Jules Lamers

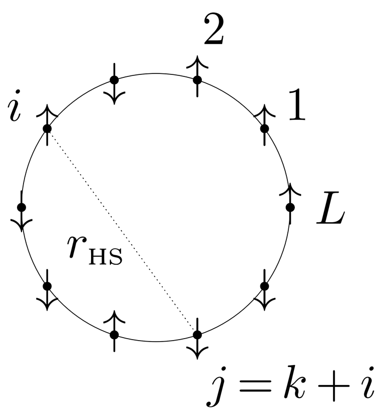
The partially isotropic Haldane–Shastry spin chain

building on Uglov [hep-th/9508145]

$J > 0$ ferromagnetic
$J < 0$ antiferromagnetic

long-range pairwise form

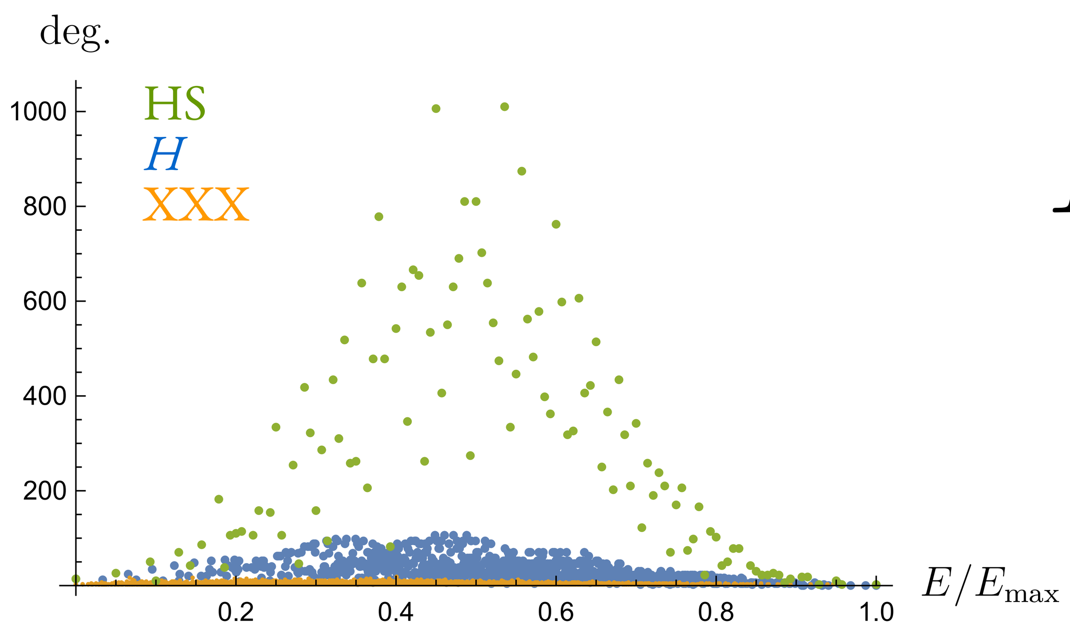
pair potential

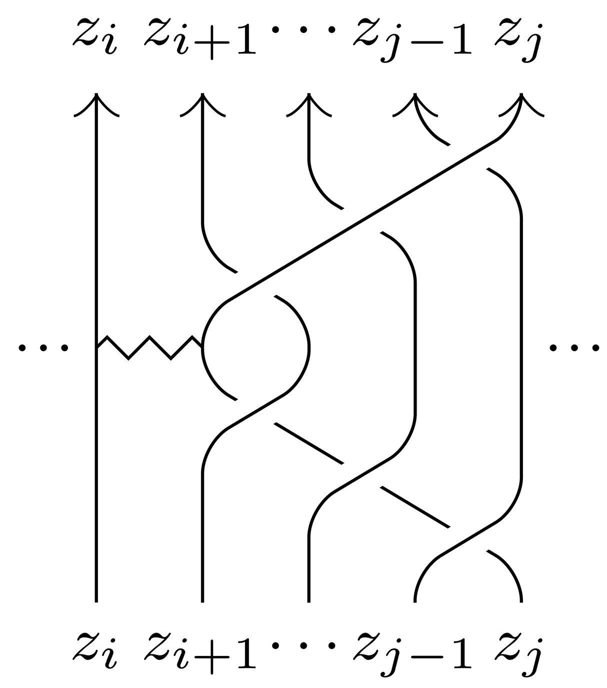


$$V = \frac{1}{r_+ r_-}$$ point splitting of $V_{\rm HS} = 1/r_{\rm HS}^2$

$$r_\pm = 2\sin\!\left(\tfrac{2\pi}{L}k \pm \mathrm{i}\,\gamma\right)$$

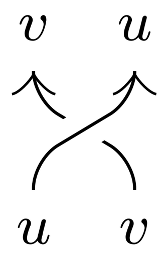
$$H = -J \sum_{i<j}^{L} V(i-j)\, S_{ij}$$



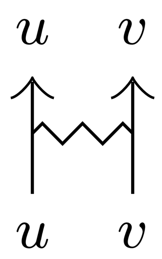
spin interactions $S_{ij} = \cdots$  $\cdots$

with $z_k = \mathrm{e}^{2\pi \mathrm{i}\, k/L}$

built from R-matrix:



