Circular functions

- Radians & degrees
- The unit circle
- Sin, cos & tan
- The unit circle
- Values of sin x, cos x & tan x
- Graphs of sin x & cos x
- Transformations of sin graphs
- Solving circular function equations
- Graphs of tan x
Radian & degrees

- The radian is another measure of angles.
- A circle with a radius of 1 has a circumference of $2\pi$ - this is the basis of the radian measure.
- It is very useful as it is a number with no unit.
- Make sure that your calculator is in radians mode & know how to change it!

\[
\pi = 180^\circ
\]

\[
1 \text{ radian} = \frac{180^\circ}{\pi}
\]

\[
1^\circ = \frac{\pi}{180^\circ}
\]
Radians & degrees

<table>
<thead>
<tr>
<th>Degrees</th>
<th>Radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>$\frac{30\pi}{180} = \frac{\pi}{6}$</td>
</tr>
<tr>
<td>45°</td>
<td>$\frac{45\pi}{180} = \frac{\pi}{4}$</td>
</tr>
<tr>
<td>60°</td>
<td>$\frac{60\pi}{180} = \frac{\pi}{3}$</td>
</tr>
<tr>
<td>90°</td>
<td>$\frac{90\pi}{180} = \frac{\pi}{2}$</td>
</tr>
<tr>
<td>120°</td>
<td>$\frac{120\pi}{180} = \frac{2\pi}{3}$</td>
</tr>
<tr>
<td>135°</td>
<td>$\frac{135\pi}{180} = \frac{3\pi}{4}$</td>
</tr>
<tr>
<td>180°</td>
<td>$\frac{180\pi}{180} = \pi$</td>
</tr>
<tr>
<td>270°</td>
<td>$\frac{270\pi}{180} = \frac{3\pi}{2}$</td>
</tr>
<tr>
<td>360°</td>
<td>$\frac{360\pi}{180} = 2\pi$</td>
</tr>
</tbody>
</table>
Sin, cos & tan

\[ \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \]
\[ \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \]
\[ \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin(\theta)}{\cos(\theta)} \]
**Sin, cos & tan - special angles**

\[
\sin(45°) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}
\]

\[
\cos(45°) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}
\]

\[
\tan(45°) = \frac{\text{opposite}}{\text{adjacent}} = 1 = 1
\]

\[
\sin(30°) = \frac{1}{2}
\]

\[
\cos(30°) = \frac{\sqrt{3}}{2}
\]

\[
\tan(30°) = \frac{1}{\sqrt{3}}
\]

\[
\sin(60°) = \frac{\sqrt{3}}{2}
\]

\[
\cos(60°) = \frac{1}{2}
\]

\[
\tan(60°) = \frac{\sqrt{3}}{1} = \sqrt{3}
\]
The unit circle

\[ y = \sin \theta \]
\[ x = \cos \theta \]

\[ \sin^2(\theta) + \cos^2(\theta) = 1 \]

\[ \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \]

\[ 0.71^2 + 0.71^2 = 1 \]
The unit circle - Symmetry

\[ \sin \left( \frac{\pi}{2} - \theta \right) = \cos(\theta) \]
\[ \cos \left( \frac{\pi}{2} + \theta \right) = -\sin(\theta) \]
\[ \cos \left( \frac{\pi}{2} - \theta \right) = \sin(\theta) \]

\[ \sin \left( \frac{\pi}{2} + \theta \right) = \cos(\theta) \]
\[ \cos \left( \frac{\pi}{2} + \theta \right) = -\sin(\theta) \]
The unit circle - Symmetry

\[ \sin(\pi - \theta) = \sin(\theta) \]
\[ \cos(\pi - \theta) = -\cos(\theta) \]
\[ \sin(\pi + \theta) = -\sin(\theta) \]
\[ \cos(\pi + \theta) = -\cos(\theta) \]
\[ \sin(2\pi - \theta) = -\sin(\theta) \]
\[ \cos(2\pi - \theta) = \cos(\theta) \]
### Values of $\sin x$, $\cos x$ & $\tan x$

