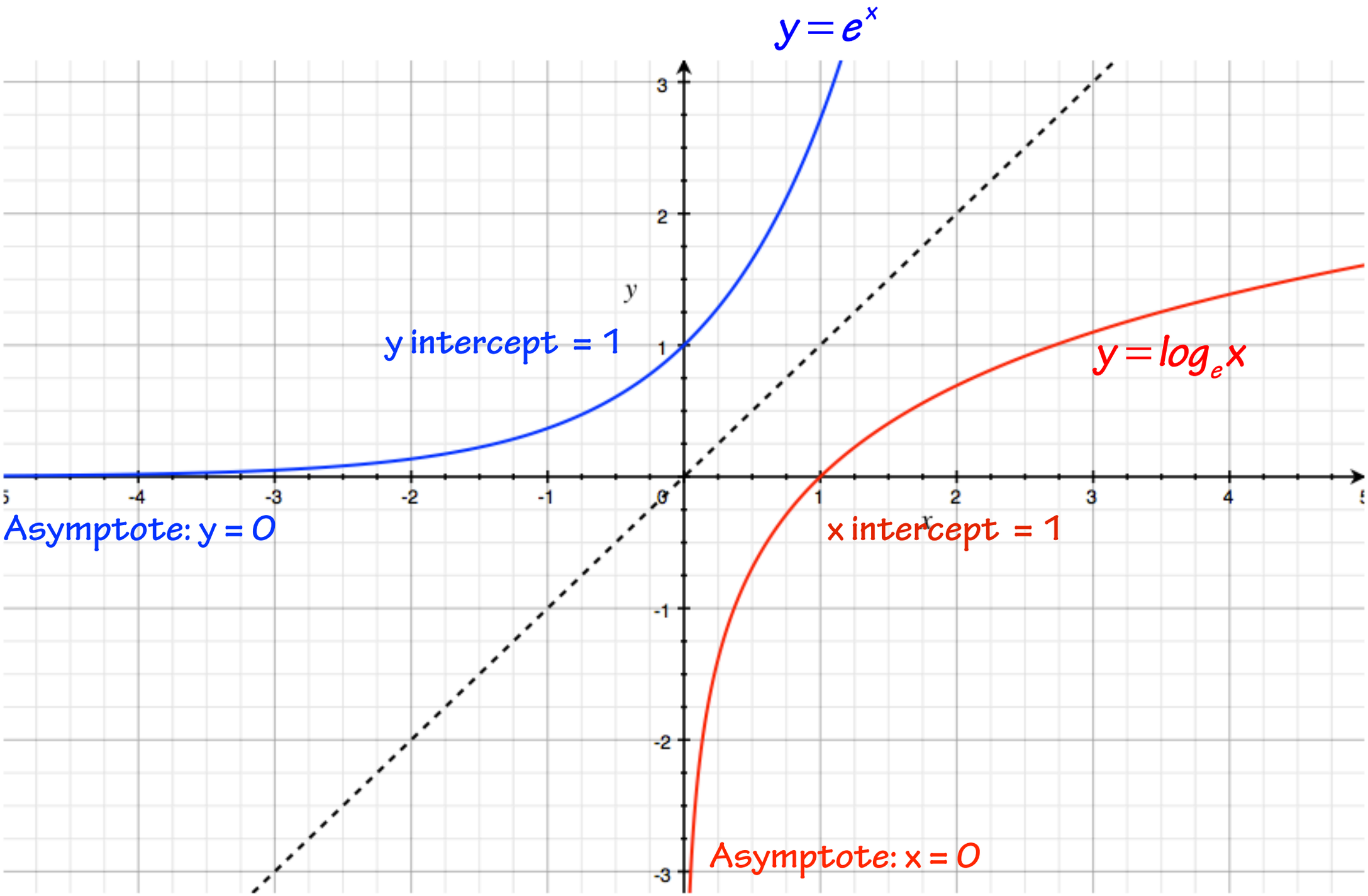
Inverse functions

- Inverse functions - graphs
- Inverse functions - rules
- Inverse functions - from equations
- Finding inverse functions - one to one.

