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On Feedback-based Iterative Learning Control for Nonlinear Systems without Global Lipschitz Continuity

Gijo Sebastian, Ying Tan, Denny Oetomo and Iven Mareels

Abstract— **Contraction mapping method is widely used in the design and analysis of feed-forward type of iterative learning control (ILC) for a class of nonlinear dynamic systems that satisfy global Lipschitz continuity condition. However, many engineering systems are only locally Lipschitz continuous. Thus contraction mapping method is not directly applicable to such systems. This paper proposes a feedback-based ILC algorithm for a class of nonlinear systems that is only locally Lipschitz continuous. In the proposed feedback-based ILC, the feed-forward controller or ILC is designed using a standard contraction mapping technique with an appropriate convergence condition while the feedback control is designed to stabilize the nonlinear dynamic systems. A novel composite energy function is proposed to show that the proposed feedback-based ILC can work even with the input saturation. Simulation results illustrate the effectiveness of the proposed method.**

I. INTRODUCTION

Iterative learning control (ILC), as introduced in [1], builds on the idea of "learning" from experience or previous task executions to ensure the perfect tracking performance without the requirement of a precise model of the engineered system. Due to its data-driven nature, ILC has been applied to many engineering systems. The various theoretical developments and applications of ILC algorithms in the last three decades have been reported, see, for example, survey papers [2]–[6] and references therein.

Contraction mapping (CM) based methods and Composite Energy Function (CEF) based techniques are two widely used design and analysis tools for ILC algorithms. Generally, CM based ILC has a feed-forward structure, in which output tracking errors in previous trials are used. With an appropriate convergence condition, it can be applied to nonlinear dynamic systems that satisfy global Lipschitz continuity (GLC) condition [7]–[9]. Although requiring GLC condition is a strong assumption, it usually leads to simpler feed-forward type ILC algorithms.

On the other hand, CEF based ILC design can easily handle nonlinear dynamic systems to track reference output or reference state using the errors from the current iteration. If there is a control Lyapunov function for the nonlinear systems, it can also handle nonlinear terms that are only locally Lipschitz continuity (LLC) [10]–[12].

These two techniques can be unified using CEF based analysis technique. For example, it is shown in [13], the P-type feed-forward ILC, which is designed using CM technique, can achieve perfect tracking performance with CEF based proof technique. Noting the fact that when the trajectories of the dynamic systems can be ensured to be bounded, the GLC condition becomes LLC. Hence, when the dynamic systems are LLC but satisfy bounded-input-bounded-state (BIBS) condition, the simple P-type feed-forward ILC can work with the consideration of input saturation [14].

As a generalization of [14], this paper investigates a new dual controller scheme to deal with nonlinear systems which satisfy neither the assumption of BIBS nor GLC. This dual controller consists of a state feedback and feed-forward ILC design. The feed-forward ILC design is based on CM method [7], [9] while the state feedback controller ensures the boundedness of the solution and robustness in the time domain. When implementing such a dual control algorithm, actuator constraints are also considered.

The contributions of this paper are summarized as follows:

1) For nonlinear dynamic systems which are neither BIBS nor GLC, we propose a feedback-based ILC with the input saturation to ensure convergence.
2) A novel CEF is used to ensure the uniform convergence of the input, state and output tracking error.
3) A completely new proof technique based on induction is proposed to show the uniform boundedness of the closed loop trajectories in the iteration domain.

The paper is organized as follows. The necessary preliminaries and problem formulation are introduced in Section II. Section III gives the convergence analysis of the proposed theorem. Section IV provides an illustrative example that demonstrates the effectiveness of the proposed control structure.

II. PRELIMINARIES AND PROBLEM FORMULATION

The following notations are used in this paper. $\mathcal{R}$ denotes the set of real numbers and $\mathcal{N}$ denotes the set of natural numbers. $\mathcal{C}^j[0,\ T_f]$ denotes the set of all continuous function in $[0,\ T_f]$ that is differentiable up to j^{th} order for any $j \in \mathcal{N}$. For any vector $\mathbf{x} \in \mathcal{R}^n$, $|\mathbf{x}|$ denotes the Euclidean norm and $|\mathbf{x}|^2 = \mathbf{x}^T\mathbf{x}$. For any matrix $A \in \mathcal{R}^{n\times m}$, $|A|$ represents the induced matrix norm. $A > 0$ indicates A is a positive definite matrix. $(\cdot)^T$ represents the transpose of a vector or a matrix. I_n denotes the identity matrix of dimension n.

Definition 1: A function $\mathbf{f}(\cdot)$ is said to be *locally Lipschitz* on a domain (open and connected set) $\mathcal{D} \subset \mathcal{R}^n$ if each point of $\mathcal{D}$ has a neighbourhood $\mathcal{D}_0$ such that $\mathbf{f}(\cdot)$ satisfies the Lipschitz condition:

$$|\mathbf{f}(\mathbf{z}_1) - \mathbf{f}(\mathbf{z}_2)| \leq L_0 \, |\mathbf{z}_1 - \mathbf{z}_2| \tag{1}$$

for all points $\mathbf{z}_1$, $\mathbf{z}_2 \in \mathcal{D}_0$ with some constant L_0 [15, p.89].

Melbourne School of Engineering, The University of Melbourne, Parkville VIC 3010 Australia {gijos, yingt, doetomo, i.mareels}@unimelb.edu.au

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Definition 2: A continuous function $\alpha : [0,\ a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$ [15, Definition 4.2].

Definition 3: The saturation function is defined as $sat(u, u^*) \triangleq sign\,(u)\,min\{u^*, |u|\}$ for any $u \in \mathcal{R}$ where $u^* > 0$ is a scalar constant. For any $\mathbf{u} \in \mathcal{R}^m$, $sat(\mathbf{u}, \mathbf{u}^*) = \left[sat(u^1, u^{1^*}), \cdots, sat(u^m, u^{m^*})\right]^T$ where $\mathbf{u}^* = [u^{1^*}, \cdots, u^{m^*}]^T$ is a vector of constants with $u^{j^*} > 0, \forall\ j \in [1,\ m]$.

