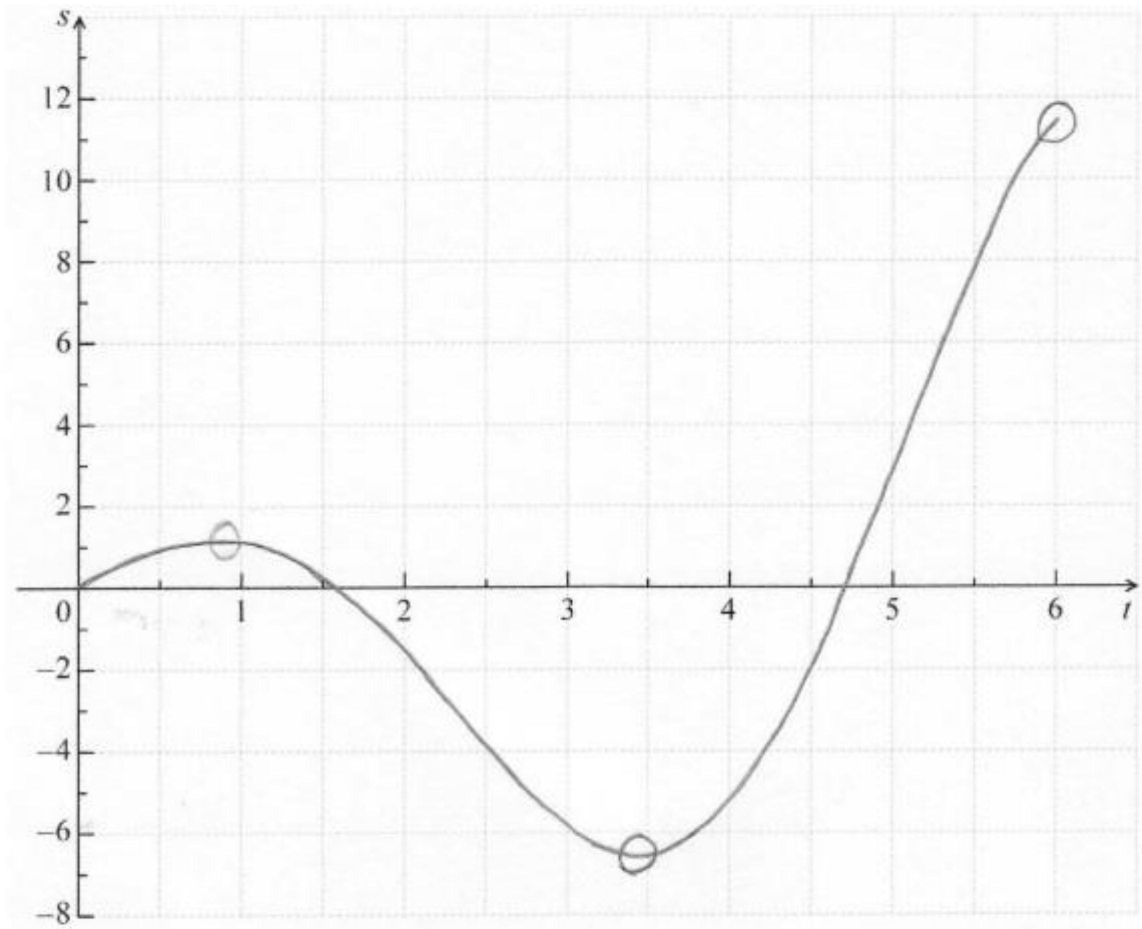
4a. *[4 marks]*

A particle's displacement, in metres, is given by $s(t) = 2t\cos t$, for $0 \le t \le 6$, where *t* is the time in seconds.

On the grid below, sketch the graph of s.



Markscheme



A1A1A

1A1 N4

Note: Award ***A1*** for approximately correct shape (do not accept line segments).

Only if this ***A1*** is awarded, award the following:

A1 for maximum and minimum within circles,

A1 for x-intercepts between 1 and 2 **and** between 4 and 5,

A1 for left endpoint at $(0, 0)$ and right endpoint within circle.

[4 marks]

4b. *[3 marks]*

Find the maximum velocity of the particle.

