Functions and relations

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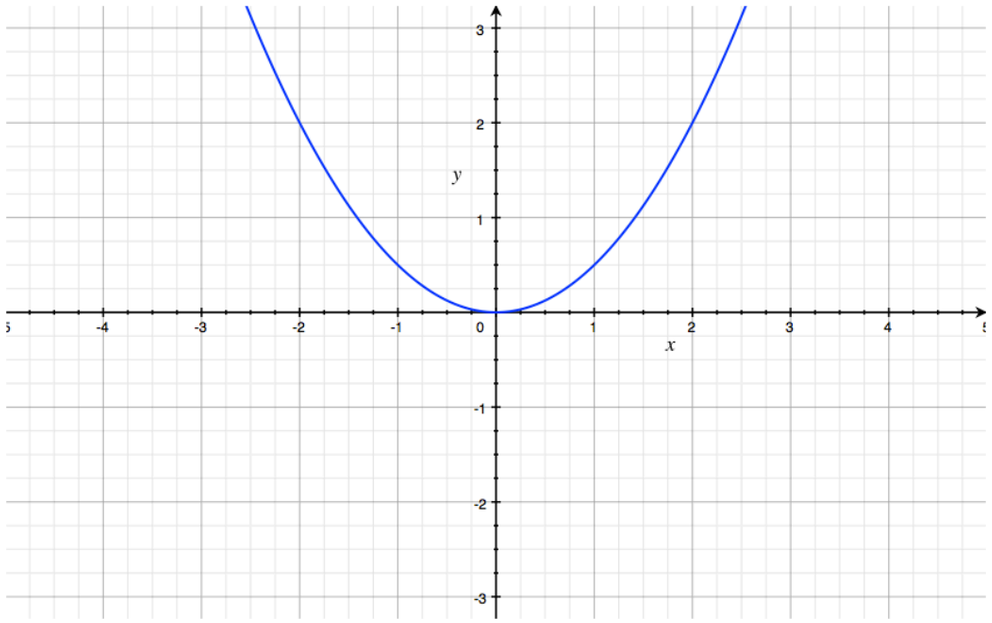
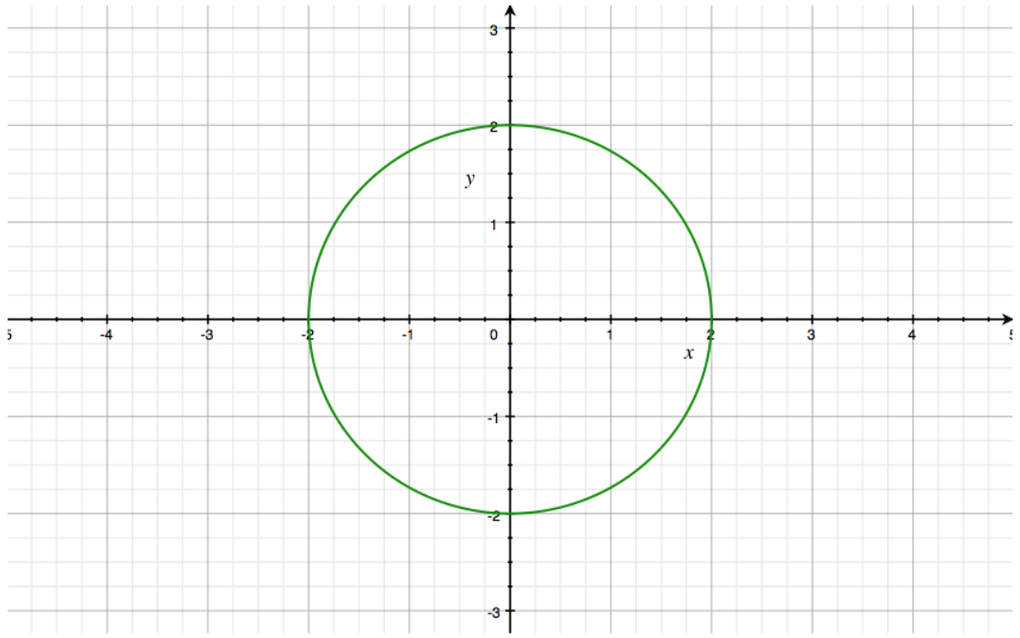
Relations

- A **relation** is a rule that links two sets of numbers: the domain & range.
- The **domain** of a relation is the set of the first elements of the ordered pairs (x values).
- The **range** of a relation is the set of the second elements of the ordered pairs (y values).
- (The range is a subset of the **co-domain** of the function.)
- Some relations exist for all possible values of x.
- Other relations have an implied domain, as the function is only valid for certain values of x.





Set rules & symbols

$A = \{1, 3, 5, 7, 9\}$ (odd numbers)

$B = \{2, 4, 6, 8, 10\}$ (even numbers)

$C = \{2, 3, 5, 7\}$ (prime numbers)

$D = \{4, 8\}$ (multiples of 4)

- $\in$: element of a set. $3 \in A$
- $\notin$: not an element of a set. $6 \notin A$
- $\cap$: intersection of two sets (in both B and C) $B \cap C = \{2\}$
- $\cup$: union of two sets (elements in set B or C) $B \cup C = \{2,3,4,5,6,7,8,10\}$
- $\subset$: subset of a set (all elements in D are part of set B) $D \subset B$
- $\setminus$: exclusion (elements in set B but not in set C) $B \setminus C = \{2,6,10\}$
- $\emptyset$: empty set $A \cap B = \emptyset$

