Rates of change

- Rates of change
- Average rate of change
- Instantaneous rates of change
- Rates of change from graphs
- Graphs of motion
Rates of change

- The rate of change measures how a variable changes with another.

Examples of rates include:

- A speed of 60 km/h
- A$ 1.05 per $ USD
- 4.9 litres of fuel per 100 km travelled
- 5.2 births per thousand people per year

Rates be found from measuring the gradient of the graph.

- Average rates of change can be calculated from an average between two points.
- Instantaneous rates of change can be calculated from the gradient of the tangent to a curve.
**Average rate of change**

- Approximating the curve with a straight line.
- The **average rate of change** is the gradient of the **chord** (straight line) between two points.

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
m = \frac{50 - 0}{10 - 0} = 5
\]

**Graph:**
- Points: (0,0), (10,50)
- Gradient: \( m = 5 \)
- Rise: 50
- Run: 10
- Equation: \( y = \frac{x^2}{2} \)
Instantaneous rates of change

• To find the instantaneous gradient at a point, a **tangent line** is drawn.
• The tangent line touches the curve at one point.
• It has the same gradient as the curve at the point of contact.

\[ \text{gradient} = \frac{-2.2}{4} = -0.55 \]

\[ \text{rise} = -2.2 \]
\[ \text{run} = 4 \]

\[ x = 2, \text{ tangent touches the curve with the same gradient.} \]
Rates of change from graphs

- The rate of change can be interpreted from the gradient of a graph.

Constant positive rate of change
Rates of change from graphs

Constant negative rate of change
Rates of change from graphs

Increasing positive rate of change
Increasing negative rate of change
Rates of change from graphs

Decreasing positive rate of change
Decreasing negative rate of change
Graphs of motion

- The rate of change of position with respect to time is the velocity.
- The rate of change of velocity with respect to time is the acceleration.
- Constant velocity in the positive direction.
Graphs of motion

- Constant velocity in the negative direction.
Graphs of motion

- Increasing velocity, moving in the positive direction. (Speeding up)
• Decreasing velocity, moving in the positive direction. (Slowing down)
• Walking in front of a motion detector (at position 0 m).