Markscheme

appropriate approach ***(M1)***

e.g. recognizing that $v = s'$, finding derivative, $a = s''$

valid method to find maximum ***(M1)***

e.g. sketch of v, $v'(t) = 0$, $t = 5.08698\ldots$

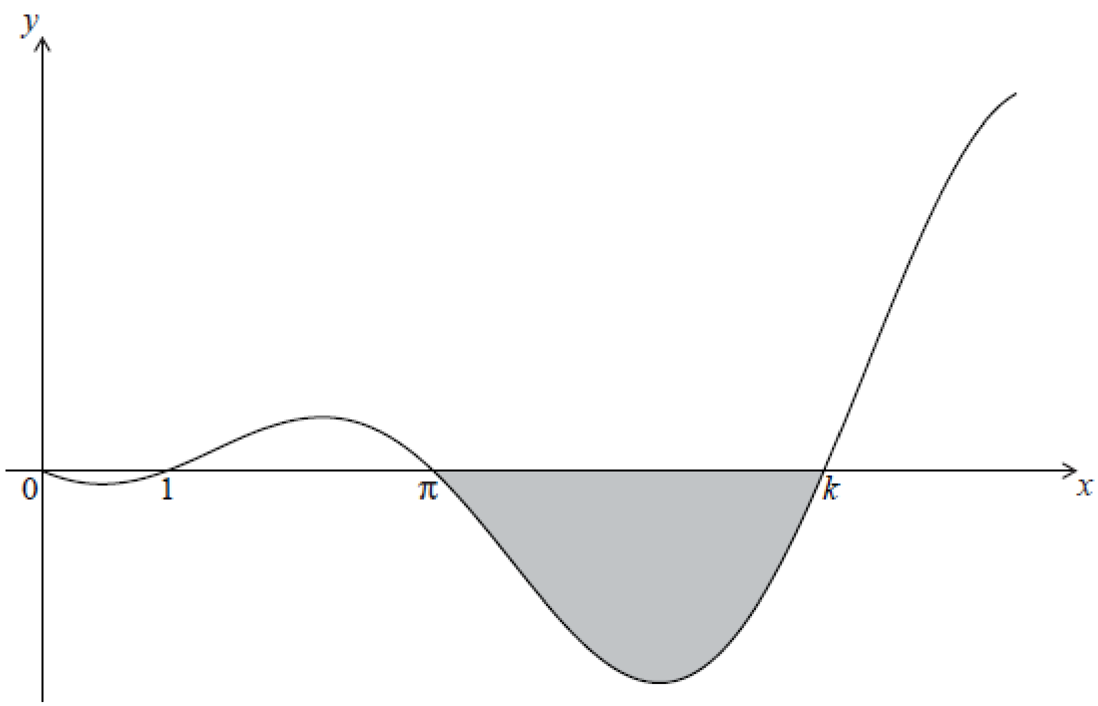
$v = 10.20025\ldots$

$v = 10.2\ [10.2, 10.3]$ ***A1*** ***N2***

[3 marks]

5a. *[2 marks]*

The graph of $y = (x-1)\sin x$, for $0 \le x \le \frac{5\pi}{2}$, is shown below.



The graph has x-intercepts at 0, 1, π and k.

Find k.

Markscheme

evidence of valid approach ***(M1)***

e.g. $y = 0$, $\sin x = 0$

$2\pi = 6.283185\ldots$

$k = 6.28$ ***A1*** ***N2***

[2 marks]

5b. *[3 marks]*

The shaded region is rotated 360° about the *x*-axis. Let *V* be the volume of the solid formed.

Write down an expression for V.

Markscheme

attempt to substitute either limits or the function into formula ***(M1)***

(accept absence of $\mathrm{d}x$)

e.g. $V = \pi \int_{\pi}^{k} (f(x))^2 \mathrm{d}x$, $\pi \int ((x-1)\sin x)^2$, $\pi \int_{\pi}^{6.28\ldots} y^2 \mathrm{d}x$

correct expression ***A2*** ***N3***

e.g. $\pi \int_{\pi}^{6.28} (x-1)^2 \sin^2 x \mathrm{d}x$, $\pi \int_{\pi}^{2\pi} ((x-1)\sin x)^2 \mathrm{d}x$

[3 marks]

5c. *[2 marks]*

The shaded region is rotated 360° about the *x*-axis. Let *V* be the volume of the solid formed.

