Robust Object Manipulation for Prosthetic Hand Applications

Wenceslao Shaw Cortez, Denny Oetomo, Chris Manzie, and Peter Choong

***Abstract*— We propose a robust control law for the manipulation of objects in light of disturbances that result from lack of a priori knowledge of the grasp scenario. An application of this control law is in cases where it is impractical to assume all properties of the object and external wrenches are known to the controller, notably in cases of object grasping and manipulation with prosthetic hands. The main contribution of the proposed approach is to guarantee semi-global asymptotic stability and exponential stability of the system. In addition, the approach lends itself to a systematic tuning method presented in the literature that improves transient performance without compromising stability of the system. Simulation results validate the effectiveness of the proposed approach.**

I. Introduction

One of the main limitations of today's powered prosthetic hands lies in the human-machine interface [1], [2]. The interface problem deals with how to acquire sufficient information from the human to control all degrees of freedom of the prosthetic device. As a result, much of the current research in prosthetics is focused solely on grasp/gesture formation, while neglecting the dexterous ability to manipulate objects.

The dexterity in the human hand allows for precise manipulation tasks to be accomplished efficiently [3]. A human hand can achieve a task such as unlocking a door by dexterously manipulating a key within the grasp. A conventional prosthetic hand requires the human to perform gross arm motions to accomplish the same task, which result in fatigue and poor performance [3], which reduces the quality of life for amputees [4]. Manipulation is an important capability, but adds more complexity in terms of how to control the dexterous hand in light of the shortcomings in the interface.

The approach taken here is to augment the human with an autonomous capability residing in the hand prosthesis. In this framework, the human performs a grasp and subsequently commands the reference object pose. The control system then determines the actuator torques to accomplish the desired manipulation motion. The idea is to let the human retain a sense of authority, while improving the performance and reducing the cognitive burden on the human [5], [6].

There exist many control systems that can manipulate objects, however most rely on problem settings that are unsuitable in this application. In the prosthetic setting, solutions are limited to on-board sensing with no a priori information of the object including weight, inertia, shape, center of mass, etc. Thus solutions that require exact knowledge of the object and other external sensors, such as cameras, are not applicable or practical. There exist some exceptional solutions for object manipulation in such a setting [7]–[10]. This work seeks to expand on those solutions by relaxing certain assumptions, for example, that neglect the effects of unknown external wrenches and uncertain object models.

External wrenches, such as from gravity, are intrinsic to everyday prosthetic use. Due to the limitations of the prosthetic setting, it is difficult to anticipate all external wrenches that may act on the hand and/or object. The human is in charge of grasp formation, thus the relative pose of the grasp to the object center of mass is unknown to the control strategy on-board the prosthesis. Also, the object center of mass may change such as when holding a glass being filled with water. Existing solutions neglect these disturbances and thus cannot provide asymptotic stability to the object's reference pose in the prosthetic setting [7], [8], [11].

In previous work, the authors developed a controller that provides object manipulation with optimal contact force distribution for prosthetic hands [12]. We extend that work, and propose a novel disturbance rejecting control law for object manipulation. The main contribution of this work is a control law that guarantees semi-global asymptotic stability in the presence of unknown disturbances. The proposed solution incorporates the same contact force distribution from [12] and retains a systematic method for gain tuning.

II. Problem Formulation

A. System Model

Consider a fully-actuated, multi-fingered hand grasping a rigid, convex object at k contact points. Each finger consists of n joints with smooth, convex fingertips. Let the finger configuration be defined by the joint angles, $\boldsymbol{q}_i \in \mathbb{R}^n$. The full hand configuration is defined by the full joint angle vector, $\boldsymbol{q} = (\boldsymbol{q}_1, \boldsymbol{q}_2, ..., \boldsymbol{q}_k)^T \in \mathbb{R}^m$, where $m = nk$. Let the inertial frame, $\mathcal{P}$, be fixed on the palm of the hand, and let each finger i have a finger base frame, $\mathcal{S}_i$, and a fingertip frame, $\mathcal{F}_i$, located at $\boldsymbol{p}_{f_i} \in \mathbb{R}^3$. The contact frame, $\mathcal{C}_i$, is located at the contact point, $\boldsymbol{p}_{c_i} \in \mathbb{R}^3$. A visual representation of the contact geometry for the ith finger is shown in Figure 1.

Let $\boldsymbol{v}_{c_i} \in \mathbb{R}^3$ denote the contact velocity at contact point $\boldsymbol{p}_{c_i}$. The joint velocities of finger i, $\dot{\boldsymbol{q}}_i$, and the contact point velocity, $\boldsymbol{v}_{c_i}$, are related via the ith hand Jacobian, $J_{h_i} \in \mathbb{R}^{3\times n}$ [13]:

$$J_{h_i} = \begin{bmatrix} I_{3\times 3} & -(\boldsymbol{p}_{fc_i})^\wedge \end{bmatrix} J_{s_i}(\boldsymbol{q}_i) \tag{1}$$

