An Iterative Learning Control Synthesis for Nonlinear Systems with Hard Input and Output Constraints

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***Abstract*— Engineered systems are always subjected to operational constraints that limit the range of feasible control input signals and their output signals. This paper proposes an iterative learning control (ILC) structure that can satisfy hard input and output constraints simultaneously for a class of nonlinear systems. This structure enables the decoupling between the design of feed-forward ILC and the output feedback. The role of feed-forward ILC is to track the desired trajectory under repetitive environment while the output feedback is added to handle output constraints with the help of a barrier Lyapunov function. The concept of virtual output constraints is proposed to ensure that the output constraints can be satisfied within the input limits by shifting and scaling the original barrier Lyapunov function. The proposed algorithm is able to ensure the perfect tracking performance and satisfaction of both input and output hard constraints. Simulation results are presented to demonstrate the effectiveness of the proposed method.**

I. INTRODUCTION

Iterative Learning Control (ILC) is an advanced tracking control algorithm that aims to achieve a perfect tracking performance without a complete knowledge of the dynamic system under repetitive environment. Such a technique finds its applications in chemical batch processes [1], precision motion control [2], energy generation [3], industrial robots [4] and so on, see for example [5], [6] and references therein.

The key idea of ILC is to learn from experience, that is, by using information obtained from experience (or past iterations), the control performance can improve over iterations. It is a prominent data-driven approach as it can work effectively even for poorly modeled systems. Many ILC algorithms have been proposed over last decades, among them, feed-forward ILC algorithms are simple to design and implement, leading to a lot of implementations in industry [7].

Operational constraints such as actuator hard constraints, sensor hard constraints always exist, though they have been long ignored in the majority of control design and analysis. When the model of dynamic system as well as the information of constraints are known *a priori*, the model-based inversion or compensation methods can be used [8]–[10]. However, in the data-driven methods such as ILC algorithms, handling hard constraints from both input and output becomes challenging.

When discrete-time systems are considered, constrained optimization based techniques can handle hard constraints by using the super vector formulation as shown in [11]–[13]. It becomes challenging for continuous-time systems as it is an optimization problem in an infinite-dimensional space. There are some ILC algorithms that are able to handle either hard input constraints or hard output constraints for continuous-time plant, [14], [15], but limited results are available that handles both input and output hard constraints for continuous-time systems.

In literature, barrier Lyapunov functions (BLF), which will approach to infinity when the output approach the constraint boundary, have been used to design an output feedback law to avoid reaching the hard output constraints [9]. The output feedback uses the derivative of some BLF to drive the system away from the output constraints. This may lead to a very large control effort due to a large derivative of the BLF close to the boundary of the constraints. When the control input is also constrained, the output signal might not be able to "stop" before it hits the constraints.

This paper proposes a new structure that is able to handle both input and output constraints separately. In order to deal with both hard input and output constraints, the concept of virtual output constraints is proposed. This virtual output constraints includes a scaling and a shifting of the original ones. The choice of scaling and shifting is dependent on the size of input constraints. By playing with the scaling and shifting gains, it is possible to show that the output feedback control, which is designed on the basis of the BLF for virtual output constraints, can satisfy both input and output constraints at the cost of a conservative design. Using a new barrier composite energy function, the convergence of ILC in the presence of constraints is achieved for a class of nonlinear affine continuous-time dynamic systems under appropriate assumptions.

It is noted that the system of interests is nonlinear affine. In order to simplify the analysis, the weak detectability assumption is used to ensure that a bounded output will lead to a bounded state without generating any unstable zero-dynamics. Under this assumption, an output feedback is sufficient to ensure the boundedness of trajectories in both finite-time domain and iteration domain. The effectiveness of the proposed method is demonstrated by a simulation example.

II. PROBLEM FORMULATION

The notations used in this work are introduced in this section. Let $\mathcal{R}$ denote the set of real numbers and $\mathcal{N}$ denote the set of natural numbers. The set of all continuous functions in the finite interval $t \in [0,\ T_f]$ that are differentiable up to j^{th} order is denoted by $\mathcal{C}^j[0,\ T_f]$ for any $j \in \mathcal{N}$. For a given positive constant ϵ, the order of magnitude in terms of ϵ is represented by $\mathcal{O}(\epsilon)$.

For a vector $\mathbf{x} \in \mathcal{R}^n$, $|\mathbf{x}|^2 \triangleq \mathbf{x}^\intercal\mathbf{x}$. A vector is called positive ($\mathbf{x} > 0$) if each element is positive. For any $\mathbf{x}(t) \in \mathcal{C}[0,\ T_f]$, the supremum norm is defined as $\|\mathbf{x}\|_s \triangleq \max_{t\in[0,\,T_f]} |\mathbf{x}(t)|_\infty$, where $|\mathbf{x}|_\infty = \max_{j\in[1,\ldots,n]} |x^j|$ and x^j denotes j^{th} element of $\mathbf{x}$. The norm $\|\mathbf{x}(t)\|_s$ is defined as $\|\mathbf{x}(t)\|_s = \max_{s\in[0,t]} |\mathbf{x}(t)|_\infty$. The $\mathcal{L}^2$ norm is defined as

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$\|\mathbf{x}\|_{\mathcal{L}^2} \triangleq \left(\int_0^{T_f} |\mathbf{x}(\tau)|^2 d\tau\right)^{\frac{1}{2}}$.

For a given matrix $A \in \mathcal{R}^{n\times m}$, $|A|$ indicates its induced matrix norm. A square matrix $A = A^\intercal > (\geq)0$ indicates that this matrix is symmetric and positive definite (positive semi-definite). For a square symmetric matrix A, $\lambda_{min}(A)$ stands for its minimum eigenvalue. The notation I_m denotes the identity matrix of dimension m.