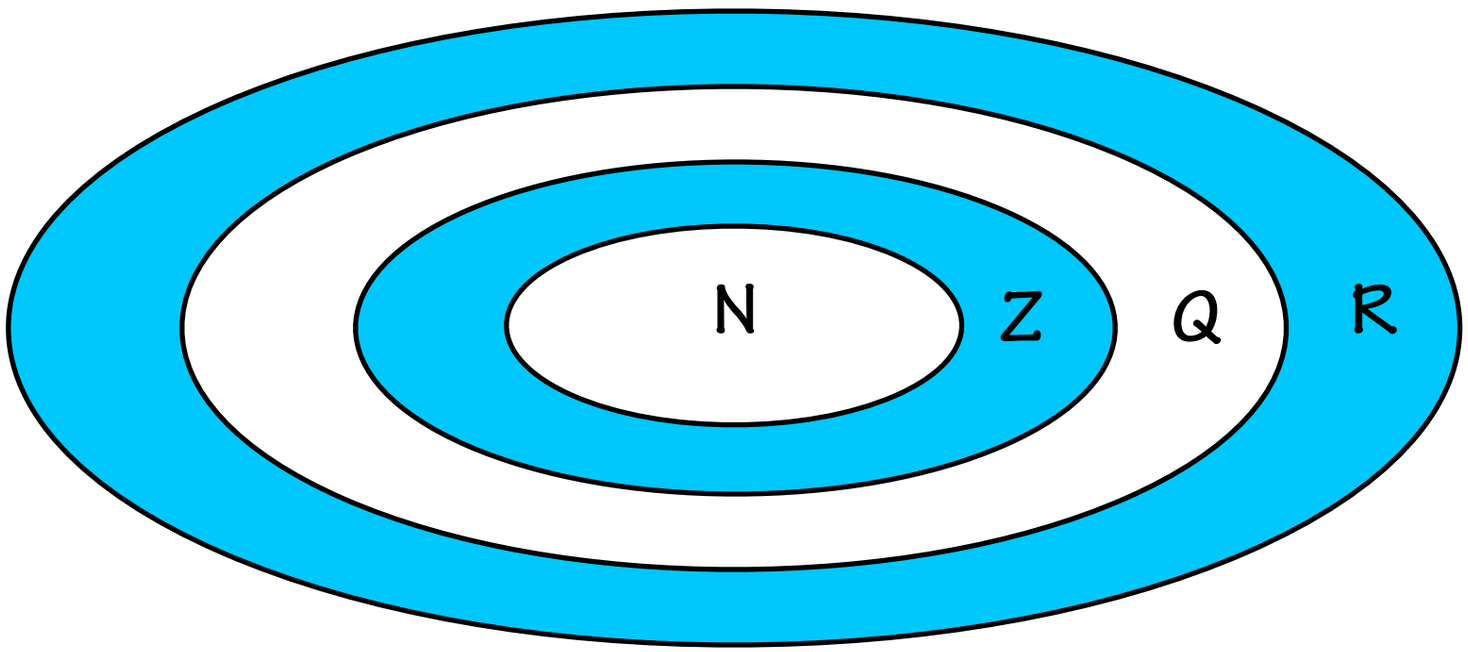
Sets of numbers

- The domain & range of a function are each a subset of a particular larger set of numbers.
- **Natural numbers (N):** {1, 2, 3,4, ……..}
- **Integers (Z):** {-2, -1, 0, 1, 2, ……..}
- **Rational numbers (Q):** Any numbers that can be made from the division of two integers (but not dividing by 0) eg 1/3, - 2.45, 5.787878…..
- **Real numbers (R):** The set of all rational and irrational numbers (includes surds, π, e)

N is a subset of Z

$N \subset Z$

$N \subset Z \subset Q \subset R$

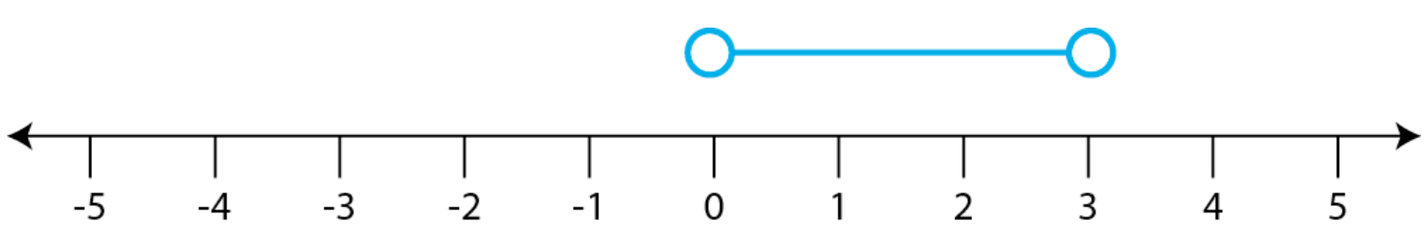


Sets & intervals

- Intervals of the real numbers can be depicted using the appropriate brackets & set notations.
- Square brackets include point, round brackets don't.