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where $I_{3\times3} \in \mathbb{R}^{3\times3}$ is the identity matrix and $\boldsymbol{p}_{fc_i} \in \mathbb{R}^3$ is the vector from ith fingertip to the ith contact point, $(\cdot)^\wedge$ denotes the skew symmetric operator, and $J_{s_i}(\boldsymbol{q}_i) \in \mathbb{R}^{6\times n}$ is the manipulator Jacobian relating $\dot{\boldsymbol{q}}_i$ with the velocity of $\boldsymbol{p}_{f_i}$. Let $\boldsymbol{p}_{fc} \in \mathbb{R}^{3k}$ denote the concatenation of all finger-to-contact point vectors, $\boldsymbol{p}_{fc_i}$, and $\boldsymbol{p}_c \in \mathbb{R}^{3k}$ the concatenation of contact points $\boldsymbol{p}_{c_i}$. The full hand Jacobian, $J_h \in \mathbb{R}^{3k\times m}$, is constructed by combining each J_{h_i} into a block diagonal matrix for $i = 1, ..., k$.

Assumption 1: The given hand has sufficient degrees of freedom such that $m = 3k$, and never reaches a singular configuration.

Remark 1: Assumption 1 ensures J_h is square and full rank, which is a common assumption in related work [7], [8], [11]. This assumption is made to not distract from the main contribution and can be relaxed by considering the null space dynamics [14].

Assumption 2: Measurements of $\boldsymbol{q}, \dot{\boldsymbol{q}}$, and $\boldsymbol{p}_{fc}$ are available from sensors on-board the robotic hand.

Each fingertip exerts a contact force on the object at contact i, denoted by $\boldsymbol{f}_{c_i} \in \mathbb{R}^3$. Let each contact force vector be concatenated into the full contact force vector, $\boldsymbol{f}_c \in \mathbb{R}^{3k}$. The dynamics of the hand are given by [14]:

$$M_h(\boldsymbol{q})\ddot{\boldsymbol{q}} + C_h(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{\tau}_e(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) = -J_h^T \boldsymbol{f}_c + \boldsymbol{u} \qquad (2)$$

where $M_h(\boldsymbol{q}) \in \mathbb{R}^{m\times m}$ is the inertia matrix, $C_h(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \mathbb{R}^{m\times m}$ is the Coriolis and centrifugal matrix, $\boldsymbol{\tau}_e(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) \in \mathbb{R}^m$ is the sum of all dissipative and non-dissipative torques acting on the joints, and $\boldsymbol{u} \in \mathbb{R}^m$ is the joint torque control input for a fully actuated hand.

Let $\boldsymbol{x}_o \in \mathbb{R}^6$ denote the object's configuration which is defined by the the object center of mass, $\boldsymbol{p}_o \in \mathbb{R}^3$, and the orientation represented by the rotation matrix, $R_{po}(\boldsymbol{x}_o) \in SO(3)$, which maps the object frame, $\mathcal{O}$, to the inertial frame, $\mathcal{P}$. Each finger contacts the object at the respective contact point, $\boldsymbol{p}_{c_i} \in \mathbb{R}^3$, and the position vector from the center of mass to said contact point is $\boldsymbol{p}_{oc_i} \in \mathbb{R}^3$.

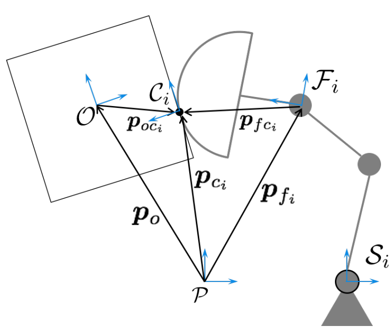


Fig. 1: A visual representation of the contact geometry for contact i.

Let the matrix $G_i \in \mathbf{R}^{6\times3}$ map the contact forces of finger i, $\boldsymbol{f}_{c_i}$, to the corresponding wrench acting on the object. The dual form, G_i^T, maps object motion to the velocity of the ith contact point. This matrix can be computed by [13]:

$$G_i^T = \begin{bmatrix} I_{3\times3} & -(\boldsymbol{p}_{oc_i})^\wedge \end{bmatrix} \qquad (3)$$

The grasp map, $G = [G_1, G_2, ..., G_k] \in \mathbb{R}^{6\times3k}$, maps the full contact force vector, $\boldsymbol{f}_c$, to the net object wrench.

Assumption 3: The given multi-fingered grasp has $k > 2$ contact points, which are non-collinear.

Remark 2: By Assumption 3, G is always full rank [13].

The object dynamics are given by [14]:

$$M_o(\boldsymbol{x}_o)\ddot{\boldsymbol{x}}_o + C_o(\boldsymbol{x}, \dot{\boldsymbol{x}}_o)\dot{\boldsymbol{x}}_o = G\boldsymbol{f}_c + \boldsymbol{w}_e(t) \qquad (4)$$

where $M_o(\boldsymbol{x}_o), C_o(\boldsymbol{x}_o, \dot{\boldsymbol{x}}_o) \in \mathbb{R}^{6\times6}$ are the object's respective inertia and Coriolis/centrifugal matrices and $\boldsymbol{w}_e(t) \in \mathbb{R}^6$ is an external wrench disturbing the object. The hand and object kinematics are related by the grasp constraint [14]:

$$\boldsymbol{v}_c = J_h\dot{\boldsymbol{q}} = G^T\dot{\boldsymbol{x}}_o \qquad (5)$$

where $\boldsymbol{v}_c \in \mathbb{R}^{3k}$ is the concatenation of all $\boldsymbol{v}_{c_i}$.