This paper considers a nonlinear multiple-input-multiple-output square[1] affine system:

$$\begin{array}{rcl} \dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{x}) + G(\mathbf{x})\mathbf{u} \\ \mathbf{u} & = & \mathrm{sat}(\mathbf{v}, \mathbf{u}^*) \\ \mathbf{y} & = & \mathbf{h}(\mathbf{x}), \end{array} \quad , \text{ with } \mathbf{x}(0) = \mathbf{x}^0 \in \mathcal{R}^n, \tag{1}$$

where $\mathbf{x} \in \mathcal{R}^n$, $\mathbf{y} \in \mathcal{R}^m$ and $\mathbf{v} \in \mathcal{R}^m$ are the system state, output and input respectively. The nonlinear mappings: $\mathbf{f}(\cdot) : \mathcal{R}^n \to \mathcal{R}^n$, $G(\cdot) : \mathcal{R}^n \to \mathcal{R}^{n\times m}$ and $\mathbf{h}(\cdot) : \mathcal{R}^n \to \mathcal{R}^m$ are all locally Lipschitz continuous[2]. Let $u \in R$ and $u^* > 0$. The saturation function is defined as $\mathrm{sat}(u, u^*) \triangleq sign(u)\, min\{u^*, |u|\}$ for any $u \in \mathcal{R}$ where $u^* > 0$ is a scalar constant. For any $\mathbf{u} \in \mathcal{R}^m$ and a positive vector $\mathbf{u}^*$, the saturation function is defined as $\mathrm{sat}(\mathbf{u}, \mathbf{u}^*) = \left[\mathrm{sat}(u^1, u^{1^*}), \cdots, \mathrm{sat}(u^m, u^{m^*})\right]^\intercal$.

The *control objective* is to find a sequence of control input $\{\mathbf{u}_i(t)\}_{i\in\mathcal{N}}$ such that the output of the system (1) converges to a given reference output $\mathbf{y}_r \in \mathcal{C}^1[0, T]$. Moreover, this system is subjected to hard input and output constraints, which are known *a priori*. The input constraint is represented by the saturation function, $\mathrm{sat}(\mathbf{v}, \mathbf{u}^*)$ with $\mathbf{u}^* > 0$ and the output constraints is defined by the norm inequality $|\mathbf{y}(t)| \leq k_b$, for all $t \in [0, T_f]$ for some given $k_b > 0$.

As the tracking problem is considered, it is convenient to change the output constraints to tracking error constraints. The tracking error, $\mathbf{e}(t)$ is defined as $\mathbf{e}(t) \triangleq \mathbf{y}_r(t) - \mathbf{y}(t)$. That indicates, for any k_b and $\mathbf{y}_r(t)$, there exists an $0 < \varepsilon_b^* < k_b$ such that for any $\varepsilon_b < \varepsilon_b^*$, if $|\mathbf{e}(t)| \leq \varepsilon_b$, the output constraint of $|\mathbf{y}(t)| \leq k_b$ is satisfied. Even though such a conversion is conservative, it can simplify the design of feedback control as discussed in [16].

To simplify the analysis, it is assumed that the system (1) satisfies the following assumptions.

Assumption 1: The system (1) has a relative degree of one. □

Remark 1: This information is needed for the design of feed-forward ILC when using Contraction Mapping methods. The discussion on the role of relative degree in ILC design can be found in [17], [18]. Under such an assumption, it is also natural to assume that there exist two positive constants (η_a, η_b), the following inequality holds for all $\mathbf{x} \in \mathcal{D}_\mathbf{x} \subseteq \mathcal{R}^n$:

$$\eta_b I_n \leq \frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}) G(\mathbf{x}) \leq \eta_a I_n. \tag{2}$$

○

Assumption 2: The system (1) is *bounded-output-bounded-state* (BOBS) stable. The state trajectories of the system (1) satisfy:

$$|\mathbf{x}(t)| \leq \gamma(\|\mathbf{y}(t)\|) + C_y \tag{3}$$

where $\gamma(\cdot)$ is a class $\mathcal{K}$ function[3] and $C_y > 0$ is a scalar constant. □

Remark 2: Assumption 2 indicates a weak detectability for the nonlinear system (1). It is weaker than the well-known output-to-state stability (OSS) [20]. This assumption indicates that unstable zero-dynamics will not happen and an output feedback is sufficient enough to ensure the uniform boundedness of the state. ○

Assumption 3: For a given compact set $\mathcal{D}_\mathbf{x} \subset \mathcal{R}^n$, there exist positive constants $C_f = C_f(\mathcal{D}_\mathbf{x})$, $C_g = C_g(\mathcal{D}_\mathbf{x})$, $C_h = C_h(\mathcal{D}_\mathbf{x})$ and $C_{h_x} = C_{h_x}(\mathcal{D}_\mathbf{x})$ such that for any $\mathbf{z}_1,\ \mathbf{z}_2 \in \mathcal{D}_\mathbf{x}$ the following inequalities hold:

$$\begin{aligned} |\mathbf{f}(\mathbf{z}_1) - \mathbf{f}(\mathbf{z}_2)| &\leq C_f\,|\mathbf{z}_1 - \mathbf{z}_2|\,, \\ |G(\mathbf{z}_1) - G(\mathbf{z}_2)| &\leq C_g\,|\mathbf{z}_1 - \mathbf{z}_2|\,, \\ |\mathbf{h}(\mathbf{z}_1) - \mathbf{h}(\mathbf{z}_2)| &\leq C_h\,|\mathbf{z}_1 - \mathbf{z}_2|\,, \\ \left|\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{z}_1) - \frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{z}_2)\right| &\leq C_{h_x}\,|\mathbf{z}_1 - \mathbf{z}_2|\,. \end{aligned} \tag{4}$$

□

Remark 3: Assumption 3 is a weak assumption which ensures the existence of solution of the nonlinear affine system (1). ○

Assumption 4: For any given $\mathbf{y}_r \in \mathcal{C}[0,\ T_f]$ satisfying $|\mathbf{y}_r| \leq k_b$ and given $\mathbf{u}^* > 0$, there exist $\mathbf{x}_r \in \mathcal{C}^1[0,\ T_f]$ and $\mathbf{u}_r \in \mathcal{C}[0,\ T_f]$ that satisfy

$$\begin{aligned} \dot{\mathbf{x}}_r &= \mathbf{f}(\mathbf{x}_r) + G(\mathbf{x}_r)\mathbf{u}_r, \\ \mathbf{y}_r &= \mathbf{h}(\mathbf{x}_r), \end{aligned} \tag{5}$$

and $\mathrm{sat}(\mathbf{u}_r(t), \mathbf{u}^*) = \mathbf{u}_r$, $\forall t \in [0,\ T_f]$. □

Remark 4: Assumption 4 ensures that there exists a reference control input that can perfectly track the given reference trajectory without violating the input and output constraints. If input constraints are not considered, this assumption can be relaxed. ○

Next assumption is very common in ILC literature (see discussion in [21]).