Set

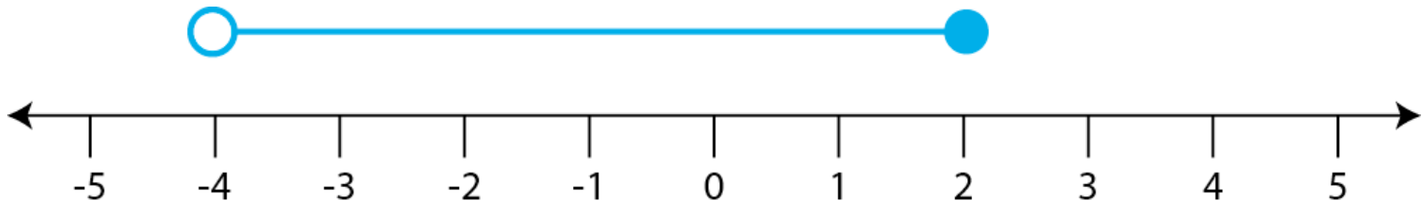
$\{x : 0 < x < 3\}$



Interval

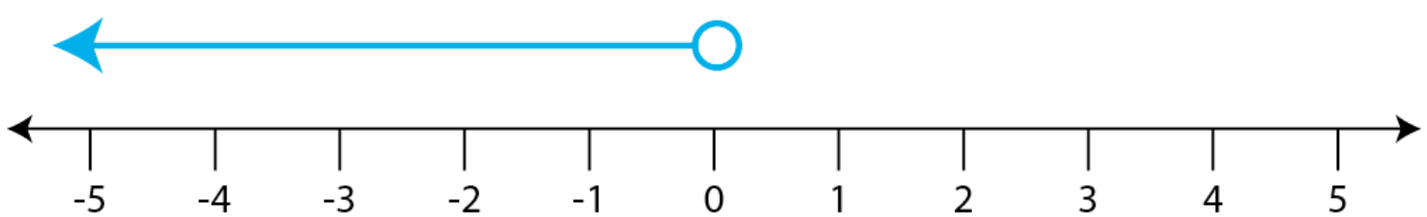
$(0,3)$

$\{x : -4 < x \leq 2\}$



$(-4,2]$

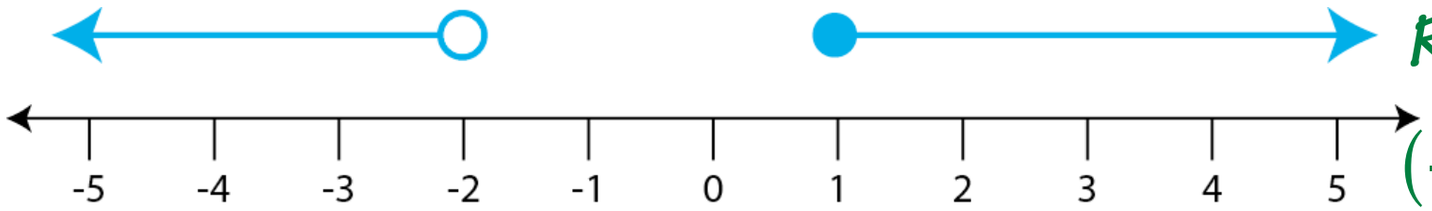
$\{x : x < 0\}$



$(-\infty,0)$

R^-

$\{x : x < -2\} \cup \{x : x \geq 1\}$

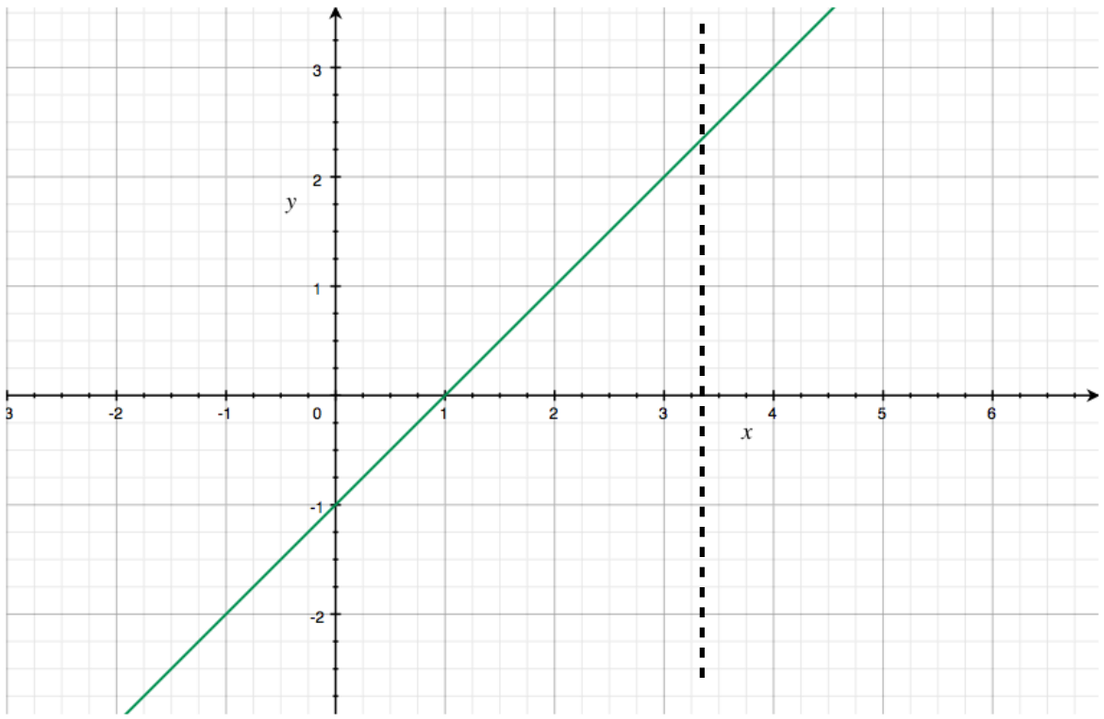


$R \setminus [-2,1)$

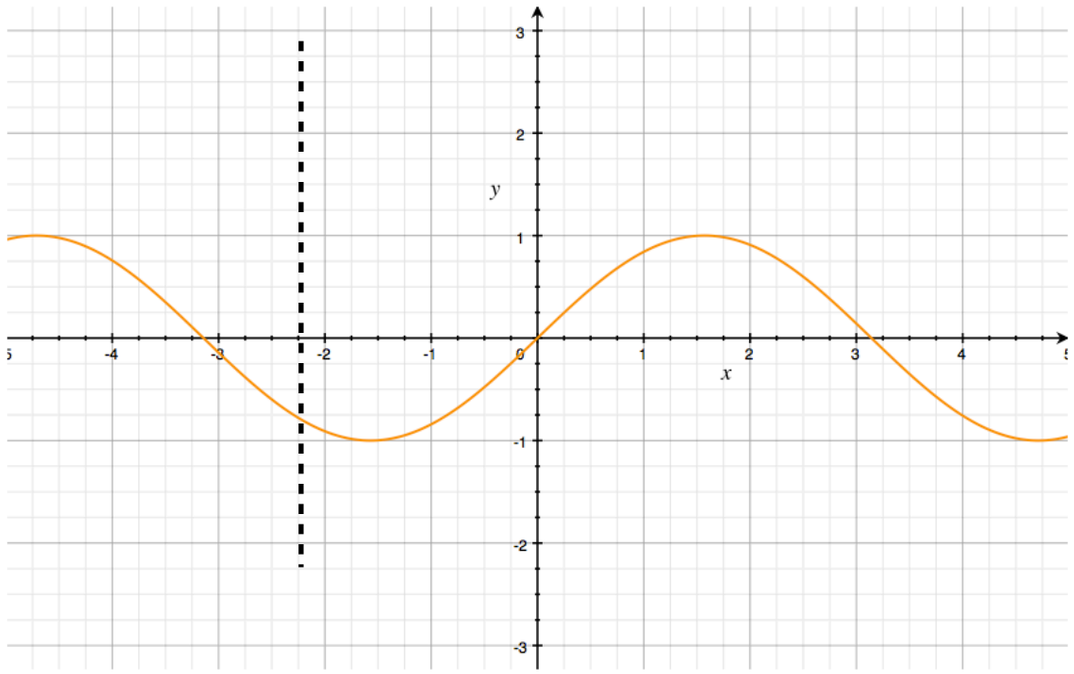
$(-\infty,-2) \cup [1,\infty)$

Functions

- A function is a type of relation where each value of x has a unique value of y.
- Functions can be **one to one** or **many to one** - where more than one x value shares the same y value.
- Examples include linear functions and circular functions (sin x, cos x)
- Functions can be determined by the **vertical line test** - any vertical line will cut through a function only once.