Definition 4: For any $\mathbf{x}(t) \in \mathcal{C}[0,\ T_f]$, the supremum norm is defined as $\|\mathbf{x}\|_s \triangleq \max_{t\in[0,\,T]} |\mathbf{x}(t)|_\infty$,where $|\mathbf{x}|_\infty = \max_{j\in[1,\,n]} |x^j|$ and x^j denotes j^{th} element of $\mathbf{x}$.

Definition 5: For any $\mathbf{x} \in \mathcal{C}[0,\ T_f]$ the $\mathcal{L}^2$ norm is defined as $\|\mathbf{x}\|_{\mathcal{L}^2} \triangleq \left(\int_0^{T_f} |\mathbf{x}(\tau)|^2 d\tau\right)^{\frac{1}{2}}$ where as the $\mathcal{L}_e^2$ norm is defined as $\|\mathbf{x}\|_{\mathcal{L}_e^2} \triangleq \left(\int_0^{T_f} e^{-\lambda\tau} |\mathbf{x}(\tau)|^2 d\tau\right)^{\frac{1}{2}}$, for any $\lambda > 0$.

For the convenience of the proof of Theorem 1, the following lemmas are used.

Lemma 1: For any given $\mathbf{u}_r,\ \mathbf{u}$ and $\mathbf{u}^* \in \mathcal{R}^m$ satisfying $sat(\mathbf{u}_r, \mathbf{u}^*) = \mathbf{u}_r$ then the following inequality holds true: $|\mathbf{u}_r - sat(\mathbf{u}, \mathbf{u}^*)|^2 \leq |\mathbf{u}_r - \mathbf{u}|^2$ [11, Property-3]. □

Lemma 2: For any $\mathbf{v},\ \mathbf{u},\ \mathbf{u}^*$ and $\mathbf{w} \in \mathcal{R}^m$, if $\mathbf{v} = sat(\mathbf{u}, \mathbf{u}^*) + \mathbf{w}$, then the following inequality holds true: $|sat(\mathbf{v}, \mathbf{u}^*) - \mathbf{v}| \leq |\mathbf{w}|$ [11, Property-4]. □

A. Problem Formulation

At the i^{th} iteration, a nonlinear multi-input-multi-output (MIMO) square[1] affine system takes the following form:

$$\begin{aligned} \dot{\mathbf{x}}_i =& \mathbf{f}(\mathbf{x}_i) + \mathbf{g}(\mathbf{x}_i)\mathbf{u}_i, \\ \mathbf{y}_i =& \mathbf{h}(\mathbf{x}_i), \end{aligned} \tag{2}$$

where $\mathbf{x}_i \in \mathcal{R}^n$, $\mathbf{u}_i \in \mathcal{R}^m$ and $\mathbf{y}_i \in \mathcal{R}^m$ are the system state, input and output vectors and i denotes the iteration number. $\mathbf{f}(\cdot)$, $\mathbf{g}(\cdot)$ and $\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\cdot)$ are LLC in their argument.

The system (2) satisfies the following assumptions.

Assumption 1: For a given compact set $\mathcal{D} \subset \mathcal{R}^n$, there are positive constants C_f, C_g, and C_{h_x} such that for any $\mathbf{z}_1,\ \mathbf{z}_2 \in \mathcal{D}$ the following inequalities hold:

$$|\mathbf{f}(\mathbf{z}_1) - \mathbf{f}(\mathbf{z}_2)| \leq C_f\,|\mathbf{z}_1 - \mathbf{z}_2|\,, \tag{3}$$

$$|\mathbf{g}(\mathbf{z}_1) - \mathbf{g}(\mathbf{z}_2)| \leq C_g\,|\mathbf{z}_1 - \mathbf{z}_2|\,, \tag{4}$$

$$\left|\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{z}_1) - \frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{z}_2)\right| \leq C_{h_x}\,|\mathbf{z}_1 - \mathbf{z}_2| \tag{5}$$

where C_f, C_g, and C_{h_x} depends upon the size of the compact set. □

Remark 1: Different from GLC conditions, Assumption 1 indicates that the nonlinear mappings $\mathbf{f}(\cdot)$ and $\mathbf{g}(\cdot)$ as well as $\mathbf{h}(\cdot)$ are LLC. Indeed, as the size of the set $\mathcal{D}$ increases, parameters C_f, C_g, C_{h_x} will increase. Moreover, this system (2) is not assumed to satisfy bounded-input-bounded-state assumption. Thus this assumption is a weaker assumption. Many nonlinear functions $\mathbf{f}(\cdot)$ and $\mathbf{g}(\cdot)$ satisfy this assumption. ∘

Assumption 2: The matrix valued functions $\mathbf{g}(\mathbf{x})$ and $\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})\mathbf{g}(\mathbf{x})$ have full column rank. Moreover, $\mathbf{g}(\mathbf{x})$ and $\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})$ are bounded $\forall \mathbf{x} \in \mathcal{D}$. □

Remark 2: Assumption 2 guarantees the uniqueness of control input $\mathbf{u}$ for a given realizable trajectory. The boundedness of $\mathbf{g}(\mathbf{x})$ and $\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})$ are reasonable in most of the practical applications. ∘

Assumption 3: For any given $\mathbf{y}_r \in \mathcal{C}[0,\ T_f]$, there exist $\mathbf{x}_r \in \mathcal{C}^1[0,\ T_f]$ and $\mathbf{u}_r \in \mathcal{C}[0,\ T_f]$ that satisfy

$$\begin{aligned} \dot{\mathbf{x}}_r =& \mathbf{f}(\mathbf{x}_r) + \mathbf{g}(\mathbf{x}_r)\mathbf{u}_r, \\ \mathbf{y}_r =& \mathbf{h}(\mathbf{x}_r), \end{aligned} \tag{6}$$

and $sat(\mathbf{u}_r, \mathbf{u}^*) = \mathbf{u}_r,\ \forall t \in [0,\ T_f]$, where $\mathbf{u}^*$ is a predefined saturation limit. □