Assumption 5: It is assumed that the system (1) executes a repetitive tracking within a finite time interval $t \in [0,\ T_f]$, and satisfies the identical initial condition: $\mathbf{x}_i(0) = \mathbf{x}_r(0)$ for any iteration $i \in \mathcal{N}$. □

In order to provide a systematic synthesis framework to deal with output constraints, the following assumption is used.

Assumption 6: There exists a continuous differentiable barrier-Lyapunov-function (BLF), $V^b(\bar{e}) : [0,\ 1) \to \mathcal{R}_{\geq 0}$ such that the following properties hold

$$\begin{aligned} &\alpha_1(\bar{e}) \;\leq\; V^b(\bar{e}) \;\leq\; \alpha_2(\bar{e}), \\ &V^b(0) = 0; \quad V^b(1) = \infty, \end{aligned} \tag{6}$$

where $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ are class $\mathcal{K}$ functions in the domain of $\bar{e} \in [0, 1)$. □

Remark 5: This assumption is a weak assumption. Many barrier functions satisfy this assumption (see for example, the discussions in [9]. In this paper, this assumption is used to generate a design framework which can be used for a large class of barrier functions. These properties in this assumption are used to simplify the convergence analysis of the proposed algorithm. ○

[1] A square system has equal number of inputs and output.

[2] A function $\mathbf{f}(\mathbf{x})$ is locally Lipschitz continuous if for any given compact $B_\Delta := \{\mathbf{x} \in \mathcal{R}^n \,|\, |\mathbf{x}| \leq \Delta\}$, there exists a positive constant $L = L(B_\Delta)$ such that $|\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{y})| \leq L\,|\mathbf{x} - \mathbf{y}|\,, \forall \mathbf{x}, \mathbf{y} \in B_\Delta$.

[3] A continuous function $\alpha : [0,\ a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$ [19, Definition 3.3].

III. CONTROLLER DESIGN

A schematics of the proposed control architecture is shown in Figure 1. The controller consists of an output feedback, which is based on one BLF satisfying Assumption 6, and a D-type feed-forward ILC. In order to deal with both hard constraints, a soft input constraint is added to feed-forward ILC, along with the output feedback.

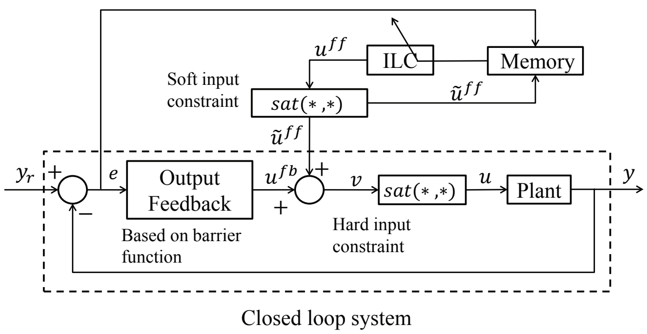

Fig. 1: Block diagram of the proposed control structure

The overall control input can be presented as

$$\begin{aligned} \mathbf{u}_i(t) &= \mathrm{sat}(\mathbf{v}_i(t),\ \mathbf{u}^*) \\ \mathbf{v}_i(t) &= \mathrm{sat}\left(\mathbf{u}_i^{ff}(t),\ \mathbf{u}^*\right) + \mathbf{u}_i^{fb}(t),\ \forall t \in [0,\ T_f], \end{aligned} \tag{7}$$

where $\mathbf{u}_i^{ff}(t)$ is the feed-forward control input which is generated by a D-type ILC algorithm and $\mathbf{u}_i^{fb}(t)$ is the output feedback control based on any BLF $V^b(\cdot)$ that satisfies Assumption 6.

As the relative degree of the system (1) is one, the feed-forward ILC control law $\mathbf{u}_i^{ff}$ employs a D-type ILC structure with a soft input saturation, which is given by

$$\mathbf{u}_{i+1}^{ff}(t) = \mathrm{sat}\left(\mathbf{u}_i^{ff}(t),\ \mathbf{u}^*\right) + \Gamma(t)\dot{\mathbf{e}}_i(t),\ \mathbf{u}_1^{ff}(t) = 0, \tag{8}$$

where $\Gamma \in \mathcal{R}^{m \times m}$ is a symmetric and positive definite learning gain matrix.

The proposed output feedback control, $\mathbf{u}_i^{fb}(t)$, takes the following form

$$\mathbf{u}_i^{fb} = \left[\frac{\partial V_i^b}{\partial \mathbf{e}_i}\right]^\intercal \tag{9}$$

where $V^b\left(\frac{\mathbf{e}^\intercal\mathbf{e}}{\varepsilon_b^2}\right)$ comes from Assumption 6.

In order to simplify the analysis, a non-dimensional positive variable $\bar{e}$ is introduced for a given output error $\mathbf{e}$ and output constraint ε_b. It has the following form:

$$\bar{e} \triangleq \frac{\mathbf{e}^\intercal\mathbf{e}}{\varepsilon_b^2}. \tag{10}$$

For a given output constraint ε_b and any choice of barrier Lyapunov function which satisfies the Assumption 6, when there is no input constraints, an output feedback has the following form:

$$\mathbf{u}^{fb} = \left[\frac{\partial V^b}{\partial \mathbf{e}}\right]^\intercal = \frac{\partial V^b}{\partial \bar{e}}\left[\frac{\partial \bar{e}}{\partial \mathbf{e}}\right]^\intercal = \frac{2}{\varepsilon_b^2}\frac{\partial V^b}{\partial \bar{e}}\mathbf{e}. \tag{11}$$

Hence the proposed feedback control is a nonlinear feedback, which depends on the choice of the BLF. Next, Proposition 1 is given to show that if the feedback control input remain within the saturation limits and the feed-forward control input is zero, then the output constraints can be satisfied for a given choice of V_b.