Assumption 4: There is no slip occurring at the contact points, and the fingertips are always in contact with the object surface. Furthermore, the effects of rolling are negligible.

Remark 3: Assumption 4 ensures (5) is always satisfied, and can be relaxed by incorporating slip prevention in the control system as will be discussed in Section III. Additionally, the simulations in Section IV include the effects of rolling to show that the proposed solution is indeed robust to rolling effects.

Let $\boldsymbol{z}(t) = (\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), \boldsymbol{x}_o(t), \dot{\boldsymbol{x}}_o(t))^T \in \mathbb{R}^{2m+12}$ be the full hand/object state. Under Assumption 1 and by incorporating the grasp constraint, (5), the hand/object dynamics can be combined into [14]:

$$M_{ho}(\boldsymbol{z})\ddot{\boldsymbol{x}}_o + C_{ho}(\boldsymbol{z})\dot{\boldsymbol{x}}_o = GJ_h^{-T}\Big(\boldsymbol{u} - \boldsymbol{\tau}_e(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)\Big) + \boldsymbol{w}_e(t) \qquad (6)$$

where

$$M_{ho}(\boldsymbol{z}) = M_o(\boldsymbol{x}_o) + GJ_h^{-T}M_h(\boldsymbol{q})J_h^{-1}G^T, \qquad (7)$$

$$C_{ho}(\boldsymbol{z}) = C_o(\boldsymbol{x}) + GJ_h^{-T}(C_h(\boldsymbol{q})J_h^{-1}G^T + M_h(\boldsymbol{z}_1)\frac{d}{dt}[J_h^{-1}G^T]), \qquad (8)$$

Note that G and J_h are functions of the hand/object state, $\boldsymbol{z}$, which has been dropped for notation.

In realistic applications, the object center of mass is uncertain/unknown and may change during the execution of a task, for example, when a prosthetic hand is used to drink from a glass. Due to the intrinsic limitations of the problem, related work use a virtual frame defined by the robotic hand configuration to define about which point the object is to be manipulated [7]–[10]. Let $\mathcal{O}_v$ be a virtual frame located at the point $\boldsymbol{p}_a \in \mathbb{R}^3$. Let $\boldsymbol{f}_a : \mathbb{R}^{3k} \rightarrow \mathbb{R}^3$ be a continuously differentiable function such that:

$$\boldsymbol{p}_a = \boldsymbol{f}_a(\boldsymbol{p}_c) \qquad (9)$$

Let $\boldsymbol{\gamma}_a \in \mathbb{R}^3$ be a local parameterization of some virtual object frame $R_{po_v} \in SO(3)$, and let $\boldsymbol{g}_a : \mathbb{R}^{3k} \rightarrow \mathbb{R}^3$ be a continuously differentiable function such that:

$$\boldsymbol{\gamma}_a = \boldsymbol{g}_a(\boldsymbol{p}_c) \qquad (10)$$

Such a local parameterization of $SO(3)$ by γ_a allows for a suitable definition of the orientation error [13].

Let $x_a = (p_a, \gamma_a)^T \in \mathbb{R}^6$ be the contact state, which by neglecting rolling effects via Assumption 4 satisfies:

$$\dot{x}_a = \begin{bmatrix} \dot{p}_a \\ \dot{\gamma}_a \end{bmatrix} = \begin{bmatrix} \frac{\partial f_a}{\partial p_c} \\ \frac{\partial g_a}{\partial p_c} \end{bmatrix} v_c = J_c v_c \tag{11}$$

where $J_c \in \mathbb{R}^{6\times 3k}$ is the contact Jacobian matrix and is a function of the hand/object state z, which is also dropped for notation. This contact Jacobian relates both object and hand motion to the contact state motion via the grasp constraint, (5):

$$\dot{x}_a = J_c G^T \dot{x}_o = J_c J_h \dot{q} \tag{12}$$

Assumption 5: J_c is full rank for all $t \geq 0$.

Remark 4: The requirement that J_c be full rank puts restrictions on the functions f_a and g_a. One solution is to use the centroid of the contact positions, $p_a = \frac{1}{k}\sum_{j=1}^{k} p_{c_j}$, and to define γ_a as the Euler angles from a virtual object frame. A virtual frame definition from [9] is used, but here it is a function of p_c, not p_f.

The dynamics of the contact state are derived by solving for $\dot{x}_o$ in (12) using Assumptions 3, 5, and differentiating:

$$\ddot{x}_o = \frac{d}{dt}[(J_c G^T)^{-1}]\dot{x}_a + (J_c G^T)^{-1}\ddot{x}_a \tag{13}$$