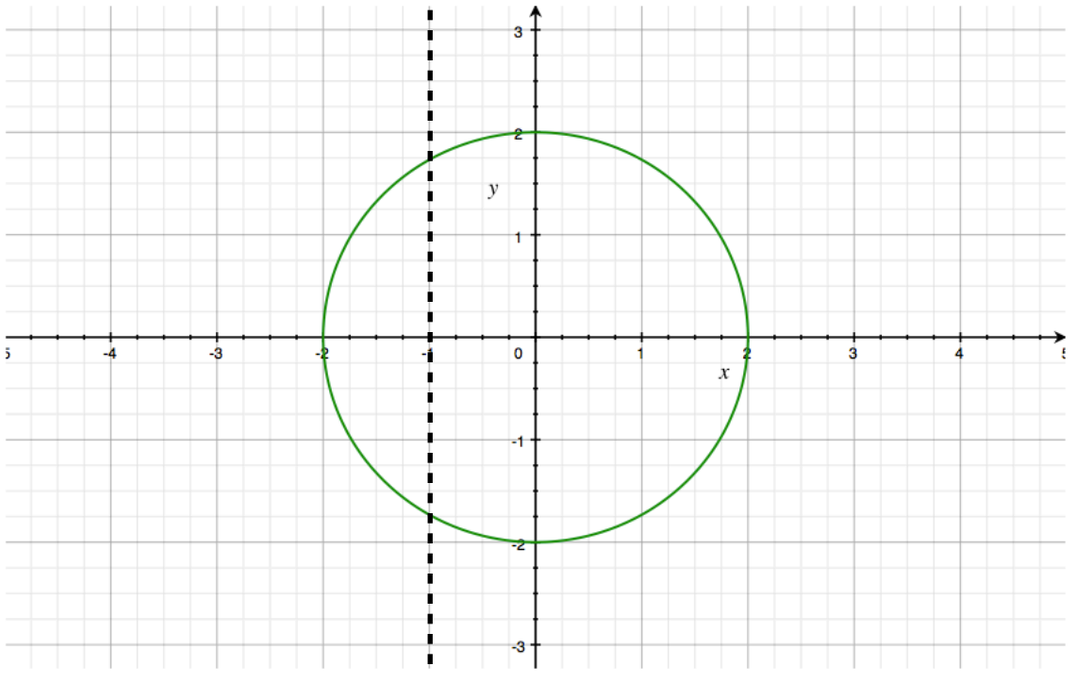
Linear function: one to one



Circular function: many to one

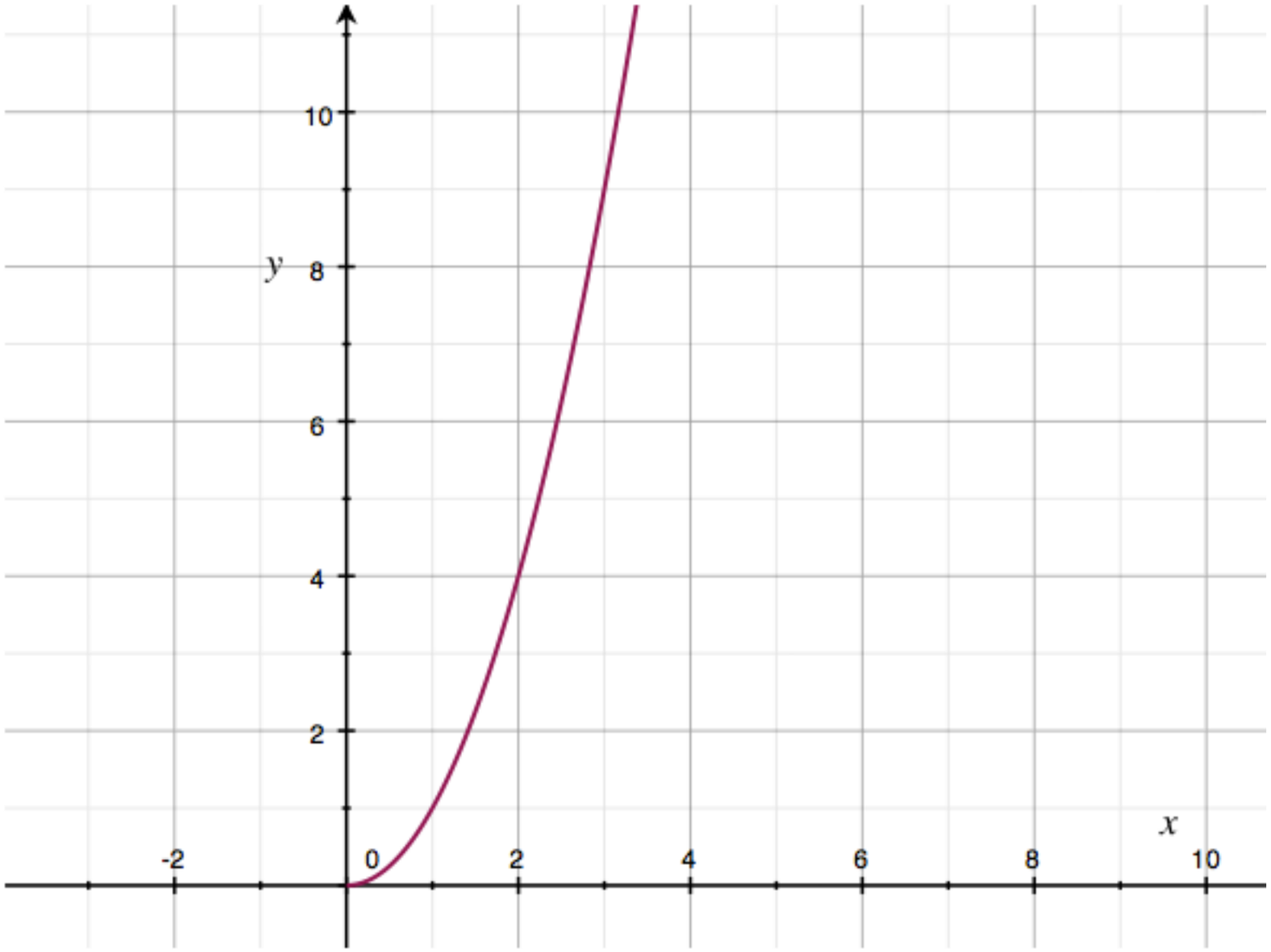
Relations

- A relation can also be **one to many** or **many to many** - where x values can have more than one y value.
- A circle is an example of this of a many to many function.
- A vertical line can cut through this graph more than once.



Function notation

- Function notation is used to describe the domain & any restrictions that might be in place.



Domain (restricted)