Proposition 1: Let $\mathbf{u}^* > 0$ and $\varepsilon_b > 0$ be given. For the system (1) satisfying Assumption 2, 3, 4 and 5, the control laws: (7) and (11) ensures the satisfaction of the output constraints if the feedback control input $\mathbf{u}^{fb}$ remains within the given saturation limit $\mathbf{u}^*$ for some BLF, V^b, satisfying Assumption 6 and the feed-forward control disappears (i.e $\mathbf{u}^{ff} = 0$). ■

The proof of Proposition 1 is given in Appendix I. In the proof, it is shown that there exists a compact set $\mathcal{D}_\mathbf{y}$ for the error trajectories $\mathbf{e}$ such that the BLF $V^b \le N_b$ for some $N_b > 0$. It is well-known that the boundedness of barrier function implies the satisfaction of output constraints. However, it might not be possible always to find a BLF that satisfies both constraints in the domain $\mathcal{D}_y$.

In order to accommodate both input and output constraints, a modification to the control law (9) is proposed in the following section.

IV. MAIN RESULTS

Due to the existence of hard input constraints, a new output feedback law is proposed to ensure that the control input is with the input limits. Consequently, it will ensure a simultaneous satisfaction of input and output constraints. This output feedback is based on a virtual BLF, which scales and shifts the original BLF which satisfies Assumption 6.

This new feedback control law is based on the following fact.

Fact 1: Let $\mathbf{u}^* > 0$ and $\varepsilon_b > 0$ be given. Let $\overline{\mathcal{D}}_y \subseteq \mathcal{D}_y$ be a compact set and V^b be a given BLF. There always exists a positive constant $\epsilon_0 = \epsilon_0(\mathbf{u}^*, \overline{\mathcal{D}}_y) \in [0, \varepsilon_b)$ such that for any ϵ_0, there exists a positive constant $k_0 = k_0(\mathbf{u}^*, \overline{\mathcal{D}}_y, \epsilon_0)$ such that for any $k \le k_0$ the following condition hold for all $\mathbf{e} \in \overline{\mathcal{D}}_y$:

$$\max_{\mathbf{e}\in\overline{\mathcal{D}}_y}\left| k\frac{\partial V^b}{\partial \mathbf{e}}\left(\frac{\mathbf{e}^\intercal\mathbf{e}}{(\varepsilon_b - \epsilon_0)^2}\right)\right|_\infty \le |\mathbf{u}^*|_\infty \tag{12}$$

That indicates that if the feedback $\mathbf{u}^{fb}$ from (9) design is modified to satisfy a virtual error constant $\varepsilon_v = \varepsilon_b - \epsilon_o$, then input constraints are always satisfied. Hence by Proposition 1 the output trajectories will never leave the compact set $\overline{\mathcal{D}}_y$ when $\mathbf{u}^{ff} = 0$. □

Fact 1 leads to the design of a new output feedback, $\overline{\mathbf{u}}^{fb}$ with the following form:

$$\overline{\mathbf{u}}^{fb} = k\left[\frac{\partial V_i^b}{\partial \mathbf{e}_i}\left(\frac{\mathbf{e}_i^\intercal\mathbf{e}_i}{\varepsilon_v^2}\right)\right]^\intercal, \tag{13}$$

where a virtual error constraint ε_v and a scaling factor k is used to ensure the input constraints for any given V^b.

Even though such a modification of feedback can ensure the satisfaction of hard constraints, the existence of feed-forward ILC can still violate both input and output constraints. Hence a careful design is still needed to ensure the satisfaction of both constraints during the learning process. It will be shown in the main result (Theorem 1) that, by using the proposed structure (see Figure 1), the virtual BLF based output feedback in combination with feed-forward ILC can ensure the perfect tracking performance and satisfaction of constraints with the help of a Barrier Composite Energy Function (BCEF).

Theorem 1: Assume that the system (1) satisfies the Assumptions 1, 2, 3, 4, and 5. For a given a BLF V^b satisfying Assumption 6, by selecting constants ϵ_o and k to satisfy Fact

1, then the closed loop system with control laws (7), (8) and (13) achieves

1) a perfect tracking performance in the presence of hard input and output constraints such that the output tracking error and the state tracking error converges uniformly, i.e $\lim_{i\to\infty} |\mathbf{e}_i(t)| = 0$ and $\lim_{i\to\infty} |\delta \mathbf{x}_i(t)| = 0$ for all $t \in [0, T_f]$;
2) uniform boundedness and $\mathcal{L}^2$ norm convergence of feed-forward control input, i.e. $\lim_{i\to\infty} \mathbf{u}_i^{ff} = \mathbf{u}_r$,

if the convergence condition:

$$\left| I_m - \Gamma \frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}) G(\mathbf{x}) \right| < 1$$

is satisfied for all $t \in [0,\ T_f]$ and $\mathbf{x} \in \mathcal{D}_{\mathbf{x}}$. ■

The proof of Theorem 1 using composite energy function based analysis is given in Appendix II. The BCEF used in the proof comprises of a quadratic term in state tracking error, the barrier Lyapunov function and $\mathcal{L}^2$ norm equivalent of the feed-forward control input. Along with Proposition 1 and the non-increasing property of BCEF along iteration, an induction based proof ensures the satisfaction of constraints and convergence in learning control.

V. AN ILLUSTRATIVE EXAMPLE

Consider the following nonlinear dynamics:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -3x_1^2 - 2x_2^2 + x_1 x_2 + (2x_2^2 + 1)u \\ y &= x_1 + x_2 \end{aligned} \tag{14}$$

The system (14) has a relative degree 1, thereby satisfying Assumption 1 and also satisfies Assumptions 2 and 3

Let the output reference trajectory be $y_r(t) = 3\sin^3\left(\frac{\pi t}{T_f}\right)\cos\left(\frac{\pi t}{T_f}\right)$ with $T_f = 5$ seconds. An allowable error bound $\varepsilon_b = 0.2$ and input saturation limit $u^* = 2$ is considered for simulation. The following barrier function, which satisfies Assumption 6 is selected,

$$V^b = \frac{k_0}{\pi} \tan\left(\frac{\pi}{2} \frac{\mathbf{e}^2}{(\varepsilon_b - \epsilon_0)^2}\right) \tag{15}$$

This leads to the following output feedback control law:

$$\overline{\mathbf{u}}^{fb} = \frac{k}{(\varepsilon_b - \epsilon_0)^2} \sec^2\left(\frac{\pi}{2} \frac{\mathbf{e}^2}{(\varepsilon_b - \epsilon_0)^2}\right) \mathbf{e} \tag{16}$$

where $k > 0$ and ϵ_0 should be selected to satisfy Fact 1.