Remark 3: In many output tracking problems, for a given reference output, some inversion based methods [7] are used to find the reference state and reference input in order to ensure that the internal state is well-behaved. Moreover, when actuator saturation is considered, Assumption 3 is needed to show that the reference input is achievable with the consideration of the input saturation. ∘

Assumption 4: The system (2) performs repetitive tracking over a finite time interval $[0,\ T_f]$ where $0 < T_f < \infty$ and an identical initial condition is satisfied at every iteration, *i.e.*, $\mathbf{x}_i(0) = \mathbf{x}_r(0),\ \forall i \in \mathcal{N}$. □

Remark 4: Assumption (4) is a standard assumption when a convergence condition is derived from the input tracking [9], [16]. If this resetting condition is not satisfied, perfect tracking performance cannot be achieved. Many ILC algorithms have been proposed to relax this assumption without achieving perfect tracking. ∘

Assumption 5: It is assumed that the system (2) has a relative degree[2] of one. Moreover, assume $\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})\mathbf{g}(\mathbf{x}) > 0, \forall \mathbf{x} \in \mathcal{R}^n$. □

Remark 5: Different from Assumption 1, the relative degree one is defined for the entire $\mathcal{R}^n$ instead of $\mathcal{D}$. First of all, this assumption is always needed in the design of ILC algorithm [9], [18]. Secondly, the boundedness of the trajectories of the system (2) cannot be guaranteed. As the system (2) is nonlinear, the finite escape phenomenon might happen [15]. ∘

The tracking error is defined as $\mathbf{e}_i(t) \triangleq \mathbf{y}_r(t) - \mathbf{y}_i(t),\quad \forall t \in [0,\ T_f]$. The control objective is to find the desired control input $\mathbf{u}_i(t)$ for system (2) performing repetitive tracking such that the output tracking error $\mathbf{e}_i(t)$ uniformly converges to zero as $i \to \infty$.

B. Controller Design - Revisiting feedback-based D-type ILC

A standard feedback-based ILC system in shown in Fig. 1. It consists of a stabilizing feedback and an ILC. The feedback ensures uniform boundedness of state and output where as an ILC finds the desired control input in iterations. We use

[1] A square system has same the dimension for input and output vectors.

[2] For the definition of *vector relative degree* for a nonlinear MIMO square system as in (2) refer [17, p.407]

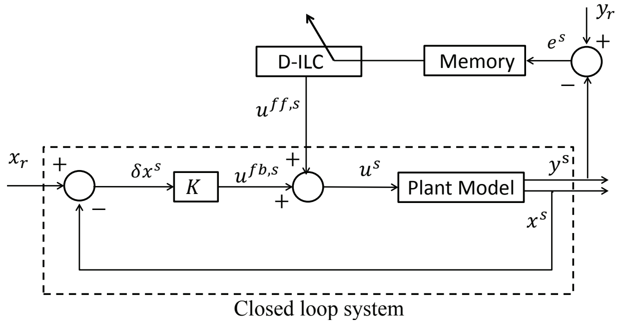

Fig. 1. Block diagram of standard feedback-based ILC

superscript s to denote control signals corresponding to the standard system. For such a system the total input is given by

$$\mathbf{u}_i^s(t) = \mathbf{u}_i^{ff,s}(t) + \mathbf{u}_i^{fb,s}(t), \tag{7}$$

where $\mathbf{u}_i^{ff,s}(t)$ represents the feed-forward input from standard D-type ILC given by (15) and $\mathbf{u}_i^{fb,s}(t)$ denotes the control input from a stabilizing feedback controller.
Next, both the controllers will be discussed in detail.

1) Feedback controller: Without any loss of generality, we assume a stabilizing feedback controller of the form:

$$\mathbf{u}^{fb,s}(t) = K\left(\mathbf{x}_r(t) - \mathbf{x}^s(t)\right), \ \forall t \in [0,\ T_f], \tag{8}$$

With this feedback control law, the closed loop of the system (2) becomes

$$\delta\dot{\mathbf{x}}^s = \mathbf{f}(\mathbf{x}_r) - \mathbf{f}(\mathbf{x}^s) + \mathbf{g}(\mathbf{x}_r)\mathbf{u}_r - \mathbf{g}(\mathbf{x}^s)K\delta\mathbf{x}^s. \tag{9}$$

where $\delta\mathbf{x}^s = \mathbf{x}_r - \mathbf{x}^s$. The feedback gain is designed such that the following assumption is satisfied:

Assumption 6: For the given feedback gain matrix K, there are a continuous differentiable Lyapunov function $V : \mathcal{D} \to R_{\geq 0}$, and class-$\mathcal{K}$ functions: α_1, α_2, α_3, and α_4 such that the following inequalities hold

$$\alpha_1(|\delta\mathbf{x}^s|) \leq V(\delta\mathbf{x}^s) \leq \alpha_2(|\delta\mathbf{x}^s|) \tag{10}$$

$$\frac{\partial V}{\partial \delta\mathbf{x}^s}(\delta\mathbf{x}^s)\left[\mathbf{f}(\mathbf{x}_r) - \mathbf{f}(\mathbf{x}^s) - \mathbf{g}(\mathbf{x}^s)K\delta\mathbf{x}^s\right] \leq -\alpha_3(|\delta\mathbf{x}^s|) \tag{11}$$

$$\left|\frac{\partial V}{\partial \delta\mathbf{x}^s}\right| \leq \alpha_4(|\delta\mathbf{x}^s|). \tag{12}$$

Remark 6: The role of the state feedback controller is to ensure uniform boundedness of the state trajectories without the input saturation. Therefore it is possible to design ILCs that are LLC. In addition, state feedback provides robustness to non-repetitive disturbances and model uncertainties. It is noted that due to the existence of input saturation, we need to show that the trajectories of the system with saturated feedback are still bounded. ◦