Co-domain

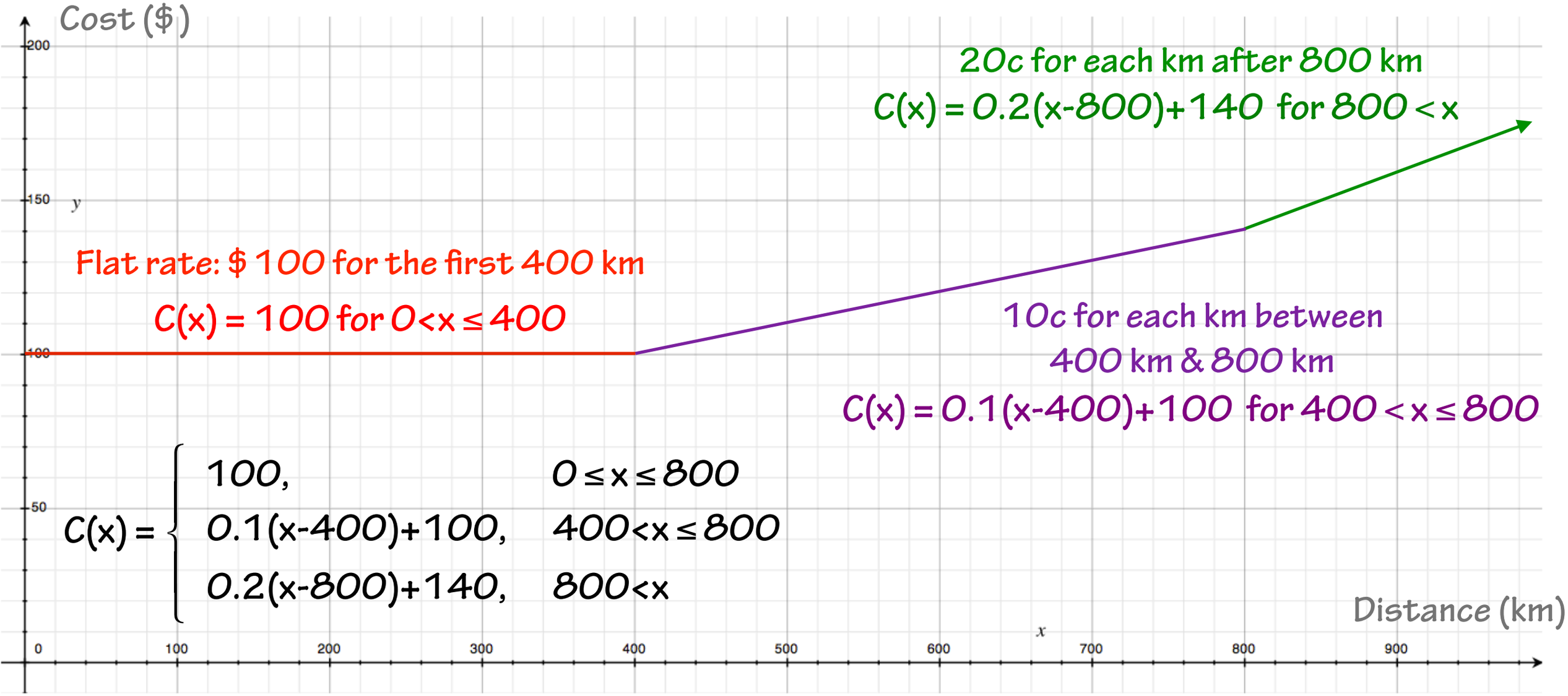
$$f:[0,\infty)\to R, f(x)=x^2$$

Rule

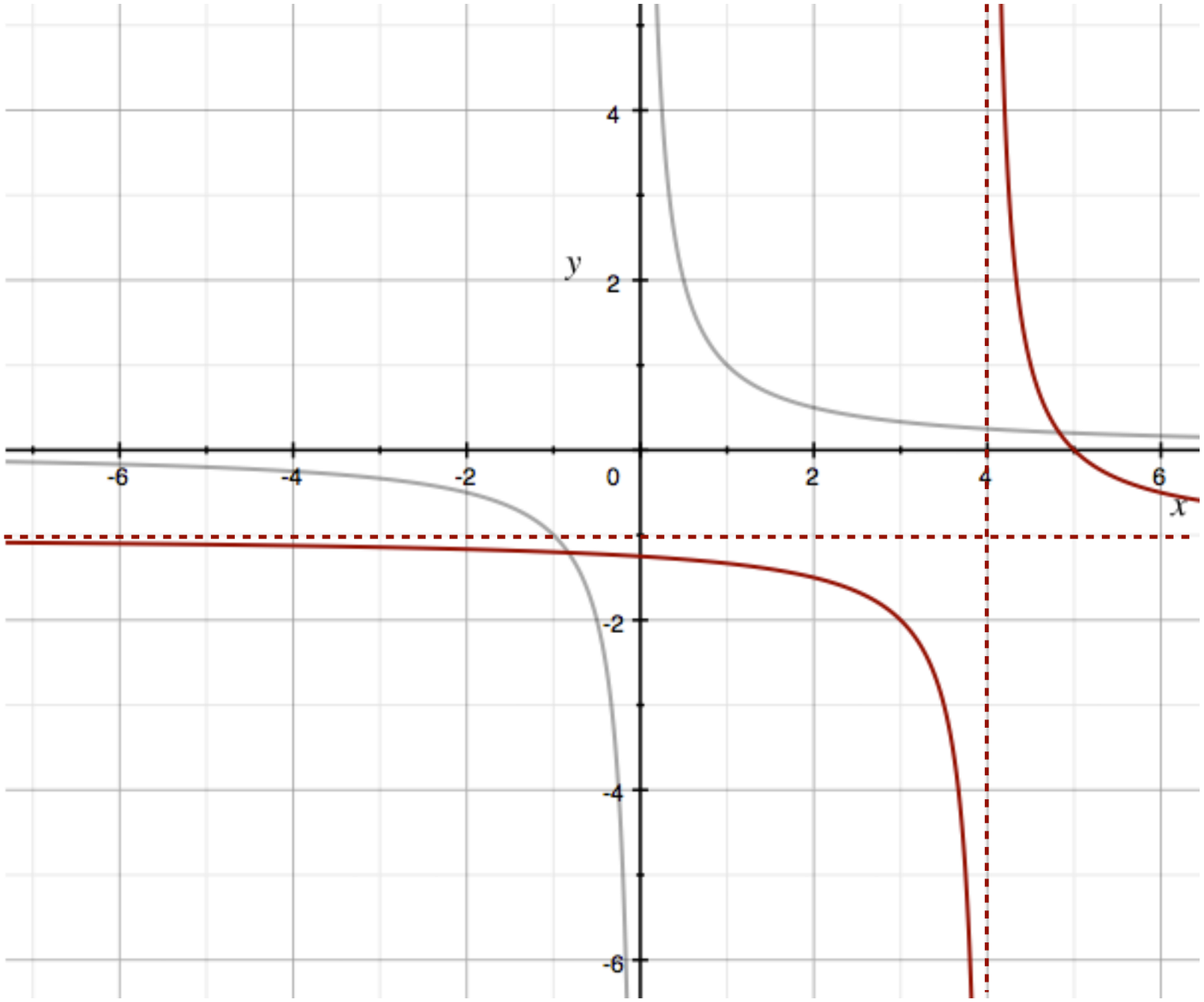
The name of the function

Hybrid functions

- A hybrid (piecewise) function is one where different rules apply over different parts of the domain.
- For example, the cost of car hire differs according to the distance traveled.