The simulation is performed for different values of $\epsilon_0 = [0,\ 0.5,\ 0.1, 0.22]$ with a constant $k_0 = 0.25$. For a given gain k, $\epsilon_0 = 0$ indicates that the actual error bound is used in the barrier function. The input and output error trajectories for all ϵ_0, which indicates four virtual error constraints, are plotted in Fig. 2 and 3 for all ϵ_0. It is verified that for $\epsilon_0 = 0.22$, the $|u^{fb}| < u^*$. This choice of ϵ_0 along with k satisfies both input and output constraints. The parameter, k give an extra degree of freedom in shaping the control input. The simulation only demonstrates that for a given choice of constraint pair $(\mathbf{u}^*, \varepsilon_b)$ there exist a virtual error constraint $\varepsilon_v = \varepsilon_b - \epsilon_0$ for a given BLF V_b that can ensure the satisfaction of both input and output constraints.

Simulation is performed with control laws : (7), (8) and (16) with $\epsilon_0 = 0.22$ and learning gain, $\Gamma = 0.2$ for 300 iterations. It is clear from Fig. 4 that the control input during the learning phase satisfies the input constraints.

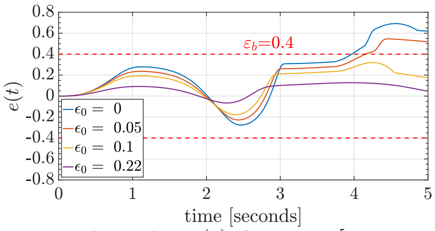

Fig. 2: Error trajectories,$e(t)$ for $\epsilon_0 = [0,\ 0.5,\ 0.1, 0.22]$

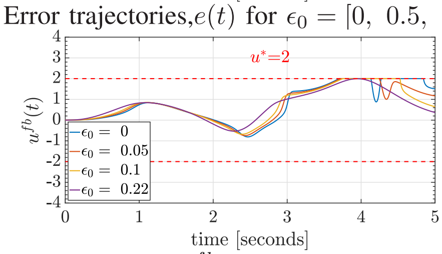

Fig. 3: Control input u^{fb} for $\epsilon_0 = [0,\ 0.5,\ 0.1, 0.22]$

The variation of supremum norm of output error along the iteration axis is shown in Fig. 5. This indicates that the output constraints are not violated in the presence of actuator saturation in any iteration.

VI. CONCLUSION

This paper presented a new output feedback-based ILC design for a general class of nonlinear affine systems. It is noted that the output feedback design is based on barrier Lyapunov function (BLF). By using the concept of virtual BLF, which scales and shifts the original BLF, it is possible to satisfy both input and output constraints. On the other hand, a Contraction Mapping based D-type ILC is used to perfectly track the desired trajectory. It is shown in the main result that this algorithm is able to handle both hard input constraints and output constraints without sacrificing the perfect tracking performance.

APPENDIX I
PROOF OF PROPOSITION 1

Proof: It is assumed in this proposition that the feedback control is not saturating and the feed-forward control input is zero. Therefore the total control input can be written as:

$$\mathbf{u} = \frac{2}{\varepsilon_b^2} \frac{\partial V^b}{\partial \bar{e}} \mathbf{e}, \tag{17}$$

As per Assumption 2, the error dynamics is BOBS. This means that for a bounded error output $\mathbf{e}$, there exists a compact set, $\mathcal{D}_{\mathbf{x}}$ for the state trajectories $\delta\mathbf{x}$. The construction of the compact set $\mathcal{D}_{\mathbf{x}}$ will ensure that for all $t \in [0, T_f]$, $\mathbf{x}$, $\delta\mathbf{x}(t)$ and $\mathbf{x}(t)$ are all in this compact set $\mathcal{D}_{\mathbf{x}}$. As for any $t \in [0, T_f]$, $\mathbf{x}(t) \in \mathcal{D}_{\mathbf{x}}$, $\mathbf{x}_r \in D_{\mathbf{x}}$, $\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})$ and $G(\mathbf{x})$ are bounded. Therefore define $\Lambda_{h_x} \triangleq \max_{\mathbf{x}\in\mathcal{D}_{\mathbf{x}}} \left|\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})\right|$ and $\Lambda_g \triangleq \max_{\mathbf{x}\in\mathcal{D}_{\mathbf{x}}} |G(\mathbf{x})|$. Denote $\Lambda_{\dot{x}_r} \triangleq \max_{t\in[0,\ T_f]} |\dot{\mathbf{x}}_r|$, and $\Lambda_{u_r} \triangleq \max_{t\in[0,\ T_f]} |\mathbf{u}_r(t)|$.

Taking the time derivative barrier function V^b yields:

$$\begin{aligned} \dot{V}^b &= \frac{2}{\varepsilon_b^2} \frac{\partial V^b}{\partial \bar{e}} \mathbf{e}^\intercal \dot{\mathbf{e}} = \frac{2}{\varepsilon_b^2} \frac{\partial V^b}{\partial \bar{e}} \mathbf{e}^\intercal \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}_r)\dot{\mathbf{x}}_r - \frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})\dot{\mathbf{x}} \right] \\ &= \frac{2}{\varepsilon_b^2} \frac{\partial V^b}{\partial \bar{e}} \mathbf{e}^\intercal \left[\left(\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}_r) - \frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}) \right) \dot{\mathbf{x}}_r + \frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}) \delta \dot{\mathbf{x}} \right] \end{aligned} \tag{18}$$