Substitution of (12) and (13) into (6) and pre-multiplication by $(J_c G^T)^{-T}$ results in the new dynamics:

$$M_a(z)\ddot{x}_a + C_a(z,\dot{z})\dot{x}_a = Ju + d(z,t) \tag{14}$$

where

$$M_a(z) = (J_c G^T)^{-T} M_{ho}(z)(J_c G^T)^{-1}, \tag{15}$$

$$C_a(z) = (J_c G^T)^{-T}\Big(M_{ho}(z)\frac{d}{dt}[(J_c G^T)^{-1}] + C_{ho}(z)(J_c G^T)^{-1}\Big) \tag{16}$$

$$J = (J_c G^T)^{-T} G J_h^{-T} \tag{17}$$

$$d(z,t) = J\tau_e(q,\dot{q},t) + (J_c G^T)^{-T} w_e(t) \tag{18}$$

Lemma 1: Under Assumptions 1, 3, 5, $M_a(z)$ is positive definite. Furthermore it is uniformly bounded such that there exists constants $m_{\min}, m_{\max} \in \mathbb{R}_{>0}$ that satisfy:

$$0 < m_{\min} \leq ||M_a(z)^{-1}|| \leq m_{\max} \tag{19}$$

Proof: Assumptions 1 and 3 ensure that $M_{ho}(z)$ is symmetric, positive definite [14]. Thus the following statement holds:

$$0 < \lambda_{\min}(M_{ho}) \leq ||M_{ho}(z)^{-1}|| \leq \lambda_{\max}(M_{ho}) \tag{20}$$

It is a well known fact that M_o and M_h are uniformly bounded, positive definite matrices [15]. Thus M_{ho} is uniformly bounded. Furthermore, under Assumption 5, M_a is also a positive definite, and is uniformly bounded by the same argument. ∎

Assumption 6: The external disturbances, $\tau_e(q,\dot{q},t)$ and $w_e(t)$, are constant at a static configuration such that $\frac{\partial}{\partial t}\tau_e(q,0,t) = 0$ and $\dot{w}_e(t) = 0$.

Let $r \in \mathbb{R}^6$ be the reference command, and let $e = x_a - r$ define the error between the reference and contact state. Let $\zeta(t) = (\zeta_1(t), \zeta_2(t))^T \in \mathbb{R}^{12}$ be the hand/virtual object state, where $\zeta_1 = e$, $\zeta_2 = \dot{e}$. The hand/virtual object system is defined by:

$$\begin{aligned} \zeta_1 &= \zeta_2 \\ \zeta_2 &= M_a(z)^{-1}\Big(-C_a(z,\dot{z})\zeta_2 + Ju + d(z,t)\Big) \end{aligned} \tag{21}$$

Assumption 7: The reference is constant: $r(t) = r$.

Remark 5: Assumption 7 is not necessarily restrictive as a piece-wise constant reference is also acceptable.

B. Control Objective

The focus of this paper is to asymptotically reach $\zeta = 0$ in the presence of disturbances. As long as the task of object manipulation is achieved, there is no specific configuration that the hand should converge to. Therefore it is sufficient to require $\dot{q} \to 0$. The control problem is defined as follows:

Problem 1: Given a hand/object system, with some initial condition $z(0)$ that satisfies Assumptions 1-7, determine a control law that semi-globally satisfies:

$$\lim_{t\to\infty}(\zeta(t), \dot{q}(t)) \to (0,0) \tag{22}$$

III. Proposed Controller

A. Stability Analysis

Due to the lack of knowledge in the prosthetic setting, the proposed controller is dependent on an approximate grasp map, $\tilde{G}$, where p_a is used in place of p_o in (3). As a result, approximation errors in $\tilde{G}$ are related to the distance between p_a and the unknown center of mass, p_o, The relation between $\tilde{G}$ and G is defined by $G = \tilde{G} + \Delta G$, where

$$\tilde{G} = \begin{bmatrix} I_{3\times3} & \cdots & I_{3\times3} \\ (p_{c_1} - p_a)^\wedge & \cdots & (p_{c_k} - p_a)^\wedge \end{bmatrix}, \tag{23}$$

$$\Delta G = \begin{bmatrix} 0_{3\times3} & \cdots & 0_{3\times3} \\ (p_a - p_o)^\wedge & \cdots & (p_a - p_o)^\wedge \end{bmatrix} \tag{24}$$

The advantage of the proposed control law is that it handles these approximation errors from ΔG in addition to rejecting external disturbances. Let the proposed control be defined in terms of two orthogonal elements:

$$u = J_h^T(\tilde{G}^\dagger \tilde{G} J_c^T u_m + k_f u_f) \tag{25}$$

where $()^\dagger$ denotes a generalized inverse, $u_m \in \mathbb{R}^6$ is the manipulation control term and $u_f \in \mathbb{R}^{3k}$ is the internal force control term. A slip prevention algorithm can be incorporated into the control to adjust the gain $k_f \in \mathbb{R}_{\geq 0}$ and relax Assumption 4, under the following assumption:

Assumption 8: The internal force control, u_f, satisfies:

$$\tilde{G}u_f = 0 \tag{26}$$

Lemma 2: Under Assumption 8, the internal force control, u_f, guarantees $Gu_f = 0$.

Proof: (26) guarantees that $\boldsymbol{u}_f$ applies a zero net wrench about any position on the object, including the center of mass. Thus from (3), it follows that $G\boldsymbol{u}_f = 0$. ■

Remark 6: By Lemma 2, Assumption 8 enforces $\boldsymbol{u}_f$ to be an element of the null space of G to regulate the internal contact forces. One such internal force controller is the optimal force distribution control term proposed in [12]. Additionally, internal force controllers related to the centroid position of the contact points is also acceptable [7]–[10].