Hyperbola



$$y = \frac{1}{x}$$

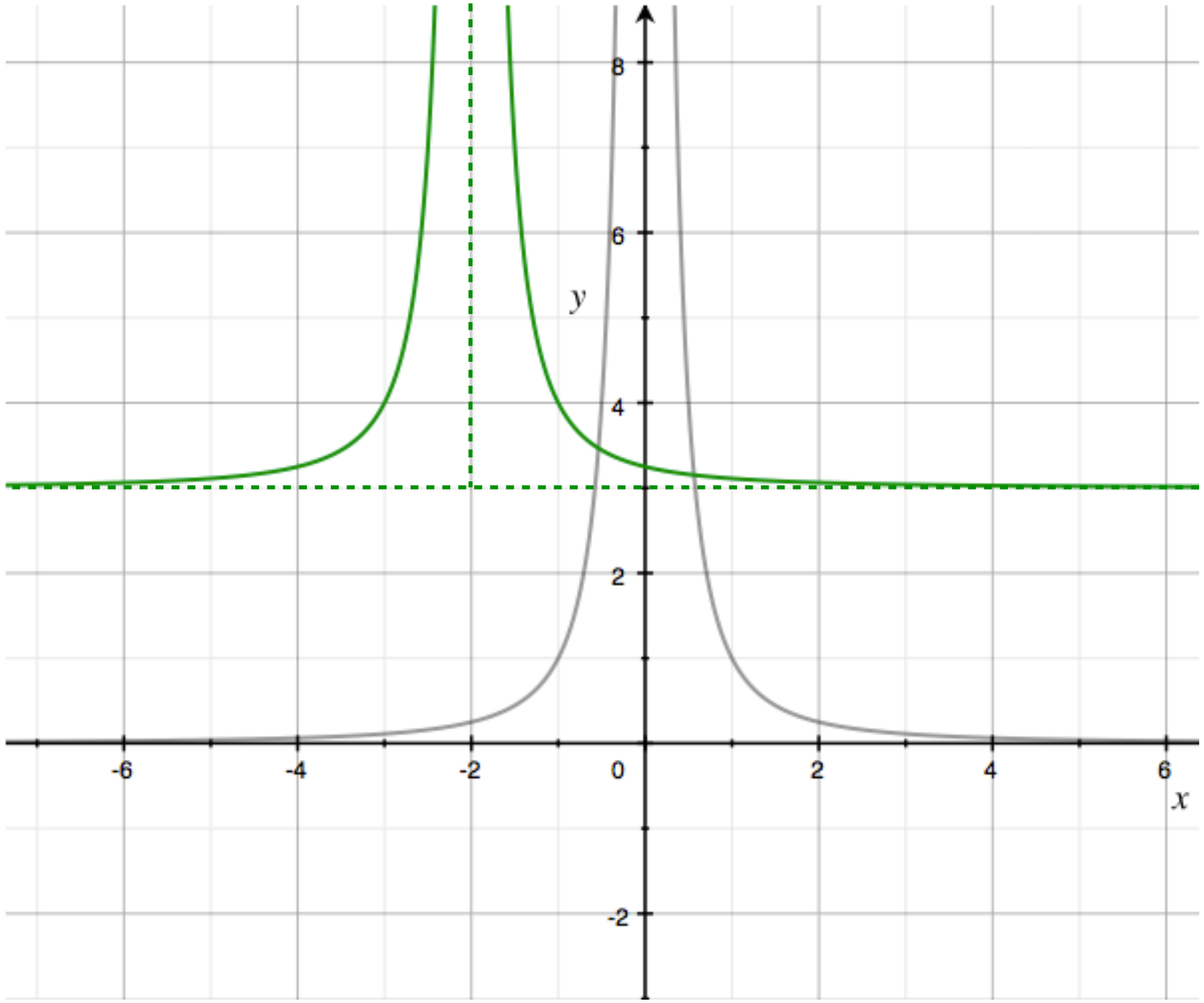
$$y = \frac{1}{(x-h)} + k$$

$$y = \frac{1}{(x-4)} - 1$$

Domain: R\{4}

Range: R\{-1}

Truncus



$$y = \frac{1}{x^2}$$

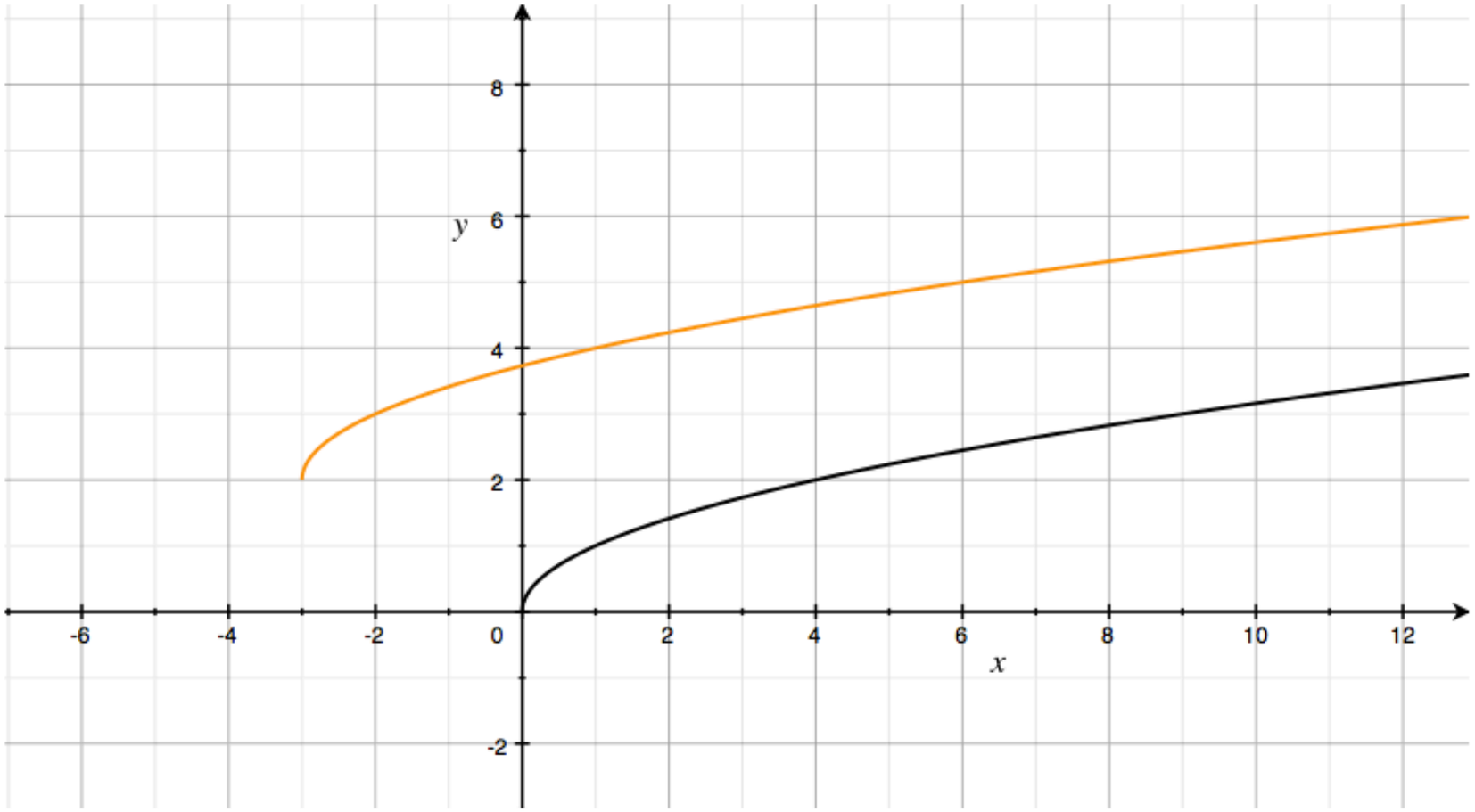
$$y = \frac{1}{(x-h)^2} + k$$

$$y = \frac{1}{(x+2)^2} + 3$$

Domain: R\{-2}

Range: (3,∞)