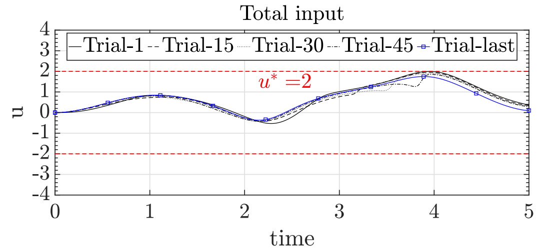

Fig. 4: Total control input at some iterations

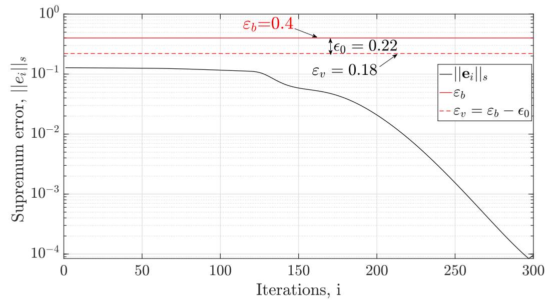

Fig. 5: Variation of supremum norm of tracking error

where $\delta\mathbf{x} = \mathbf{x}_r - \mathbf{x}$. Substituting (1) and (5) into (18) yields:

$$\begin{aligned}\dot{V}^b = & \frac{2}{\varepsilon_b^2}\frac{\partial V^b}{\partial \bar{e}}\mathbf{e}^\intercal\left(\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}_r) - \frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})\right)\dot{\mathbf{x}}_r \\ & + \frac{2}{\varepsilon_b^2}\frac{\partial V^b}{\partial \bar{e}}\mathbf{e}^\intercal\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})\left(\mathbf{f}(\mathbf{x}_r) - \mathbf{f}(\mathbf{x}) + G(\mathbf{x}_r)\mathbf{u}_r\right) \\ & - \frac{2}{\varepsilon_b^2}\frac{\partial V^b}{\partial \bar{e}}\mathbf{e}^\intercal\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})G(\mathbf{x})\mathbf{u} \qquad (19)\end{aligned}$$

Using the Lipschitz inequalities from (4) and substituting (17) back into (19) leads to:

$$\dot{V}^b \leq -\hat{\eta}_b\left[\frac{\partial V^b}{\partial \bar{e}}\right]^2|\mathbf{e}|^2 + C_{th}\frac{\partial V^b}{\partial \bar{e}}|\mathbf{e}||\delta\mathbf{x}| + \hat{\eta}_a\frac{\partial V^b}{\partial \bar{e}}|\mathbf{e}| \qquad (20)$$

where $C_{th} \triangleq \frac{2}{\varepsilon_b^2}\left(C_{h_x}\Lambda_{\dot{x}_r} + C_g\Lambda_{u_r}\Lambda_{h_x} + \Lambda_{h_x}C_f\right)$, $\hat{\eta}_a \triangleq \frac{2}{\varepsilon_b^2}\eta_a\lambda_{u_r}$ and $\hat{\eta}_b = 4\eta_b\frac{1}{\varepsilon_b^4}$, Using Young's inequality relation, there exists a positive constant φ_1 such that:

$$\frac{\partial V^b}{\partial \bar{e}}|\mathbf{e}||\delta\mathbf{x}| \leq \frac{\varphi_1}{2}\left[\frac{\partial V^b}{\partial \bar{e}}\right]^2|\mathbf{e}|^2 + \frac{1}{2\varphi_1}|\delta\mathbf{x}|^2. \qquad (21)$$

Using (21), the equation (20) yields:

$$\begin{aligned}\dot{V}^b \leq & -\left(\hat{\eta}_b - \frac{1}{2}C_{th}\varphi_1\right)\left[\frac{\partial V^b}{\partial \bar{e}}\right]^2|\mathbf{e}|^2 + \hat{\eta}_a\frac{\partial V^b}{\partial \bar{e}}|\mathbf{e}| \\ & + C_{th}\frac{1}{2\varphi_1}|\delta\mathbf{x}|^2 \qquad (22)\end{aligned}$$

Notice that because of Assumption 5, $V^b(0) = 0$ as $\mathbf{e}(0) = 0$. It is not difficult to find an invariant compact set if $|\delta\mathbf{x}|$ is bounded. Because of Assumption 2, $|\delta\mathbf{x}|$ is bounded if $|\mathbf{e}|$ is bounded. For any positive constants $\underline{\nu}$ and $\overline{\nu}$, there exists a constant φ_1 such that $\underline{\nu} \leq |\mathbf{e}| \leq \overline{\nu}$

This indicates the existence of a compact set, $\mathcal{D}_\mathbf{y}$ for the output error trajectories $\mathbf{e}$. Therefore there exists a positive constant $N_b < \infty$ such that $V^b(t) \leq N_b$ within the compact set. The boundedness of barrier function V^b ensures the satisfaction of output constraint, $|\mathbf{e}(t)| < \varepsilon_b$. This completes the proof. ■

Appendix II
Proof of Theorem 1

Proof: Consider the following barrier composite energy function (BCEF):

$$E_i(t) = e^{-\lambda t}W_{i-1}(t) + \int_0^t e^{-\lambda\tau}\delta\mathbf{u}_i^{ff\intercal}(\tau)\delta\mathbf{u}_i^{ff}(\tau)d\tau \qquad (23)$$
$$\forall t \in [0, T_f], i \in \mathcal{N}, \lambda > 0,\ W_0(t) = 0,$$

where $W_{i-1} = \frac{1}{2}\delta\mathbf{x}_{i-1}^\intercal\delta\mathbf{x}_{i-1} + V_{i-1}^b$, $\delta\mathbf{x} \triangleq \mathbf{x}_r - \mathbf{x}$, $\delta\mathbf{u}^{ff} \triangleq \mathbf{u}_r - \mathbf{u}^{ff}$,

To simplify the notation, assume $\widetilde{\mathbf{u}}^{ff} \triangleq \mathrm{sat}\left(\mathbf{u}_i^{ff}(t),\ \mathbf{u}^*\right)$ and $\delta\widetilde{\mathbf{u}}^{ff} \triangleq \mathbf{u}_r - \widetilde{\mathbf{u}}^{ff}$.

Induction based proof technique can be employed to ensure the satisfaction of constraints in every iteration.

The following lemmas, related to the properties of saturation function $\mathrm{sat}(\cdot,\cdot)$, are needed to facilitate this proof.