The proposed manipulation control term is the linear PID control for the hand/virtual object system:

$$\boldsymbol{u}_m = -K_p\boldsymbol{\zeta}_1 - K_i \int_0^t \boldsymbol{\zeta}_1 dt - K_d\boldsymbol{\zeta}_2 \tag{27}$$

where $K_p, K_i, K_d \in \mathbb{R}^{6\times 6}$ are the respective proportional, integral, and derivative gain matrices. This approach is in line with the robot manipulator controller proposed in [16], in which the authors re-define the PID-controlled system as a singularly perturbed system to prove semi-global asymptotic stability.

Theorem 1: Under Assumptions 1-8, and given a $\Delta \in \mathbb{R}_{>0}$ such that $\forall ||(\boldsymbol{x}(0), \dot{\boldsymbol{x}}(0))|| \leq \Delta$, $\exists K_p^*, K_i^*, K_d^*$ such that (21) with control law (25), (27) is semi-globally asymptotically stable with respect to the origin, $\boldsymbol{\zeta} = 0$, and $\dot{\boldsymbol{q}} \to 0$.

Proof: (21) can be written as:

$$\begin{aligned}\dot{\boldsymbol{\zeta}}_1 &= \boldsymbol{\zeta}_2,\\ \dot{\boldsymbol{\zeta}}_2 &= M_a(\boldsymbol{z})^{-1}\Big(- C_a(\boldsymbol{z}, \dot{\boldsymbol{z}})\boldsymbol{\zeta}_2 + \boldsymbol{u}_m + \boldsymbol{d}(\boldsymbol{z}, t) + \boldsymbol{\sigma}(\boldsymbol{\zeta}, \boldsymbol{z})\Big)\end{aligned} \tag{28}$$

where $\boldsymbol{\sigma}(\boldsymbol{\zeta}, \boldsymbol{z}) \in \mathbb{R}^6$ is the approximation error that arises when canceling terms from J with their equivalent inverse approximations from (25):

$$\boldsymbol{\sigma}(\boldsymbol{\zeta}, \boldsymbol{z}) = (J_c G^T)^{-T}\Delta G(\tilde{G}^{\dagger}G - I)J_c^T\boldsymbol{u}_m \tag{29}$$

Noting that (12) $\implies$ $C_a(\boldsymbol{z}, \boldsymbol{\zeta})$, let $\boldsymbol{\psi}(\boldsymbol{\zeta}, \boldsymbol{z}, t)$ be defined by:

$$\boldsymbol{\psi}(\boldsymbol{\zeta}, \boldsymbol{z}, t) = -C_a(\boldsymbol{z}, \boldsymbol{\zeta})\boldsymbol{\zeta}_2 + \boldsymbol{d}(\boldsymbol{z}, t) + \boldsymbol{\sigma}(\boldsymbol{\zeta}, \boldsymbol{z}) \tag{30}$$

Substituting (30) into (28) results in:

$$\begin{aligned}\dot{\boldsymbol{\zeta}}_1 &= \boldsymbol{\zeta}_2,\\ \dot{\boldsymbol{\zeta}}_2 &= M_a(\boldsymbol{z})^{-1}(\boldsymbol{\psi}(\boldsymbol{\zeta}, \boldsymbol{z}, t) + \boldsymbol{u}_m)\end{aligned} \tag{31}$$

Differentiation of $\boldsymbol{\psi}(\boldsymbol{\zeta}, \boldsymbol{z}, t)$ with respect to time results in:

$$\dot{\boldsymbol{\psi}}(\boldsymbol{\zeta}, \boldsymbol{z}, t) = \frac{\partial\boldsymbol{\psi}(\boldsymbol{\zeta}, \boldsymbol{z}, t)}{\partial\boldsymbol{\zeta}}\dot{\boldsymbol{\zeta}} + \frac{\partial\boldsymbol{\psi}(\boldsymbol{\zeta}, \boldsymbol{z}, t)}{\partial\boldsymbol{z}}\dot{\boldsymbol{z}} + \frac{\partial\boldsymbol{\psi}(\boldsymbol{\zeta}, \boldsymbol{z}, t)}{\partial t} \tag{32}$$

In [16], a similar system of the form (31) is re-defined as a singularly perturbed system, such that a parameter $\varepsilon \in \mathbb{R}_{>0}$ can be chosen sufficiently small to provide asymptotic stability for a given Δ. To obtain similar results here, it is necessary to show that $\dot{\boldsymbol{\psi}}(0, \boldsymbol{z}, t) = 0$ when $\dot{\boldsymbol{\zeta}} = 0$, which guarantees that the origin is an equilibrium point of the singularly perturbed system as defined in [16]. By Assumptions 1, 3, 4, 5, and 7, (12) (13) ensure that $\dot{\boldsymbol{\zeta}} = 0 \implies (\dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \dot{\boldsymbol{x}}_0, \ddot{\boldsymbol{x}}_0) = 0$. This relation implies that at the origin, both the object and hand are at rest, and so $\dot{\boldsymbol{\zeta}} = 0 \implies \dot{\boldsymbol{z}} = 0$. Finally, Assumption 6 guarantees that $\frac{\partial}{\partial t}\boldsymbol{\psi}(0, \boldsymbol{z}, t) = 0$, and thus $\dot{\boldsymbol{\psi}}(0, \boldsymbol{z}, t) = 0$ when $\dot{\boldsymbol{\zeta}} = 0$.