Square root



$$y = \sqrt{x}$$

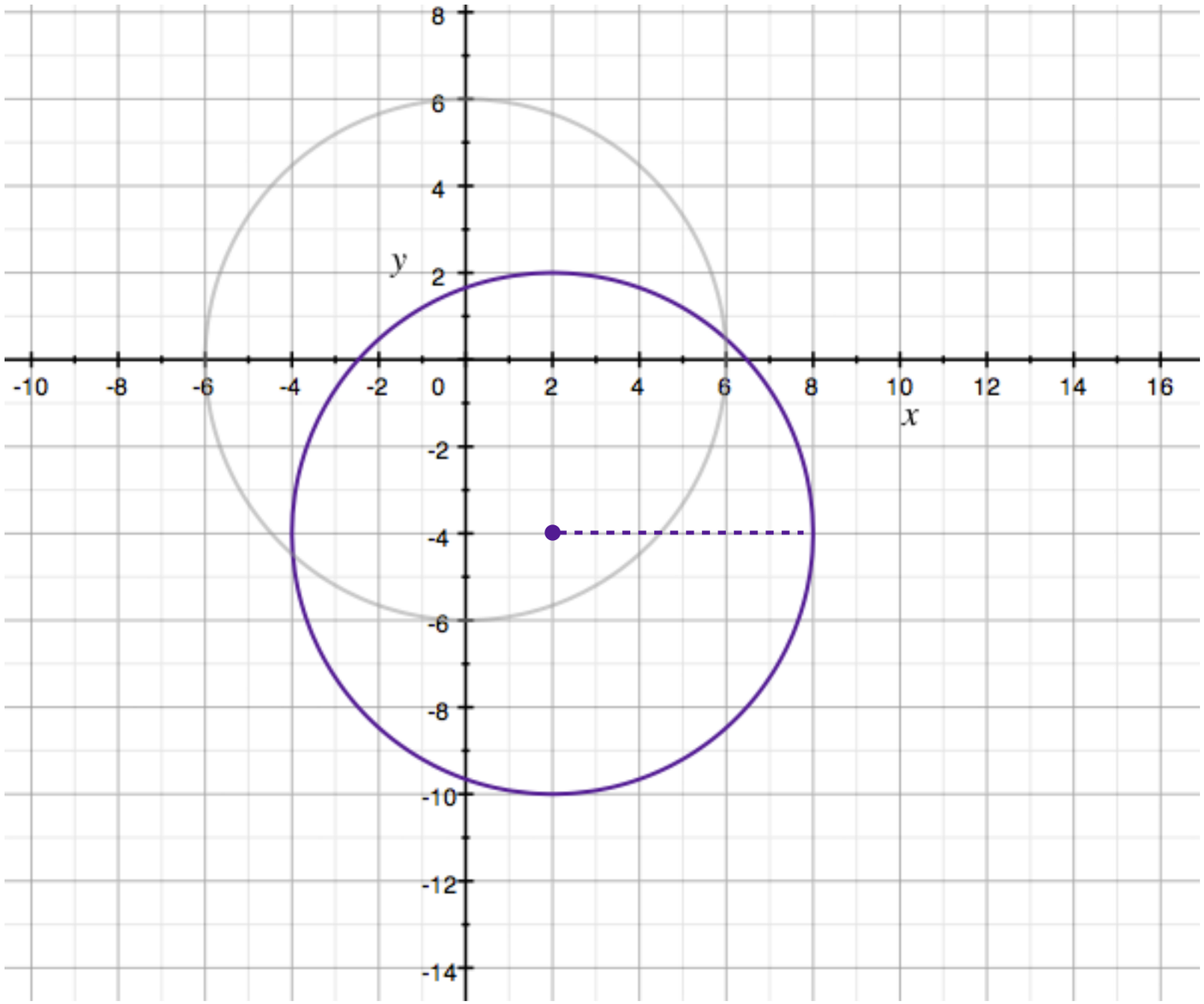
$$y = \sqrt{x-h}+k$$

$$y = \sqrt{x+3}+2$$

Domain: [-3, ∞)

Range: [2,∞)

Circle



$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 36$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y+4)^2 = 36$$

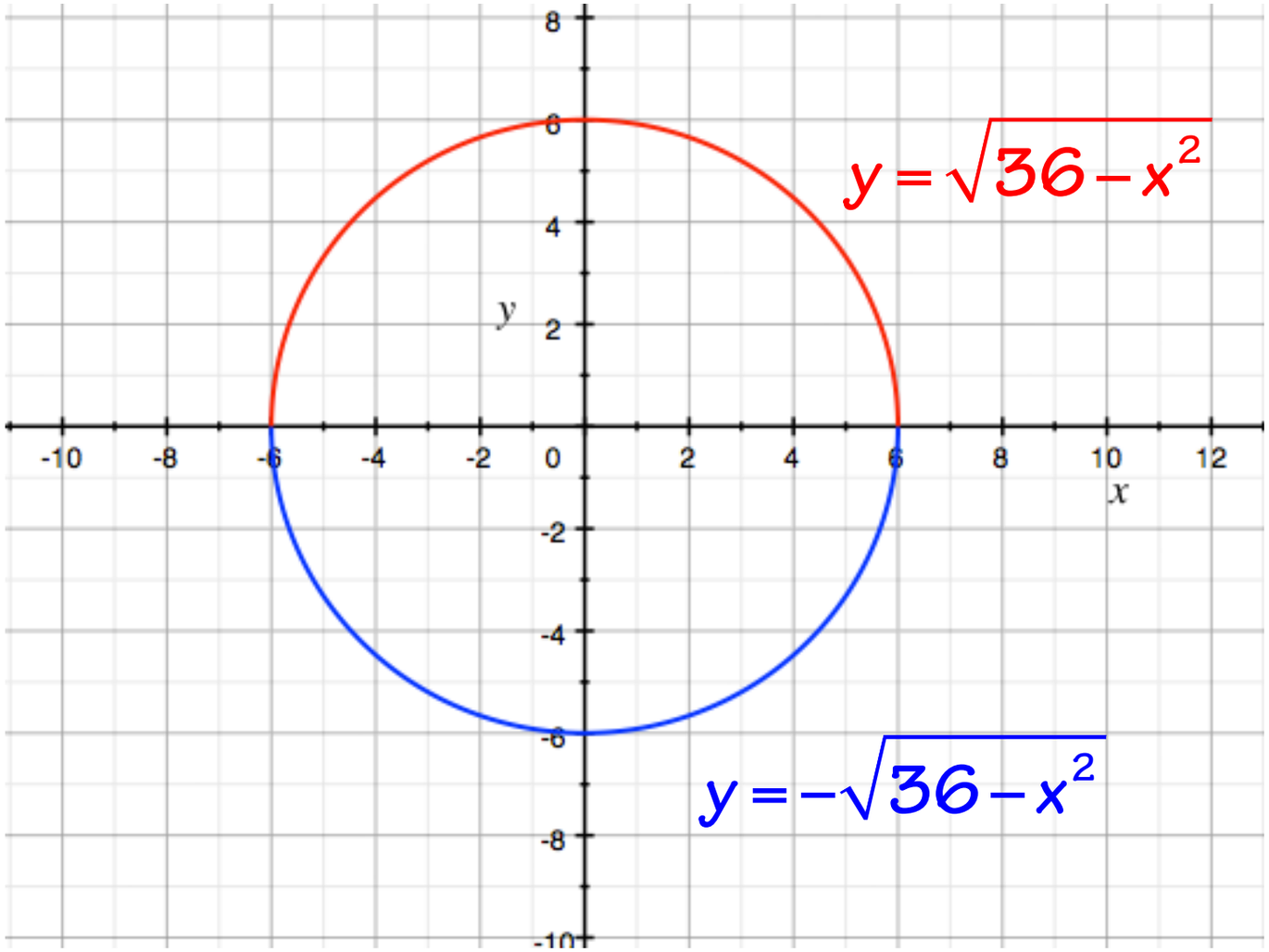
Domain: [-4,8]