Proposition 1: Assume that Assumption 3 and 6 hold. There exists a domain of attraction $\mathcal{D}$ such that the trajectories of the following dynamic system:

$$\delta\dot{\mathbf{x}}^s = \mathbf{f}(\mathbf{x}_r) - \mathbf{f}(\mathbf{x}^s) + \mathbf{g}(\mathbf{x}_r)\mathbf{u}_r - \mathbf{g}(\mathbf{x}^s)sat\left(K\delta\mathbf{x}^s, \mathbf{u}^*\right) \tag{13}$$

will stay in $\mathcal{D}$ if the initial condition $\mathbf{x}^0 \in \mathcal{D}$

Proof:The matrix K can be represented as $K = [\mathbf{k}_1\ \mathbf{k}_2\ \cdots \mathbf{k}_n]$ where $\mathbf{k}_j \in \mathcal{R}^m$, $\forall j = 1, \cdots, m$. It is noted that $\frac{\partial V}{\partial \delta\mathbf{x}^s}(\delta\mathbf{x}^s)\mathbf{g}(\mathbf{x}^s) \in \mathcal{R}^{1\times m}$ while $K\delta\mathbf{x}^s \in \mathcal{R}^m$. Let $\mathcal{D}_1$ be a compact set $|\mathbf{k}_j\delta x_j| \leq u_j^*,\ \forall j = 1, \cdots, m$. This indicates that saturation do not happen. It follows that the Lyapunov function V from Assumption 6 along the trajectories of the system (13) satisfy

$$\frac{\partial V}{\partial \delta\mathbf{x}^s}(\delta\mathbf{x}^s)\left[\mathbf{f}(\mathbf{x}_r) - \mathbf{f}(\mathbf{x}^s) - \mathbf{g}(\mathbf{x}^s)K\delta\mathbf{x}^s\right] + \frac{\partial V}{\partial \delta\mathbf{x}^s}\mathbf{g}(\mathbf{x}_r)\mathbf{u}_r \\ \leq -\alpha_3(|\delta\mathbf{x}^s|) + \alpha_4|\delta\mathbf{x}^s|B_r, \tag{14}$$

where $B_r = \max_{t\in[0,T]} |g(\mathbf{x}_r(t))\mathbf{u}_r(t)|$. Based on the properties of $\alpha_3(\cdot)$ and $\alpha_4(\cdot)$, it is always possible to find a compact set $\mathcal{D}$ such that $\frac{dV}{dt} \leq 0$. This indicates the boundedness of the trajectories of the system (13). This completes the proof. □

Remark 7: Note that the compact set $\mathcal{D}$ takes two possible forms. The first one is a ball in $\mathcal{R}^n$, which contains origin. The second one has the following format:

$$\mathcal{D} := \{\delta\mathbf{x}^s \mid \nu \leq |\delta\mathbf{x}^s| \leq \Delta\}$$

for some positive constants Δ and ν. However, in both cases, the trajectories are bounded. ◦

Remark 8: Note that due to Assumption 4, the initial error $\delta\mathbf{x}^s(0) = \mathbf{0}$, the feedback law will always work in a small interval of initial time instant. This can prevent the finite escape from happening. Under such a situation, if the input signal is bounded, there exists a compact set $\tilde{\mathcal{D}} \subset \mathcal{D}$, which depends on the bound of input signal, so that the trajectories of system (13) are bounded. ◦

2) D-type ILC: The standard D-type ILC is given by

$$\mathbf{u}_{i+1}^{ff,s}(t) = \mathbf{u}_i^{ff,s}(t) + q\Gamma(t)\dot{\mathbf{e}}_i^s(t), \\ \mathbf{u}_1^{ff,s}(t) = 0, \forall t \in [0,\ T_f],\ i = 1, 2, \cdots \tag{15}$$

where $\mathbf{e}_i^s(t) \triangleq \mathbf{y}_r(t) - \mathbf{y}_i^s(t)$, $\Gamma(t) \in \mathcal{R}^{m\times m}$ is a positive symmetric matrix gain and $q > 0$ is the learning rate.
For system (2) with control law (7), a sufficient condition for convergence is given by

$$\left|I_m - q\Gamma\frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\mathbf{x})\mathbf{g}(\mathbf{x})\right| < 1, \quad \forall \mathbf{x} \in \mathcal{D}, \tag{16}$$

which is independent of $\mathbf{f}(\mathbf{x})$ and feedback gain K. It can be shown using CM methods that the tracking error converges uniformly provided that the nonlinear mappings $\mathbf{f}(\cdot)$, $\mathbf{g}(\cdot)$, and $\mathbf{h}(\cdot)$ are GLC [7], [9], [11].

This work tries to incorporate a feedback control law (8) with a feed-forward D-type ILC without GLC in the presence of input saturation. It is worthwhile to highlight that the technical difficulties are:

- Even though feedback control law is saturated, it can ensure the bounded trajectories of the system (2). When both feedback and feed-forward controllers exist, due to the existence of input saturation, the boundedness of trajectories cannot be always guaranteed.
- A standard D-type feed-forward ILC design is based on CM method, which requires GLC condition of the system. Unless the trajectories of the system (2) are bounded, the D-type ILC cannot ensure the convergence of the tracking error.

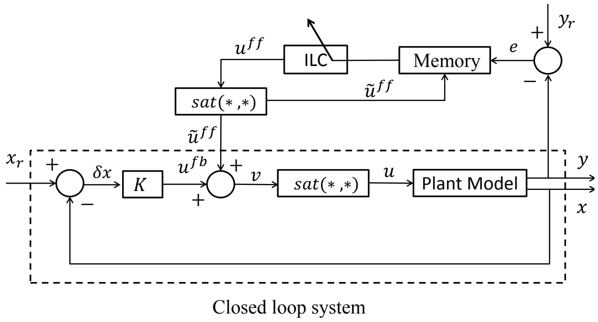

Fig. 2. Block diagram of a proposed control configuration

III. Proposed Control Configuration and Convergence Analysis

The proposed control structure consists of a modified D-type ILC with control saturation and an actuator saturation at the total control input as shown in Fig. 2.The same saturation limit, $\mathbf{u}^*$ is assumed for both the saturation functions. The total control input is given by:

$$\begin{aligned}\mathbf{u}_i(t) =& sat(\mathbf{v}_i(t),\ \mathbf{u}^*)\\ \mathbf{v}_i(t) =& \widetilde{\mathbf{u}}_i^{ff}(t) + \mathbf{u}_i^{fb}(t),\ \forall t \in [0,\ T_f],\ i \in \mathcal{N}.\end{aligned} \tag{17}$$

where $\widetilde{\mathbf{u}}_i^{ff}(t) = sat(\mathbf{u}_i^{ff}(t),\ \mathbf{u}^*)$ represents the modified input from the ILC control law given by:

$$\mathbf{u}_{i+1}^{ff}(t) = \widetilde{\mathbf{u}}_i^{ff}(t) + q\Gamma(t)\dot{\mathbf{e}}_i(t),\ \mathbf{u}_1^{ff}(t) = 0, \tag{18}$$

and $\mathbf{u}_i^{fb}(t)$ is the stabilizing feedback control given by:

$$\mathbf{u}_i^{fb}(t) = K\left(\mathbf{x}_r(t) - \mathbf{x}_i(t)\right), \tag{19}$$

which satisfies Assumption 6.
This leads to the following closed loop system with the consideration of the system (2) and the system (6):

$$\begin{aligned}\delta\dot{\mathbf{x}}_i =& \mathbf{f}(\mathbf{x}_r) - \mathbf{f}(\mathbf{x}_i) + \mathbf{g}(\mathbf{x}_r)\mathbf{u}_r - \mathbf{g}(\mathbf{x}_i)\mathbf{u}_i\\ =& \delta\mathbf{f}(\mathbf{x}_i) + \delta\mathbf{g}(\mathbf{x}_i)\mathbf{u}_r + \mathbf{g}(\mathbf{x}_i)\delta\mathbf{u}_i,\ i = 1, \ldots\end{aligned} \tag{20}$$

where $\delta\mathbf{x}_i \triangleq \mathbf{x}_r - \mathbf{x}_i$, $\delta\mathbf{u}_i \triangleq \mathbf{u}_r - \mathbf{u}_i$, $\delta\mathbf{f}(\mathbf{x}_i) \triangleq \mathbf{f}(\mathbf{x}_r) - \mathbf{f}(\mathbf{x}_i)$ and $\delta\mathbf{g}(\mathbf{x}_i) \triangleq \mathbf{g}(\mathbf{x}_r) - \mathbf{g}(\mathbf{x}_i)$.

Theorem 1: The system (2) with the control laws (17), (18), (19) satisfying the Assumptions 1–6 and the convergence condition (16)

1) can achieve zero output tracking error and state tracking error such that $\mathbf{e}_i(t)$ and $\delta\mathbf{x}_i(t)$ converges to zero uniformly;
2) can ensure uniform boundedness of state $\mathbf{x}_i(t)$ and feed forward input $\mathbf{u}_i^{ff}(t)$;
3) and the feed forward input $\mathbf{u}_i^{ff}(t)$ converges to reference input $\mathbf{u}_r(t)$ in $\mathcal{L}^2$ norm as $i \to \infty$.

Proof: The induction method will be used to show the boundedness of the trajectories of the closed loop system (20). In the proof of convergence, the following CEF is considered:

$$\begin{aligned}E_i(t) =& \frac{1}{2}e^{-\lambda t}\delta\mathbf{x}_{i-1}^T\delta\mathbf{x}_{i-1} + \int_0^t e^{-\lambda\tau}\delta\mathbf{u}_i^{ff^T}(\tau)\delta\mathbf{u}_i^{ff}(\tau)d\tau,\\ &\forall t \in [0, T_f], i \in \mathcal{N}, \mathbf{x}_0(t) = 0,\end{aligned} \tag{21}$$

for some positive constant λ. The CEF at current iteration consists of a time weighted state tracking error from previous iteration and $\mathcal{L}_e^2$ norm of feed forward control input error from current iteration.

Let Δ_1 be any positive constant. For a given positive constant λ and any given interval $[0, T_f]$ with $e^{-\lambda t}\left|\delta\mathbf{x}(t)\right|^2 \leq \Delta_1$, there exists a positive constant Δ_2 such that $\|\delta\mathbf{x}\|_s^2 \leq \Delta_2$. For this constant Δ_2, we can find a compact set $\mathcal{D}$ such that if $\|\delta\mathbf{x}\|_s^2 \leq \Delta_2$ is satisfied, the desired $\mathbf{x}_r(t)$, $\mathbf{x}$ and $\delta\mathbf{x}$ will stay in this compact set.

The proof is completed by using the Mathematical Induction method. Two steps are needed.

Step one will show that $E_1(t)$, $\delta\mathbf{x}_0(t)$ $\delta\mathbf{x}_1(t)$ are uniformly bounded by a compact set $\mathcal{D}$, for any $t \in [0, T_f]$.

Note that $\delta\mathbf{x}_0(t) = \mathbf{x}_r(t)$ is bounded and $\mathbf{u}_1^{ff}(t) = \mathbf{0}$, $E_1(t)$ is bounded. It is assumed that $E_1(t)$ is in a compact set $\mathcal{D}$.

It is noted that by applying Proposition 1, we can conclude that the trajectories of the first iteration is uniformly bounded. It is assumed that $\delta\mathbf{x}_1(t) \in \mathcal{D}$.

Step 2 will show that if $E_k(t)$ are bounded in the compact set, we can show that $E_{k+1}(t)$ will be in the same bound $\mathcal{D}$. From Remark 7, it is noted that $\delta\mathbf{x}_i(t)$ is uniformly bounded for any iteration.

Assume that at the k^{th} iteration, $\max_{t\in[0,T_f]} E_k(t) \leq \Delta_1$ and $\|\delta\mathbf{x}_{k-1}\|_s^2 \leq \Delta_2$. We will show that at the $(k+1)^{th}$ iteration, $\max_{t\in[0,T_f]} E_{k+1}(t) \leq \Delta_1$ and $\|\delta\mathbf{x}_k\|_s^2 \leq \Delta_2$. The construction of the compact set $\mathcal{D}$ will ensure that for all $t \in [0, T_f]$, $\mathbf{x}_k(t)$, $\delta\mathbf{x}_k(t)$ and $\mathbf{x}_r(t)$ are all in this compact set $\mathcal{D}$.