With Lemma 1, the proof for semi-global asymptotic stability for $\boldsymbol{\zeta}(t)$ follows from [16], such that $(\boldsymbol{\zeta}, \dot{\boldsymbol{q}}) \to (0, 0)$. ■

Remark 7: Notice the approximation error (29), which shows the disturbance arising from offset between $\boldsymbol{p}_a$ and $\boldsymbol{p}_o$. Conventional controllers cannot account for this disturbance arising from the uncertain grasp scenario, whereas the proposed controller provides semi-global asymptotic stability of the system. Furthermore the proposed control does not restrict the bound on any of the disturbances. In practice, the hardware capabilities of the robotic hand is the main limitation to what magnitude disturbances can be rejected.

B. Gain Tuning

The proposed control lends itself to a systematic method of gain tuning presented in [16]. This approach facilitates the design of the controller to improve transient performance without compromising stability. Let the K_p, K_i, K_d gains be defined by:

$$K_p = \bar{M}(K_1 + \frac{1}{\varepsilon}K_2) \tag{33a}$$

$$K_i = \frac{1}{\varepsilon}\bar{M}K_1 \tag{33b}$$

$$K_d = \bar{M}(K_2 + \frac{1}{\varepsilon}I_{6\times 6}) \tag{33c}$$

where $\bar{M} \in \mathbb{R}^{6\times 6}$ is a positive definite matrix, $K_1, K_2 \in \mathbb{R}^{6\times 6}$ are positive definite gain matrices, and $\varepsilon \in \mathbb{R}_{>0}$ is a tuning parameter. The structure defined in (33) facilitates the choice of each gain parameter. The gains K_1 and K_2 relate to the behavior of a linear system, and can be chosen based on the desired closed loop time constant and damping coefficient [16]. The parameter $\bar{M}$ is a constant estimate of M_a. It can be defined by an upper bound on the eigenvalues of M_a^{-1}, which is guaranteed by Lemma 1. In this case only an upper bound on $||M_a^{-1}||$ is required, which is desirable for the prosthetic setting where exact knowledge of the object is unknown.

Finally, the PID gains as defined in (33) are parameterized by ε, which dictates the size of the region of attraction. Thus once K_1, K_2 are defined, ε is solely responsible for the system's transient response.

Corollary 1: Under the same assumptions as Theorem 1, there exists a $\varepsilon^* > 0$, such that for $0 < \varepsilon < \varepsilon^*$, the origin is exponentially stable.

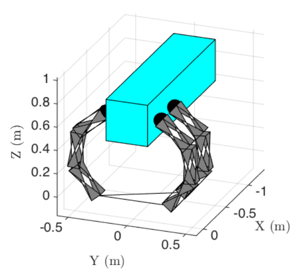
Proof: Under the same approach used in Theorem 1, the structure of the proof follows from [16] albeit with the new system (21) with the control law defined by (25), (27). ■

Remark 8: Exponential stability with respect to the single parameter ε allows a simple, systematic way to improve the robustness of the system. However, in practice signal noise will provide a lower bound, ε^{**}, on ε. In the case that noise levels are sufficiently high, the set defined by $\varepsilon^{**} < \varepsilon < \varepsilon^*$ may be empty.

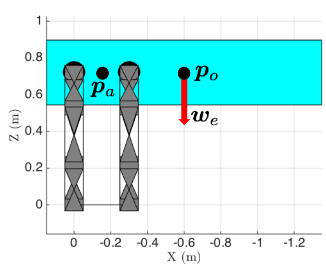
IV. RESULTS

In the prosthetic setting, the hand prosthesis is subject to unknown disturbances during everyday use. To provide object manipulation capabilities, the control solution must be able to handle these unknown disturbances despite the offset between the grasp pose and unknown object center of mass. The effect of this offset is to amplify the disturbance that acts at the center of mass. Conventional controllers [8]–[12] that can provide object manipulation lack the ability to reject such disturbances and result in a steady state error away from the human commanded object pose. This simulation compares a conventional controller to the proposed controller that can handle unknown disturbances with zero steady state error. Also, the proposed control with the aforementioned tuning method is capable of improving transient behavior, subject to actuator limitations, by simply adjusting one variable, ε.

The simulation consists of a three-fingered hand model with 16 degrees of freedom and hemispherical fingertips. Two revolute joints are located at the finger base with one revolute joint between the remaining links of the finger. The links all share the same dimensions and mass properties. The object being grasped by the hand is a rectangular prism. The given grasp is purposefully offset from the object center of mass, and a force is applied at said center of mass. Here $\boldsymbol{x}_a$ is defined as from Remark 4, where $\boldsymbol{p}_a$ is defined to be the centroid of the contact points. The hand and object configurations are depicted in Figure 2, and the full simulation parameters are listed in Table I. It is important to note that rolling effects are incorporated into the simulation.



(a) Isometric view



(b) Side view depictingthe grasp centroid, $\boldsymbol{p}_a$, object center of mass, $\boldsymbol{p}_o$, and external force, $\boldsymbol{w}_e$.