Find V.

Markscheme

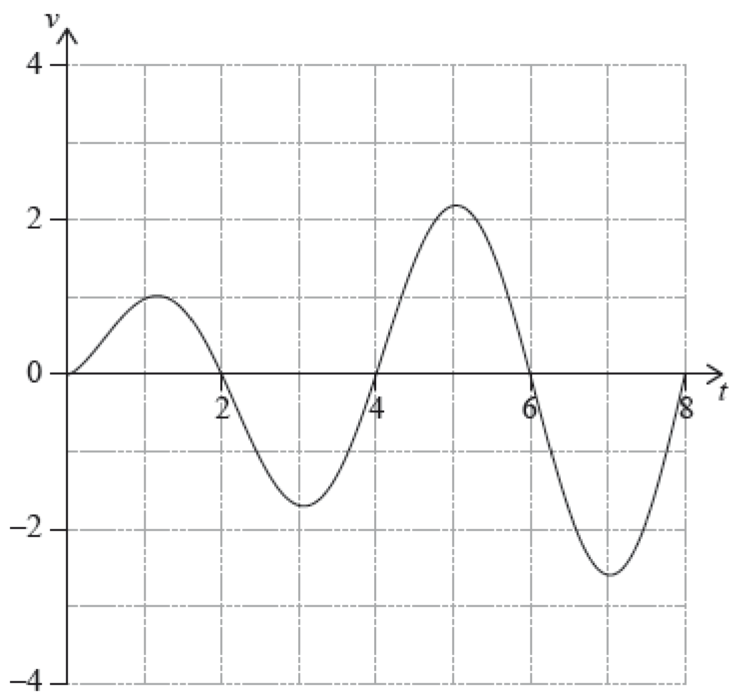
$V = 69.60192562\ldots$

$V = 69.6$ ***A2*** ***N2***

[2 marks]

6a. *[1 mark]*

A particle P moves along a straight line. Its velocity $v_{\mathrm{P}}\ \mathrm{m\,s^{-1}}$ after t seconds is given by $v_{\mathrm{P}} = \sqrt{t}\sin\left(\frac{\pi}{2}t\right)$, for $0 \leqslant t \leqslant 8$. The following diagram shows the graph of v_{P}.



Write down the first value of t at which P changes direction.

Markscheme

$t = 2$ ***A1 N1***

[1 mark]

6b. *[2 marks]*

Find the **total** distance travelled by P, for $0 \leqslant t \leqslant 8$.

Markscheme

substitution of limits or function into formula or correct sum ***(A1)***

eg $\int_0^8 |v|\,\mathrm{d}t,\ \int |v_Q|\,\mathrm{d}t,\ \int_0^2 v\mathrm{d}t - \int_2^4 v\mathrm{d}t + \int_4^6 v\mathrm{d}t - \int_6^8 v\mathrm{d}t$

9.64782

distance $= 9.65\ (\text{metres})$ ***A1 N2***

[2 marks]

6c. *[4 marks]*

A second particle Q also moves along a straight line. Its velocity, $v_{\mathrm{Q}}\ \mathrm{m\,s^{-1}}$ after t seconds is given by $v_{\mathrm{Q}} = \sqrt{t}$ for $0 \leqslant t \leqslant 8$. After k seconds Q has travelled the same total distance as P.

Find k.

Markscheme

correct approach ***(A1)***

eg $s = \int \sqrt{t},\ \int_0^k \sqrt{t}\mathrm{d}t,\ \int_0^k |v_{\mathrm{Q}}|\,\mathrm{d}t$

correct integration ***(A1)***

eg $\int \sqrt{t} = \frac{2}{3}t^{\frac{3}{2}} + c,\ \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^k,\ \frac{2}{3}k^{\frac{3}{2}}$

equating their expression to the distance travelled by their P ***(M1)***

eg $\frac{2}{3}k^{\frac{3}{2}} = 9.65,\ \int_0^k \sqrt{t}\mathrm{d}t = 9.65$

5.93855

5.94 (seconds) ***A1*** ***N3***

[4 marks]