As for any $t \in [0, T_f]$, $\mathbf{x}_k(t) \in \mathcal{D}$, $\mathbf{x}_r \in D$, $\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)$ and $\mathbf{g}(\mathbf{x}_k)$ are bounded as per Assumption 2. Therefore define $\bar{\Lambda}_{h_x} \triangleq \max_{\mathbf{x}_k\in\mathcal{D}}\left|q\Gamma\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\right|$ and $\Lambda_g \triangleq \max_{\mathbf{x}_k\in\mathcal{D}}|\mathbf{g}(\mathbf{x}_k)|$. Denote $\Lambda_{\dot{x}_r} \triangleq \max_{\mathbf{x}\in\mathcal{D},\ t\in[0,\ T_f]}|\mathbf{f}(\mathbf{x}_r) + \mathbf{g}(\mathbf{x}_r)\mathbf{u}_r(t)|$, and $\Lambda_{u_r} \triangleq \max_{t\in[0,\ T_f]}|\mathbf{u}_r(t)|$.

At the $(k+1)^{th}$ iteration, the difference of energy function between two consecutive iterations is given by $\Delta E_{k+1} = E_{k+1} - E_k$, therefore

$$\begin{aligned}\Delta E_{k+1} =& \frac{1}{2}e^{-\lambda t}\left(\left|\delta\mathbf{x}_k\right|^2 - \left|\delta\mathbf{x}_{k-1}\right|^2\right)\\ &+ \int_0^t e^{-\lambda\tau}\left(\left|\delta\mathbf{u}_{k+1}^{ff}\right|^2 - \left|\delta\mathbf{u}_k^{ff}\right|^2\right)d\tau.\end{aligned} \tag{22}$$

By Assumption 4 and equations (20), (3) and (4), the first part of first term in equation (22) can be expanded as:

$$\begin{aligned}&\frac{1}{2}e^{-\lambda t}\delta\mathbf{x}_k^T\delta\mathbf{x}_k\\ &= -\frac{\lambda}{2}\int_0^t e^{-\lambda\tau}\delta\mathbf{x}_k^T\delta\mathbf{x}_k d\tau + \int_0^t e^{-\lambda\tau}\delta\mathbf{x}_k^T\delta\dot{\mathbf{x}}_k d\tau\\ &\leq -\left(\frac{\lambda}{2} - C_f - C_g\Lambda_{u_r}\right)\int_0^t e^{-\lambda\tau}\left|\delta\mathbf{x}_k\right|^2 d\tau\\ &\quad + \Lambda_g\int_0^t e^{-\lambda\tau}\left|\delta\mathbf{x}_k\right|\left|\delta\mathbf{u}_k\right|d\tau.\end{aligned} \tag{23}$$

By using completion of squares, we can show that there exists an $\alpha > 0$ such that:

$$|\delta\mathbf{x}_k| \left|\delta\widetilde{\mathbf{u}}_k^{ff}\right| = \sqrt{\alpha}\,|\delta\mathbf{x}_k| \frac{1}{\sqrt{\alpha}} \left|\delta\widetilde{\mathbf{u}}_k^{ff}\right| \leq \frac{\alpha}{2}|\delta\mathbf{x}_k|^2 + \frac{1}{2\alpha}\left|\delta\widetilde{\mathbf{u}}_k^{ff}\right|^2. \quad (24)$$

Using Lemma 2 on $|\delta\mathbf{u}_k|$ yields:

$$\begin{aligned}|\delta\mathbf{u}_k| &= |\mathbf{u}_r - \mathbf{v}_k - (sat(\mathbf{v}_k,\mathbf{u}^*) - \mathbf{v}_k)| \\ &\leq \left|\mathbf{u}_r - sat(\mathbf{u}_k^{ff},\mathbf{u}^*) - \mathbf{u}_k^{fb}\right| + |sat(\mathbf{v}_k,\mathbf{u}^*) - \mathbf{v}_k| \\ &\leq \left|\delta\widetilde{\mathbf{u}}_k^{ff} - \mathbf{u}_k^{fb}\right| + \left|\mathbf{u}_k^{fb}\right| \\ &\leq \left|\delta\widetilde{\mathbf{u}}_k^{ff}\right| + 2\left|\mathbf{u}_k^{fb}\right| = \left|\delta\widetilde{\mathbf{u}}_k^{ff}\right| + 2|K|\,|\delta\mathbf{x}_k|. \quad (25)\end{aligned}$$

where $\delta\widetilde{\mathbf{u}}_k^{ff} = \mathbf{u}_r - sat(\mathbf{u}_k^{ff},\mathbf{u}^*)$. Substituting (25) back into (23), followed by (24) gives

$$\frac{1}{2}e^{-\lambda t}\delta\mathbf{x}_k^T\delta\mathbf{x}_k \leq -\left(\frac{\lambda}{2} - \Lambda_a\right)\int_0^t e^{-\lambda\tau}|\delta\mathbf{x}_k|^2 d\tau + \frac{\Lambda_g}{2\alpha}\int_0^t e^{-\lambda\tau}\left|\delta\widetilde{\mathbf{u}}_k^{ff}\right|^2 d\tau, \quad (26)$$

where $\Lambda_a = C_f + C_g\Lambda_{u_r} + 2\Lambda_g|K| + \frac{\alpha}{2}\Lambda_g$.