<table>
<thead>
<tr>
<th>Angle</th>
<th>Sin $\theta$</th>
<th>Cos $\theta$</th>
<th>Tan $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>30°</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
</tr>
<tr>
<td>45°</td>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{1}{\sqrt{2}}$</td>
<td>$\frac{1}{\sqrt{2}}$</td>
</tr>
<tr>
<td>60°</td>
<td>$\frac{\pi}{3}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>90°</td>
<td>$\frac{\pi}{2}$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>120°</td>
<td>$\frac{2\pi}{3}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>135°</td>
<td>$\frac{3\pi}{4}$</td>
<td>$\frac{1}{\sqrt{2}}$</td>
<td>$-\frac{1}{\sqrt{2}}$</td>
</tr>
<tr>
<td>180°</td>
<td>$\pi$</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>270°</td>
<td>$\frac{3\pi}{2}$</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>360°</td>
<td>$2\pi$</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Graphs of \( \sin x \) & \( \cos x \)

\[
\begin{align*}
\sin \frac{\pi}{2} &= 1 \\
\cos \frac{\pi}{2} &= 0 \\
\sin \pi &= 0 \\
\cos \frac{3\pi}{2} &= 0 \\
\sin \frac{3\pi}{2} &= -1 \\
\cos \pi &= -1 \\
\sin 2\pi &= 0 \\
\cos 2\pi &= 1
\end{align*}
\]

\[
\begin{align*}
\cos \left( x \right) &= \sin \left( x + \frac{\pi}{2} \right) \\
\sin \left( x \right) &= \cos \left( x - \frac{\pi}{2} \right)
\end{align*}
\]
Transformations of sin graphs

\[ y = 2 \sin x \]
Transformations of sin graphs

$y = \sin 2x$
Transformations of sin graphs

$y = \sin x + 2$
Transformations of sin graphs - summary

\[ y = a \sin b x + c \]

- \( a \): amplitude of graph
  - The height that the graph goes above & the midpoint.

- \( b \): period factor
  - The period of the function is found from \( \frac{2\pi}{b} \).

- \( c \): vertical translation
  - The graph is shifted up by \( c \) units.
Solving circular function equations

$2\sin 4x = 1 \quad (0 < x < \pi)$

Next two solutions: one period after the first two

$\frac{\pi}{24} - \frac{\pi}{24} = \frac{6\pi}{24} - \frac{\pi}{24}$

$x = \frac{5\pi}{24} \quad (37.5^\circ)$

$x = \frac{\pi}{24} \quad (7.5^\circ)$

$\frac{\pi}{24} + \frac{\pi}{24} = \frac{\pi}{24} + \frac{12\pi}{24} = \frac{13\pi}{24} \quad (97.5^\circ)$

$\frac{5\pi}{24} + \frac{\pi}{24} = \frac{5\pi}{24} + \frac{12\pi}{24} = \frac{17\pi}{24} \quad (127.5^\circ)$
Graphs of $\tan x$

- Vertical asymptote: $\frac{\pi}{2}$
- $y = \tan x$
- Period $= \pi$
- $\tan \frac{\pi}{4} = 1$
- $\tan \theta = \frac{\sin \theta}{\cos \theta}$

As $\theta \to \frac{\pi}{2}$, $\cos \theta \to 0$, $\tan \theta \to \infty$
Graphs of $\tan x$

$y = 2\tan x$

This graph is dilated from the x-axis by a factor of 2.

The period is still the same: $\pi$
Graphs of $\tan x$

This graph now has a period of $\pi/2$. 

$y = \tan 2x$
Transformations of sin graphs - summary

\[ y = a \tan bx + c \]

\[ a = \text{dilation factor of graph} \]
\[ a > 1, \text{the graph is steeper.} \]
\[ 0 < a < 1, \text{the graph is less steep.} \]

\[ b = \text{period factor} \]

\[ \text{The period of the tan function is found from } \frac{\pi}{b}. \]

\[ c = \text{vertical translation} \]
\[ \text{The graph is shifted up by } c \text{ units.} \]