$$= \check{R}\!\left(\frac{v}{u}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{(q-q^{-1})\,v}{q\,v-q^{-1}u} & \frac{v-u}{q\,v-q^{-1}u} & 0 \\ 0 & \frac{v-u}{q\,v-q^{-1}u} & \frac{(q-q^{-1})\,u}{q\,v-q^{-1}u} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

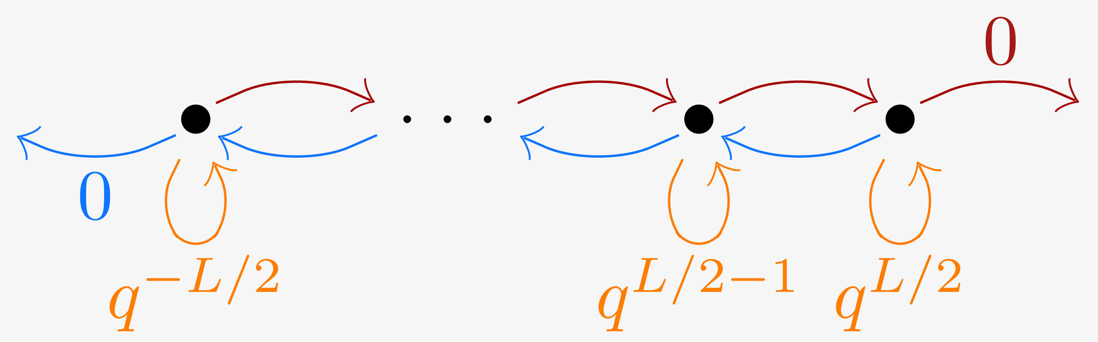


$$= (q-q^{-1})\,\check{R}'(1) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -q^{-1} & 1 & 0 \\ 0 & 1 & -q & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

cf XXZ: $u = \mathrm{e}^{2\lambda},\, q = \mathrm{e}^{\gamma},\, \Delta = \frac{q+q^{-1}}{2}$

Key properties

- isotropic limit $q = \mathrm{e}^{\gamma} \to 1$: *Haldane–Shastry*
- for all q *partial isotropy* $[S^z, H] = 0$
- for all L highly degenerate spectrum due to *infinitely many symmetries*: $U_q(\widehat{\mathfrak{sl}_2})$, e.g. containing q-analogue of spin: $U_q(\mathfrak{sl}_2)$