From the ILC update law (18) we have

$$\delta\mathbf{u}_{k+1}^{ff} = \delta\widetilde{\mathbf{u}}_k^{ff} - q\Gamma\dot{\mathbf{e}}_k, \quad (27)$$

Using the plant dynamics (2) and reference dynamics (6), we get

$$\begin{aligned}\dot{\mathbf{e}}_k &= \frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_r)\dot{\mathbf{x}}_r - \frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\dot{\mathbf{x}}_k \\ &= \left(\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_r) - \frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\right)\dot{\mathbf{x}}_r + \frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\delta\dot{\mathbf{x}}_k \quad (28)\end{aligned}$$

Substituting (28) and back into (27), by using (20) provides,

$$\delta\mathbf{u}_{k+1}^{ff} = \mathrm{P}(\mathbf{x}_k)\delta\widetilde{\mathbf{u}}_k^{ff} + \boldsymbol{\zeta}_k, \quad (29)$$

where $\mathrm{P}(\mathbf{x}_k) = I_m - q\Gamma\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\mathbf{g}(\mathbf{x}_k)$, and

$$\begin{aligned}\boldsymbol{\zeta}_k = &-q\Gamma\left(\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_r) - \frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\right)\dot{\mathbf{x}}_r \\ &- q\Gamma\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\delta\mathbf{f}(\mathbf{x}_k) - q\Gamma\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\delta\mathbf{g}(\mathbf{x}_k)\mathbf{u}_r \\ &+ q\Gamma\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\mathbf{g}(\mathbf{x}_k)\left(\delta\widetilde{\mathbf{u}}_k^{ff} - \delta\mathbf{u}_k\right) \quad (30)\end{aligned}$$

As the convergence condition (16) is satisfied, it is possible to find $\Gamma > 0$ and $q > 0$ such that $|\mathrm{P}(\mathbf{x}_k)| < \rho < 1$.

From (29), we can show that

$$\begin{aligned}&\delta\mathbf{u}_{k+1}^{ff\,T}\delta\mathbf{u}_{k+1}^{ff} - \delta\widetilde{\mathbf{u}}_k^{ff\,T}\delta\widetilde{\mathbf{u}}_k^{ff} \\ &= -\delta\widetilde{\mathbf{u}}_k^{ff\,T}(I_m - |\mathrm{P}(\mathbf{x}_k)|^2)\delta\widetilde{\mathbf{u}}_k^{ff} + |\boldsymbol{\zeta}_k|^2 + 2\boldsymbol{\zeta}_k^T\mathrm{P}(\mathbf{x}_k)\delta\widetilde{\mathbf{u}}_k^{ff} \\ &\leq -\lambda_p\left|\delta\widetilde{\mathbf{u}}_k^{ff}\right|^2 + |\boldsymbol{\zeta}_k|^2 + 2|\boldsymbol{\zeta}_k|\left|\mathrm{P}(\mathbf{x}_k)\delta\widetilde{\mathbf{u}}_k^{ff}\right|, \quad (31)\end{aligned}$$

where $\lambda_p = \left|I_m - |\mathrm{P}(\mathbf{x}_k)|^2\right|$. As $|\mathrm{P}(\mathbf{x}_k)| < \rho < 1$, we have $\lambda_p > 0$. The second term in (31) can be written as:

$$\begin{aligned}|\boldsymbol{\zeta}_k| \leq &\left|-q\Gamma\left(\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_r) - \frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\right)\right||\dot{\mathbf{x}}_r| \\ &+ \left|-q\Gamma\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\right|(|\delta\mathbf{f}(\mathbf{x}_k)| + |\delta\mathbf{g}(\mathbf{x}_k)|\,|\mathbf{u}_r|) \\ &+ \left|q\Gamma\frac{\partial\mathbf{h}}{\partial\mathbf{x}}(\mathbf{x}_k)\right||\mathbf{g}(\mathbf{x}_k)|\left|\left(\delta\widetilde{\mathbf{u}}_k^{ff} - \delta\mathbf{u}_k\right)\right| \\ = &\Lambda_z|\delta\mathbf{x}_k| \quad (32)\end{aligned}$$

where $\Lambda_z = q|\Gamma|C_{h_x}\Lambda_{\dot{x}_r} + \bar{\Lambda}_{h_x}(C_f + C_g\Lambda_{u_r} + 2\Lambda_g|K|)$. Substituting (32) back into (31) and using (24) yields,

$$\begin{aligned}&\left|\delta\mathbf{u}_{k+1}^{ff}\right|^2 - \left|\delta\widetilde{\mathbf{u}}_k^{ff}\right|^2 \\ &\leq -\lambda_p\left|\delta\widetilde{\mathbf{u}}_k^{ff}\right|^2 + \Lambda_z^2|\delta\mathbf{x}_k|^2 + 2\Lambda_z\rho|\delta\mathbf{x}_k|\left|\delta\widetilde{\mathbf{u}}_k^{ff}\right| \\ &\leq -\left(\lambda_p - \frac{1}{\alpha}\Lambda_z\rho\right)\left|\delta\widetilde{\mathbf{u}}_k^{ff}\right|^2 + \left(\Lambda_z^2 + \alpha\Lambda_z\rho\right)|\delta\mathbf{x}_k|^2 \quad (33)\end{aligned}$$

In the second term of equation (22) applying Lemma 1 followed by substituting (33) yields

$$\begin{aligned}&\int_0^t e^{-\lambda\tau}\left(\left|\delta\mathbf{u}_{k+1}^{ff}\right|^2 - \left|\delta\mathbf{u}_k^{ff}\right|^2\right)d\tau \\ &\leq \int_0^t e^{-\lambda\tau}\left(\left|\delta\mathbf{u}_{k+1}^{ff}\right|^2 - \left|\delta\widetilde{\mathbf{u}}_k^{ff}\right|^2\right)d\tau \\ &\leq -\left(\lambda_p - \frac{1}{\alpha}\Lambda_z\rho\right)\int_0^t e^{-\lambda\tau}\left|\delta\widetilde{\mathbf{u}}_k^{ff}\right|^2 d\tau \\ &\quad + \left(\Lambda_z^2 + \alpha\Lambda_z\rho\right)\int_0^t e^{-\lambda\tau}|\delta\mathbf{x}_k|^2 d\tau. \quad (34)\end{aligned}$$