Fig. 2: A given multifingered grasp.

The simulation was performed using Matlab's ode45 integrator, with a simulation time of 15 seconds. The value of k_f was empirically chosen such that the contact points do

TABLE I: Simulation Parameters

Link length	0.3 m
Link mass	0.05 kg
Link moment of inertia	$\text{diag}([0.385, 0.021, 0.385]) \times 10^{-3}\text{kgm}^2$
Fingertip radius	0.06 m
Object dimensions	1.5 m $\times$ 0.3538 m $\times$ 0.3538 m
Object mass	0.0018 kg
Object moment of inertia	$\text{diag}([0.0375, 0.356, 0.356]) \times 10^{-3}\text{kgm}^2$
Initial $\boldsymbol{q}_i, \forall i = 1, ..., k$	$[0, 0.873, 0.785, 0.785]^T$ rad
Initial $\boldsymbol{p}_o$	$[-0.6, 0, 0.722]^T$ m
Initial $\boldsymbol{p}_a$	$[-0.1, 0.059, 0.722]^T$ m
Initial $\boldsymbol{\gamma}_a$	$[0, 0, 0]^T$rad
$\boldsymbol{\tau}_e$	$-.001 * I_{m\times m}\dot{\boldsymbol{q}}$ Nm
$\boldsymbol{w}_e$	$[0, 0, -1, 0, 0, 0]^T$ N
k_f	10
K_p	$6.25 * I_{6\times 6}$
K_i	$2.50 * I_{6\times 6}$
K_d	$2.51 * I_{6\times 6}$

not lose contact with the object. In future work k_f will be determined by slip prevention algorithms.

A. Conventional Controller

Figure 3 shows the response of the conventional controller from [12] to the force acting on the grasped object. The plots show that the conventional controller is only able to arrest the motion resulting from the disturbance, but $\boldsymbol{e}$ reaches a steady-state error instead of asymptotically returning to the origin. In the prosthetic sense, this compromises the ability to perform routine tasks in the presence of external disturbances including gravity. Note that in the figures $\boldsymbol{e}_i$ refers to the ith element of $\boldsymbol{e}$.

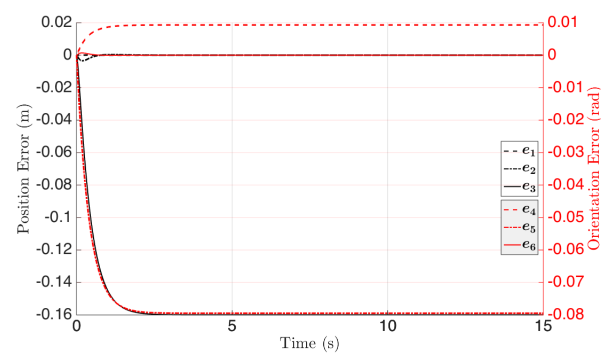
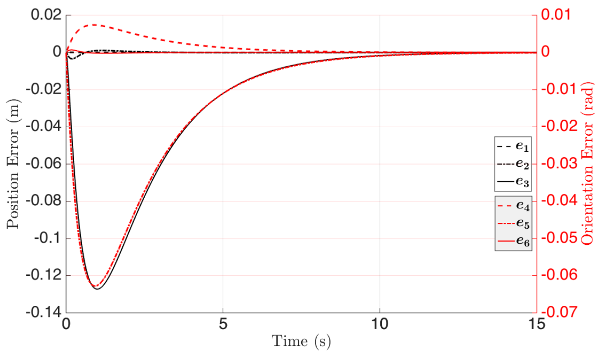


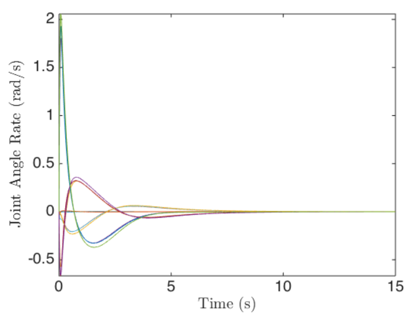
Fig. 3: System response to step disturbance with the conventional controller [12].

B. Proposed Controller

The gains, K_p, K_i, K_d in Table I were determined by (33), with $\bar{M}, K_1, K_2$ defined as in [16], with values $\bar{M} = 0.003 * I_{6\times 6}$, $K_1 = 1.0 * I_{6\times 6}$, and $K_2 = 2.5 * I_{6\times 6}$. The tuning parameter, ε, was chosen empirically to be 0.001. Figure 4 shows the resulting trajectories as the proposed control system responds to the step disturbance. Figure 4a shows the object pose asymptotically returning to the initial pose configuration. Figure 4b shows the joint angle rates converging to zero as the hand reaches a static configuration. The proposed controller is able to reject the unknown disturbance, and thus provides a sense of robust object manipulation for prosthetic hands. Furthermore, the effect of rolling does not impede the controllers performance, which suggests that the controller is robust to rolling effects.



(a) pose error.



(b) Joint rate trajectory.

Fig. 4: System response to step disturbance with proposed controller.