Lemma 1: [22, Property-3] For any given $\mathbf{u}_r$, $\mathbf{u}$ and $\mathbf{u}^* \in \mathcal{R}^m$ satisfying $\mathrm{sat}(\mathbf{u}_r, \mathbf{u}^*) = \mathbf{u}_r$ then the following inequality holds: $|\mathbf{u}_r - \mathrm{sat}(\mathbf{u}, \mathbf{u}^*)|^2 \leq |\mathbf{u}_r - \mathbf{u}|^2$. □

Lemma 2: [22, Property-4] For any $\mathbf{u}$, $\mathbf{u}^*$ and $\mathbf{w} \in \mathcal{R}^m$ satisfying $\mathbf{u}^* > 0$, if $\mathbf{v} = \mathrm{sat}(\mathbf{u}, \mathbf{u}^*) + \mathbf{w}$, then the following inequality holds: $|\mathrm{sat}(\mathbf{v}, \mathbf{u}^*) - \mathbf{v}| \leq |\mathbf{w}|$. □

Firstly, E_1 is finite as $W_0 = 0$ and $\mathbf{u}_1^{ff} = 0$. By Proposition 1, the constraints are satisfied at first iteration. This leads to a finite E_2.

Assume that there exist a positive constant Δ_m such that $\max_{t\in[0,T_f]} E_j \leq \Delta_m$, which indicates that the trajectories satisfy the output constraints for any j^{th} iteration. If $E_{j+1} \leq E_j$, this indicates that trajectories belong to the same set. This will lead to the conclusion that $\mathbf{e}_{j+1}$ satisfy output constraint.

The difference in BCEF between two consecutive iterations is given by $\Delta E_{j+1} = E_{j+1} - E_j$.

From (23), it follows:

$$\begin{aligned}\Delta E_{j+1} = & e^{-\lambda t}\left(W_j - W_{j-1}\right) \\ & + \int_0^t e^{-\lambda\tau}\left(\left|\delta\mathbf{u}_{j+1}^{ff}\right|^2 - \left|\delta\mathbf{u}_j^{ff}\right|^2\right)d\tau. \qquad (24)\end{aligned}$$

The first term in (24) can be written as:

$$e^{-\lambda t}W_j = \int_0^t e^{-\lambda\tau}\dot{W}_j d\tau - \lambda\int_0^t e^{-\lambda\tau}W_j d\tau. \qquad (25)$$

But $\dot{W}_j = \delta\mathbf{x}_j^\intercal\delta\dot{\mathbf{x}}_j + \frac{2}{\varepsilon_v^2}\frac{\partial V_j^b}{\partial \bar{e}_j}\mathbf{e}_j^\intercal\dot{\mathbf{e}}_j$ can be rearranged by substituting for $\dot{\mathbf{e}}$ as similar to (18), leading to the following inequality relation:

$$\begin{aligned}\dot{W}_j \leq\ & C_{fu}|\delta\mathbf{x}_j|^2 + \Lambda_g|\delta\mathbf{x}_j||\delta\mathbf{u}_j| \\ & + C_{fr}|\mathbf{e}||\delta\mathbf{x}_j| + \hat{\eta}_a\frac{\partial V_j^b}{\partial \bar{e}_j}|\mathbf{e}_j||\delta\mathbf{u}_j| \qquad (26)\end{aligned}$$

where $C_{fu} = C_f + C_g\Lambda_{u_r}$ and $C_{fr} = \frac{2}{\varepsilon_v^2}\frac{\partial V^b}{\partial \bar{e}}\left(\Lambda_{h_x}C_{fu} + C_{hx}\Lambda_{\dot{x}_r}\right)$. Note that in this analysis the $\bar{e}$ is represented in terms of virtual error bound ε_v, i.e $\bar{e} = \frac{|\mathbf{e}|^2}{\varepsilon_v^2}$

Using Lemma 2 on $|\delta\mathbf{u}_j|$ yields:

$$\begin{aligned}|\delta\mathbf{u}_j| = & |\mathbf{u}_r - \mathrm{sat}(\mathbf{v}_j, \mathbf{u}^*)| = |\mathbf{u}_r - \mathbf{v}_j - (\mathrm{sat}(\mathbf{v}_j, \mathbf{u}^*) - \mathbf{v}_j)| \\ \leq & \left|\delta\widetilde{\mathbf{u}}_j^{ff}\right| + 2\left|\mathbf{u}_j^{fb}\right| \leq \left|\delta\widetilde{\mathbf{u}}_j^{ff}\right| + \frac{4}{\varepsilon_v^2}\frac{\partial V_j^b}{\partial \bar{e}_j}|\mathbf{e}_j| \qquad (27)\end{aligned}$$

Note that due to (4), $|\mathbf{e}| = |\mathbf{h}(\mathbf{x}_r) - \mathbf{h}(\mathbf{x})| \leq C_h |\delta\mathbf{x}|$. Substituting (27) back into (26) yields:

$$\dot{W}_j \leq \hat{C}_{fu} |\delta\mathbf{x}_j|^2 + \hat{\Lambda}_g |\delta\mathbf{x}_j| \left|\delta\mathbf{u}_j^{ff}\right| \tag{28}$$

where $\hat{C}_{fu} = C_{fu} + C_h \frac{\partial V_j^b}{\partial \bar{e}_j}\left[C_{fr} + \Lambda_g \frac{4}{\varepsilon_b^2} + \hat{\eta}_a \frac{\partial V_j^b}{\partial \bar{e}_j}\right]$ $\hat{\Lambda}_g = \Lambda_g + \hat{\eta}_a C_h^2 \frac{\partial V_j^b}{\partial \bar{e}_j}$. Note that upon assumption that the trajectories are bounded on j^{th} iteration, $\frac{\partial V_j^b}{\partial \bar{e}_j}$ is also bounded.