Range: [-10,2]

(Diameter = 12)

Circle from functions

- Circles are described by a relation, not a function.
- They can be defined by combining two functions together.



$$x^2+y^2=36$$

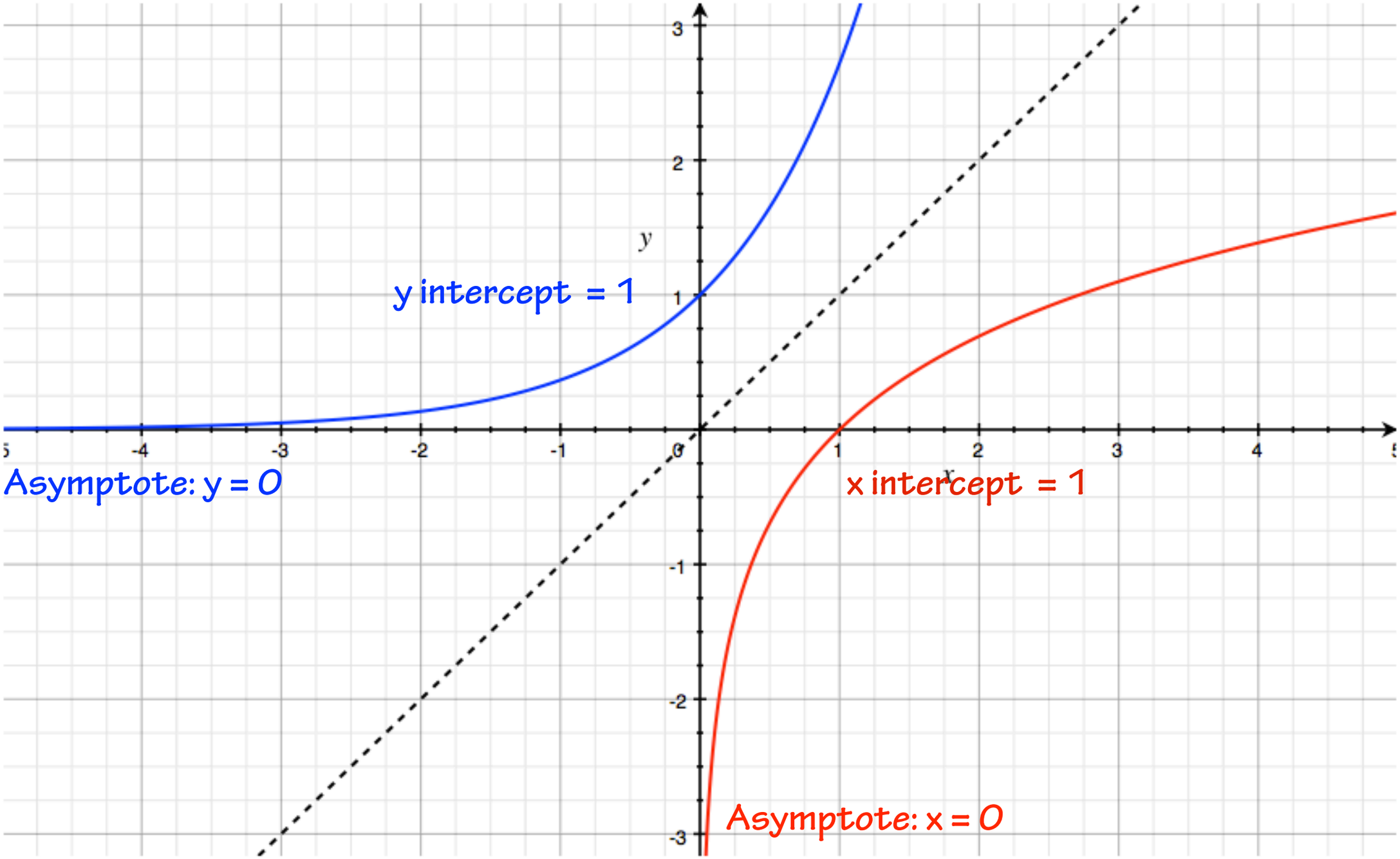
$$y^2=36-x^2$$

$$y=\pm\sqrt{36-x^2}$$

Inverse functions

- Every one to one function has an **inverse function**, $f^{-1}(x)$.
- Inverses are used to work backwards & solve equations.
- The graph of an inverse function can be found from mirroring the graph around the line $y = x$.
- The domain of the inverse $f^{-1}(x)$ is the range of $f(x)$.
- The range of the inverse $f^{-1}(x)$ is the domain of $f(x)$.
- The x intercept of $f^{-1}(x)$ is the y intercept of $f(x)$.
- The y intercept of $f^{-1}(x)$ is the x intercept of $f(x)$.

Inverse functions - from graphs



Inverse functions - from equations

$f(x): y = 2x + 6$

$f^{-1}(x): x = 2y + 6$

$2y = x - 6$

$y = \frac{x}{2} - 3$