Figure 5 demonstrates the effectiveness of the gain tuning results where decreasing one parameter, ε, attenuates the overshoot and settling time of the position error. Note that this behavior is seen in all elements of $\boldsymbol{e}$, but the related plots have been omitted for clarity. Along with the semi-global property, ε can be sufficiently decreased to guarantee a desired transient behavior regardless of the disturbance magnitude. Note that in practice ε is lower bounded by noise sensitivities, and the magnitude of disturbances that can be rejected is limited by actuation capabilities.

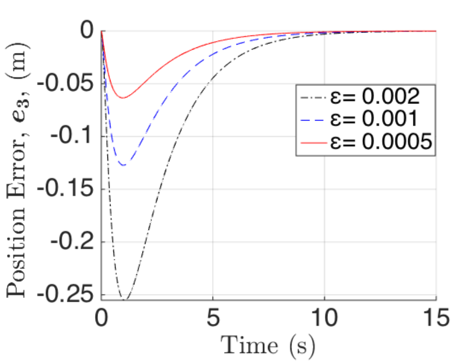


Fig. 5: Effect of ε on transient behavior of position error $\boldsymbol{e}_3$

V. CONCLUSION

In this paper a control law was proposed to provide robust object manipulation for prosthetic hands. The proposed controller handles unknown disturbances acting on the hand and object, which is highly relevant in the context of prosthetics. The stability analysis shows how such disturbances are handled without exact knowledge of the hand/object model. Simulation results demonstrate the disturbance rejection and simple tuning properties of the proposed controller.

REFERENCES

[1] J. S. Schofield, K. R. Evans, J. P. Carey, and J. S. Hebert, "Applications of sensory feedback in motorized upper extremity prosthesis: a review," *Expert Review of Medical Devices*, vol. 11, no. 5, pp. 499–511, 2014.

[2] U. Wijk and I. Carlsson, "Forearm amputees' views of prosthesis use and sensory feedback," *Journal of Hand Therapy*, vol. 28, no. 3, pp. 269–278, 2015.

[3] I. M. Bullock, R. R. Ma, and A. M. Dollar, "A hand-centric classification of human and robot dexterous manipulation," *IEEE Transactions on Haptics*, vol. 6, no. 2, pp. 129–144, 2013.

[4] C. H. Jang, H. S. Yang, H. E. Yang, S. Y. Lee, J. W. Kwon, B. D. Yun, J. Y. Choi, S. N. Kim, and H. W. Jeong, "A survey on activities of daily living and occupations of upper extremity amputees," *Annals of Rehabilitation Medicine*, vol. 35, no. 6, pp. 907–921, 2011.

[5] M. Markovic, S. Dosen, D. Popovic, B. Graimann, and D. Farina, "Sensor fusion and computer vision for context-aware control of a multi degree-of-freedom prosthesis," *Journal of Neural Engineering*, vol. 12, no. 6, 2015.

[6] C. Cipriani, F. Zaccone, S. Micera, and M. C. Carrozza, "On the shared control of an emg-controlled prosthetic hand: analysis of user-prosthesis interaction," *IEEE Transactions on Robotics*, vol. 24, no. 1, pp. 170–184, 2008.

[7] T. Wimböck, C. Ott, A. Albu-Schäffer, and G. Hirzinger, "Comparison of object-level grasp controllers for dynamic dexterous manipulation," *International Journal of Robotics Research*, vol. 31, no. 1, pp. 3–23, 2012.

[8] A. Kawamura, K. Tahara, R. Kurazume, and T. Hasegawa, "Dynamic grasping of an arbitrary polyhedral object," *Robotica*, vol. 31, no. 4, pp. 511–523, 2013.

[9] K. Tahara, S. Arimoto, and M. Yoshida, "Dynamic object manipulation using a virtual frame by a triple soft-fingered robotic hand," *IEEE International Conference on Robotics and Automation*, pp. 4322–4327, 2010.

[10] J. H. Bae, S. W. Park, D. Kim, M. H. Baeg, and S. R. Oh, "A grasp strategy with the geometric centroid of a groped object shape derived from contact spots," *IEEE International Conference on Robotics and Automation*, pp. 3798–3804, 2012.

[11] A. Caldas, A. Micaelli, M. Grossard, M. Makarov, P. Rodriguez-Ayerbe, and D. Dumur, "Object-level impedance control for dexterous manipulation with contact uncertainties using an LMI-based approach," in *IEEE International Conference on Robotics and Automation*, 2015, pp. 3668–3674.

[12] W. Shaw-Cortez, D. Oetomo, C. Manzie, and P. Choong, "Towards dynamic object manipulation with tactile sensing for prosthetic hands," *Proceedings of the 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2016.

[13] A. Cole, J. Hauser, and S. Sastry, "Kinematics and control of multifingered hands with rolling contact," *IEEE Transactions on Automatic Control*, vol. 34, no. 4, pp. 398–404, 1989.

[14] R. Murray, Z. Li, and S. Sastry, *A mathematical introduction to robotic manipulation*. CRC Press: Boca Raton, FL, USA, 1994.

[15] M. W. Spong, *Robot Dynamics and Control*. Wiley, 1989.

[16] J. Alvarez-Ramirez, I. Cervantes, and R. Kelly, "Pid regulation of robot manipulators: stability and performance," *Systems & Control Letters*, vol. 41, no. 2, pp. 73–83, 2000.