Using Young's inequality relation, there exists a positive constant φ_2 such that:

$$|\delta\mathbf{x}_j| \left|\delta\widetilde{\mathbf{u}}_j^{ff}\right| \leq \frac{\varphi_2}{2} |\delta\mathbf{x}_j|^2 + \frac{1}{2\varphi_2} \left|\delta\widetilde{\mathbf{u}}_j^{ff}\right|^2, \tag{29}$$

Substituting (29) back into (28) yields:

$$\dot{W}_j \leq \mathcal{O}_1(\varphi_2) |\delta\mathbf{x}_j|^2 + \mathcal{O}_2\left(\varphi_2^{-1}\right) \left|\delta\widetilde{\mathbf{u}}_j^{ff}\right|^2 \tag{30}$$

where $\mathcal{O}_1(\varphi_2) = \hat{C}_{fu} + \hat{\Lambda}_g \frac{\varphi_2}{2}$ and $\mathcal{O}_2\left(\varphi_2^{-1}\right) = \hat{\Lambda}_g \frac{1}{2\varphi_2}$.

From the ILC update law (8) we have

$$\delta\mathbf{u}_{j+1}^{ff} = \delta\widetilde{\mathbf{u}}_j^{ff} - \Gamma\dot{\mathbf{e}}_j = \mathrm{P}(\mathbf{x}_j)\delta\widetilde{\mathbf{u}}_j^{ff} - \boldsymbol{\zeta}_j, \tag{31}$$

where $\mathrm{P}(\mathbf{x}_j) = I_m - \Gamma\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}_j)G(\mathbf{x}_j)$, and $\boldsymbol{\zeta}_j = \Gamma\left(\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}_r) - \frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}_j)\right)\dot{\mathbf{x}}_r + \Gamma\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}_j)\left(\delta G(\mathbf{x}_j)\mathbf{u}_r + \delta\mathbf{f}(\mathbf{x}_j)\right) - \Gamma\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x}_j)G(\mathbf{x}_j)\left(\delta\widetilde{\mathbf{u}}_j^{ff} - \delta\mathbf{u}_j\right)$.

Using Lemma 2 on $\left|\delta\widetilde{\mathbf{u}}_j^{ff} - \delta\mathbf{u}_j\right|$ yields

$$\begin{aligned}\left|\delta\widetilde{\mathbf{u}}_j^{ff} - \delta\mathbf{u}_j\right| &= \left|\mathbf{u}_j - \widetilde{\mathbf{u}}_j^{ff}\right| = \left|\mathrm{sat}(\mathbf{v}_j, \mathbf{u}^*) - \mathbf{v}_j + \mathbf{u}_j^{fb}\right| \\ &\leq 2\left|\mathbf{u}_j^{fb}\right| \leq \frac{4}{\varepsilon_v^2} C_h \frac{\partial V_j^b}{\partial \bar{e}_j} |\delta\mathbf{x}_j|. \end{aligned} \tag{32}$$

It is possible to show that: $\left|\boldsymbol{\zeta}_j\right| \leq C_{bd} |\delta\mathbf{x}_j|$ where $C_{bd} = |\Gamma|\left[C_{fr} + 2C_h\hat{\eta}_a \frac{\partial V_j^b}{\partial \bar{e}_j}\right]$.

$$\begin{aligned}\left|\delta\mathbf{u}_{j+1}^{ff}\right|^2 - \left|\delta\widetilde{\mathbf{u}}_j^{ff}\right|^2 \leq &-\lambda_p \left|\delta\widetilde{\mathbf{u}}_j^{ff}\right|^2 + \left|\boldsymbol{\zeta}_j\right|^2 \\ &+ 2\left|\boldsymbol{\zeta}_j\right| \left|\mathrm{P}(\mathbf{x}_j)\delta\widetilde{\mathbf{u}}_j^{ff}\right| \end{aligned} \tag{33}$$

where $\lambda_p = \lambda_{\min}\left(I_m - \mathrm{P}(\mathbf{x}_j)^{\intercal}\mathrm{P}(\mathbf{x}_j)\right)$. If $|\mathrm{P}(\mathbf{x}_j)| < \rho < 1$, we have $\lambda_p > 0$. Using Young's inequality relation from:

$$\begin{aligned}&\left|\delta\mathbf{u}_{j+1}^{ff}\right|^2 - \left|\delta\widetilde{\mathbf{u}}_j^{ff}\right|^2 \\ &\leq -\left(\lambda_p - \mathcal{O}_3(\varphi_2^{-1})\right)\left|\delta\widetilde{\mathbf{u}}_j^{ff}\right|^2 + \mathcal{O}_4(\varphi_2) |\delta\mathbf{x}_j|^2 \end{aligned} \tag{34}$$

where $\mathcal{O}_3(\varphi_2^{-1}) = \dfrac{\rho C_{bd}}{\varphi_3}$, $\mathcal{O}_4(\varphi_2) = C_{bd}^2 + C_{bd}\rho\varphi_3$.

Substituting (25), (30), (34) back into (24) yields

$$\begin{aligned}\Delta E_{j+1} \leq &-\int_0^t \left[\frac{\lambda}{2} - \mathcal{O}_5(\varphi_2)\right] e^{-\lambda\tau} |\delta\mathbf{x}_j|^2 d\tau \\ &- \int_0^t \left[\lambda_p - \mathcal{O}_6\left(\varphi_2^{-1}\right)\right] e^{-\lambda\tau} \left|\delta\widetilde{\mathbf{u}}_j^{ff}\right|^2 d\tau \\ &- \lambda\int_0^t e^{-\lambda\tau} V_j^b d\tau - e^{-\lambda\tau} W_{j-1} \end{aligned} \tag{35}$$

where $\mathcal{O}_5(\varphi_2) = \mathcal{O}_1(\varphi_2) + \mathcal{O}_4(\varphi_2)$, $\mathcal{O}_6\left(\varphi_2^{-1}\right) = \mathcal{O}_2\left(\varphi_2^{-1}\right) + \mathcal{O}_3\left(\varphi_2^{-1}\right)$. For a given λ_p there exists a φ_2 such that $\lambda_p > \mathcal{O}_6\left(\varphi_2^{-1}\right)$. For that φ_2 there exists a $\lambda > 2\mathcal{O}_5(\varphi_2)$ such that $\Delta E_{j+1} \leq 0$. This leads to $\mathbf{e}_{j+1} \in \mathcal{D}_y$. Hence by induction the constraints are satisfied for any $j \in \mathcal{N}$.

As BCEF is non-increasing along the iteration axis, using the standard arguments from [21], we can show that tracking error converges uniformly and feed-forward control converges in $\mathcal{L}^2$ norm sense. This completes the proof.

■

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