Inverse functions - graphs



Inverse functions - rules

- Every one to one function has an **inverse function**, $f^{-1}(x)$.
- Knowing about inverses helps to work backwards & solve equations.
- The graph of an inverse function can be found from mirroring the original graph around the line $y = x$.
- The domain of the inverse $f^{-1}(x)$ is the range of $f(x)$.
- The range of the inverse $f^{-1}(x)$ is the domain of $f(x)$.
- The x intercept of $f^{-1}(x)$ is the y intercept of $f(x)$.
- The y intercept of $f^{-1}(x)$ is the x intercept of $f(x)$.
- A horizontal asymptote for $f(x)$ produces a vertical asymptote for $f^{-1}(x)$.
- A vertical asymptote for $f(x)$ produces a horizontal asymptote for $f^{-1}(x)$.

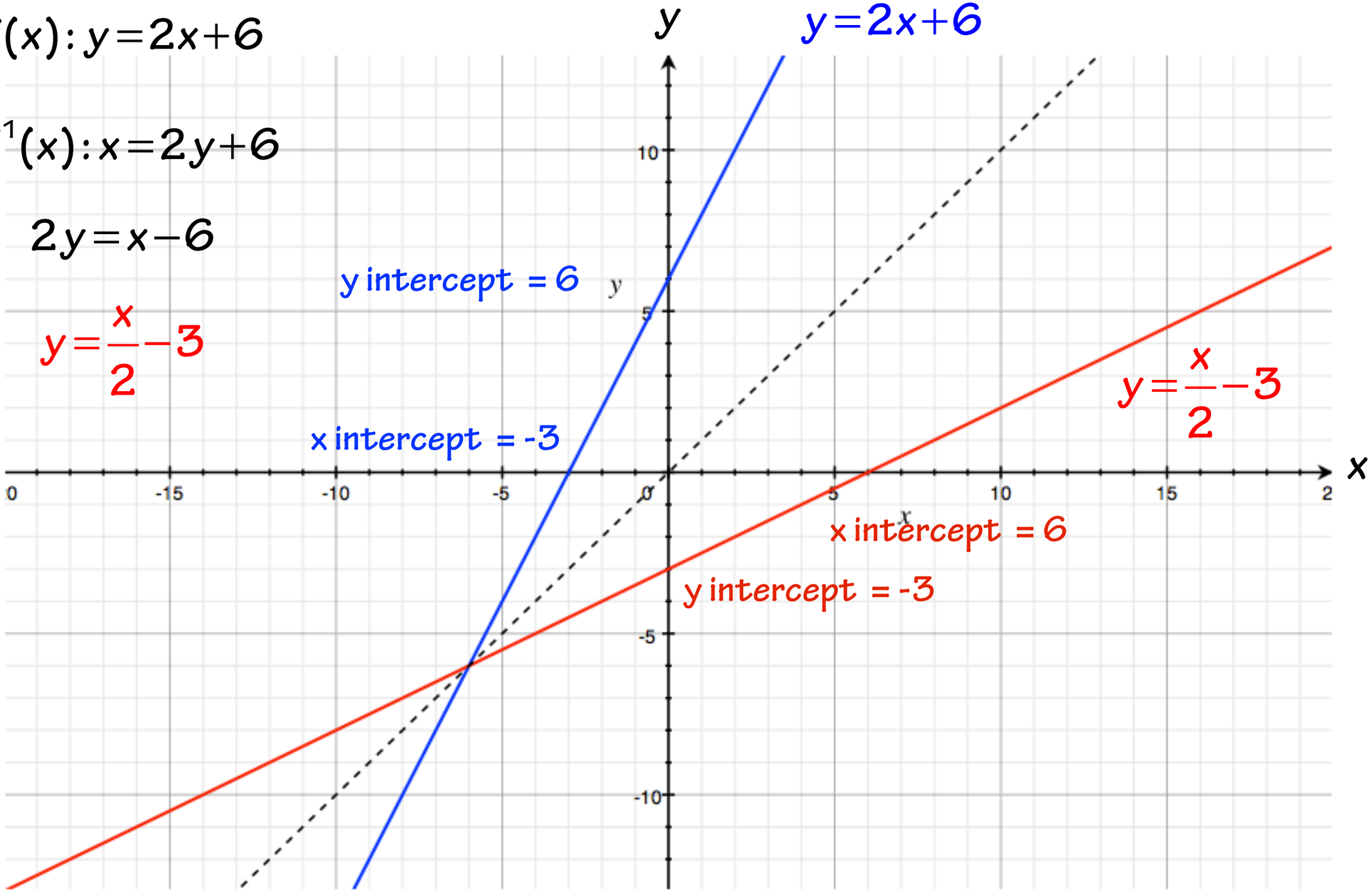
Inverse functions - from equations

$f(x): y = 2x + 6$

$f^{-1}(x): x = 2y + 6$

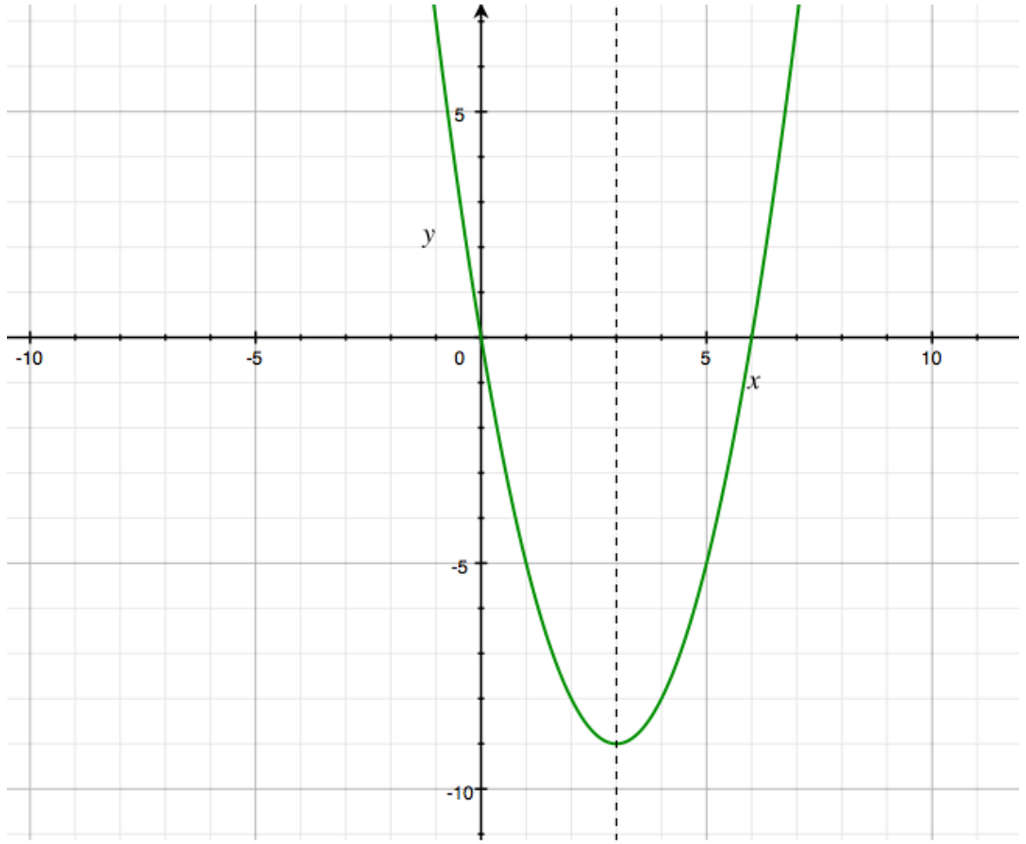
$2y = x - 6$

$y = \frac{x}{2} - 3$

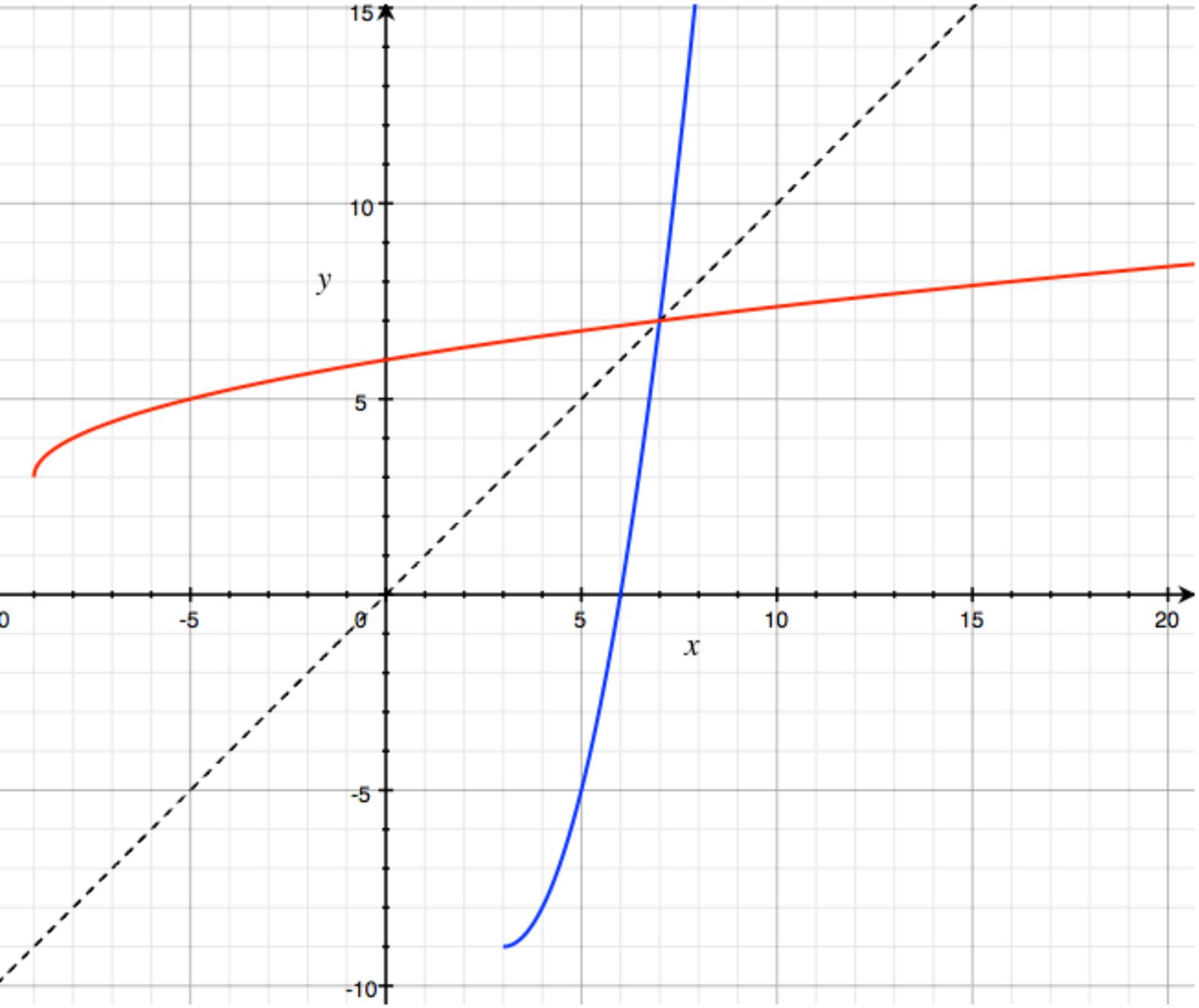


Finding inverse functions - one to one.

- Let $f:[a,\infty) \rightarrow R$, where $f(x)=x^2-6x$.
- If a is the smallest real number such that f has an inverse function, find the value of a and the inverse $f^{-1}(x)$.
- This is a quadratic function (many to one).
- Either side of the turning point, the function will be one to one.



Finding inverse functions - one to one.



Finding a:

$y = x^2 - 6x$

$y = x^2 - 6x + 9 - 9$

$y = (x-3)^2 - 9$

$a = 3$

$f : [3, \infty) \to R, x^2 - 6x$

Finding the inverse:

$x = (y-3)^2 - 9$

$x + 9 = (y-3)^2$

$\sqrt{x+9} = y - 3$

$y = \sqrt{x+9} + 3$

$f : [-9, \infty) \to R, \sqrt{x+9} + 3$