Transformation of functions

- Translations
- Dilations (from the x axis)
- Dilations (from the y axis)
- Reflections (in the x axis)
- Reflections (in the y axis)
- Summary
- Applying transformations
- Finding equations from transformation (graphs)
- Finding equations from transformations (from points)
**Translations**

- **Translations** move individual points horizontally or vertically.
- **Translations** can be applied to the graph of functions.
- Moving right 1 unit, up 2 units:

  \[ y = (x - 1)^2 + 2 \]

Each point creates an image:

Original function: \( y = x^2 \)

Each point creates an image:

- \( x' = x + h \)
- \( y' = y + k \)
- \( x = x' - h \)
- \( y = y' - k \)

\[
\begin{align*}
  y' - k &= (x' - h)^2 \\
  y' &= (x' - h)^2 + k \\
  \text{Translated function:} & \quad y = f(x - h) + k
\end{align*}
\]

\( h = \) horizontal translation
\( k = \) vertical translation
Dilations (from the x axis)

- **Dilations** are multiplications that stretch the graph away from an axis.
- Dilations can be from the x or y axis.
- A dilation of $a = 3$ from the x axis: stretches vertically by a factor of 3.

Each point creates an image:

Original function: $y = x^2$

Dilated function: $y = af(x)$

$a = \text{dilation from the } x \text{ axis}$

$y' = a(x')^2$  \hspace{1cm}  $y = \frac{y'}{a}$

$x = x'$  \hspace{1cm}  $y' = a \times y$
Dilations (from the y axis)

- A dilation of $b = 3$ from the y axis: stretches horizontally by a factor of 3.

Original function: $y = x^2$

Each point creates an image:

$$x' = b \cdot x$$

$$y' = y$$

$$x = \frac{x'}{b}$$

$$y = y'$$

Dilated function:

$$y' = \left(\frac{x'}{b}\right)^2$$

$$y = \left(\frac{x}{b}\right)^2$$

$b = \text{dilation from the y axis}$
Reflections (in the x axis)

- **Reflections** flip the graph around the x or y axis.
- Reflections keep the shape of the graph the same.
- A reflection in the x axis: signs are changed for y values.

Original function: \( y = x^3 + 2 \)

Each point creates an image:

\[
\begin{align*}
x' &= x \\
y' &= -y \\
x &= x' \\
y &= -y' \\
-y' &= (x')^3 + 2 \\
y' &= -(x')^3 - 2
\end{align*}
\]

Reflected function: \( y = -f(x) \)
Reflections (in the y axis)

- **Reflections** flip the graph around the x or y axis.
- Reflections keep the shape of the graph the same.
- A reflection in the y axis: signs are changed for x values.

Original function: \( y = x^3 + 2 \)

Each point creates an image:

- \( x' = -x \)
- \( y' = y \)
- \( x = -x' \)
- \( y = y' \)

\[ y' = (-x')^3 + 2 \]

Reflected function: \( y = f(-x) \)
Summary

Translated $h$ units right: $y = f(x - h)$

Translated $k$ units up: $y = f(x) + k$

Dilation by $a$ from the $x$ axis: $y = af(x)$

Dilation by $b$ from the $y$ axis: $y = f\left(\frac{x}{b}\right)$

Reflection about $x$ axis: $y = -f(x)$

Reflection about $y$ axis: $y = f(-x)$

Translations & dilations involving $x$ (inside the function) will always be opposites operations to $y$. 
Applying transformations

- The order in which transformations are applied will determine the final equation.

Transforming: \( y = x^2 \)

1. Translation of 3 units to the right
   \[ x, y \]
   \[ \downarrow \]
   \[ x+3, y \]

2. Dilation by 2 from the x axis:
   \[ x+3, 2y \]
   \[ \downarrow \]
   \[ x+3, 2y \]

3. Reflection about x axis:
   \[ x+3, -2y \]
   \[ \downarrow \]
   \[ x+3, -2y \]

4. Translation of 4 units up:
   \[ x+3, -2y+4 \]
Applying transformations: step by step

• The order in which transformations are applied will determine the final equation.

1. Translation of 3 units to the right.
   \[ y = (x - 3)^2 \]

2. Dilation by 2 from the x axis.
   \[ y = 2(x - 3)^2 \]

3. Reflection about x axis.
   \[ y = -2(x - 3)^2 \]

4. Translation of 4 units up.
   \[ y = -2(x - 3)^2 + 4 \]
Finding equations from transformation (graphs)

- The equations of transformed functions can be found from graphs.
- For every unknown constant, one piece of information will be required to help to find them.
- Points, stationary points and asymptotes are used.

\[
y = \frac{a}{(x-h)^2} + k
\]

\[
y = \frac{a}{(x-3)^2} - 1
\]

Substitute in a point: (4,1)

\[1 = \frac{a}{(4-3)^2} - 1\]

\[2 = \frac{a}{(1)^2}\]

\[a = 2\]

\[
y = \frac{2}{(x-3)^2} - 1
\]
The equations of transformed functions can be found from points.

For every unknown constant one piece of information will be required to help to find them.

Simultaneous equations are used to find the unknowns from points.

An equation of the form: \( y = \sqrt{ax+b} \)

Passes through the points: \((7,6)\) and \((3,4)\)

\[
\begin{align*}
4 &= \sqrt{3a+b} \\
6 &= \sqrt{7a+b} \\
16 &= 3a+b \\
36 &= 7a+b
\end{align*}
\]

\[
\begin{align*}
20 &= 4a \\
16 &= 3a+b \\
16 - 3 \times 5 &= b \\
\end{align*}
\]

\[
\begin{align*}
a &= 5 \\
b &= 1
\end{align*}
\]

\[y = \sqrt{5x+1}\]