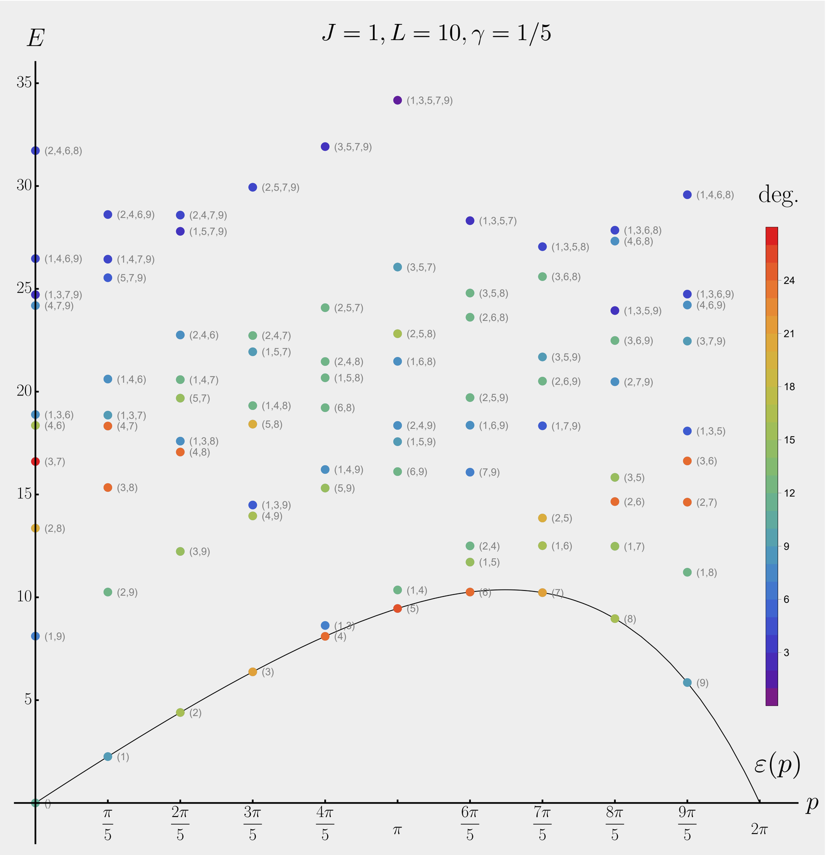


S_q^+, S_q^- } q-ladder operators

$S_q^z = q^{S^z}$

Exact spectrum

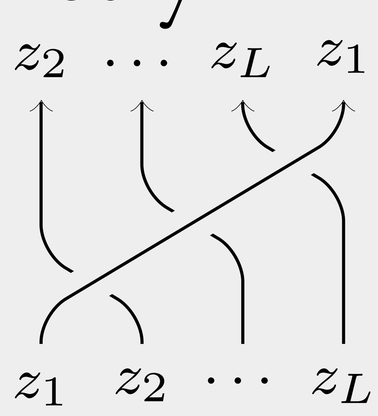
- allowed quasimomenta: simple combinatorics
- additive energy
- additive q-momentum
- dispersion q-deforms $\varepsilon_{\rm HS}(m) = J\,m\,(L-m)/2$



'motif': $L-1 \times$ ○ or ●; ~~●●~~		E	p mod 2π	$U_q(\widehat{\mathfrak{sl}_2})$-irrep	q-spin	multiplet
$(\,)$	○○○○○○○○○	0	0	□□□□□□□□□	$\frac{L}{2}$	q-ferromagnetic
(m)	○○●○○○○○○ (m)	$\varepsilon(m)$	$\frac{2\pi}{L}\,m$	□ ⊗ ⊟ ⊗ □□□□□	$(\frac{L}{2}-1) \oplus \cdots$	(affine) q-magnon
$\vdots$	○○●○○●○○ (m_1 m_2); ○●○●○○●○○ (m_1 m_2 m_3)	$\sum_r \varepsilon(m_r)$	$\frac{2\pi}{L}\sum_r m_r$	● ⇝ ⊟; ○⋯○ ⇝ □⋯□ } ⊗	$\vdots$	higher excitations; q-spinons
$(1,\cdots,L-1)$	●○●○●○●○●	E/J max	$\frac{L\pi}{2}$	⊟ ⊗ ⊟ ⊗ ⊟ ⊗ ⊟	0	q-antiferromagnetic

Quirks

- hermitian for $q \in \mathbb{R}$
- multi-spin interactions
- no translational symmetry yet q-homogeneous: commutes with q-shift operator



- chiral: no left/right symmetry

more info?
julesl@chalmers.se
CHALMERS