The following inequality can be obtained upon substituting (26) and (34) into (22), and considering the positiveness of $\frac{1}{2}e^{-\lambda t}|\delta\mathbf{x}_{k-1}|^2$,

$$\Delta E_{k+1} \leq -N_\lambda\int_0^t e^{-\lambda\tau}|\delta\mathbf{x}_k|^2 d\tau - N_\alpha\int_0^t e^{-\lambda\tau}\left|\delta\widetilde{\mathbf{u}}_k^{ff}\right|^2 d\tau, \quad (35)$$

where $N_\lambda = \frac{\lambda}{2} - \Lambda_a - \Lambda_z(\Lambda_z + \alpha\rho)$; $N_\alpha = \lambda_p - \frac{1}{2\alpha}(2\Lambda_z\rho + \Lambda_g)$. There exists $\alpha > 0$ and $\lambda > 2\Lambda_a + \Lambda_z(\Lambda_z + \alpha\rho)$ such that $N_\alpha > 0$ and $N_\lambda > 0$, which leads to $\Delta E_{k+1}(t) \leq 0$. Therefore, we can conclude $E_{k+1} \leq \Delta_1$ as well.

By using induction, for all $k = 1, 2, \ldots$, we can conclude the boundedness of the trajectories over the finite time $[0, T_f]$.

Moreover E_{k+1} is non-increasing along the iteration axis and satisfies (35). Hence

$$\begin{aligned}&\lim_{k\to\infty}\int_0^{T_f} e^{-\lambda\tau}\left[N_\lambda|\delta\mathbf{x}_k|^2 + N_\alpha\left|\delta\widetilde{\mathbf{u}}_k^{ff}\right|^2\right]d\tau = 0, \\ &\lim_{k\to\infty}\|\delta\mathbf{x}_k\|_{\mathcal{L}_e^2} = 0 \text{ and } \lim_{k\to\infty}\left\|\delta\widetilde{\mathbf{u}}_k^{ff}\right\|_{\mathcal{L}_e^2} = 0. \quad (36)\end{aligned}$$

Thus point-wise convergence is obtained. Convergence in the sense of $\mathcal{L}_e^2$ norm is equivalent to that of $\mathcal{L}^2$ norm. Using the Assumption 3 with the LLC condition of $\mathbf{f}(\cdot)$, $\mathbf{g}(\cdot)$ and the boundedness of $\mathbf{x}_i$, and $\mathbf{u}_i$, from equation (20), the boundedness of $\delta\dot{\mathbf{x}}_i$ can be assured. Therefore $\delta\mathbf{x}_i$ is uniformly continuous. Hence uniform convergence of $\delta\mathbf{x}_i$ is ensured when $i \to \infty$. This completes the proof. □

IV. An illustrative example

A simple scalar nonlinear system is used to illustrate the idea of this paper. Consider the system :

$$\begin{aligned}\dot{x} &= -3x + (1 + 2x^2)(u + 3), \\ y &= x, \quad (37)\end{aligned}$$

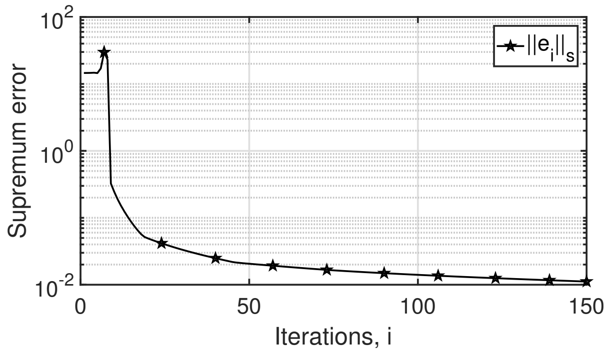

Fig. 3. Convergence of the supremum norm of the error

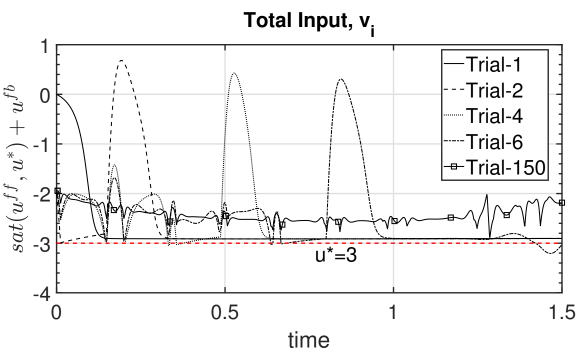

Fig. 4. Variation of total control input before saturation

does not satisfy BIBS condition.

When $x(0) = 2$ and $u(t) = -2$, the solution of the system is given by $x(t) = \frac{3-e^t}{3-2e^t}$ which is unbounded. Therefore the system is not BIBS. In addition, finite escape may happen as well. In particular, when there is no feedback and the feed-forward ILC is selected as (18), the trajectories of the system might be unbounded with a finite time interval.

The feedback controller is taken as $u^{fb}(t) = 0.2(x_r(t) - x(t))$. The simulation is performed with $x_r(t) = 2\cos(2t) + 3\sin(t)$ for $t = [0,\ 1.5]$ with the ILC update rate $q = 0.1$ and $u^* = 3$. With these chosen parameters, Assumptions 1–6 and the convergence condition (16) are satisfied for this example. The convergence of supremum norm of error in the iteration domain is shown in Fig.3. The control input $\mathbf{v}_i$ from (17) at the first few iterations and the last iteration are shown in Fig.4. Clearly, though input saturation happens, the tracking error still converges. This demonstrates the effectiveness of the proposed method.

V. CONCLUSION

For contraction mapping based design of feedback-based iterative learning control (ILC) schemes, global Lipschitz continuity is generally required. This paper proposes a feedback-based D-type ILC algorithm. Although ILC design is based on contraction mapping method, by introducing the feedback, the proposed feedback-based ILC algorithm is applicable to nonlinear dynamic system that satisfy neither global Lipschitz continuity condition nor bounded-input-bounded-state condition. Moreover, input saturation is also considered to address the constraints from actuators. A new composite energy function is proposed to prove the convergence of this control scheme with the help of novel induction based proof technique.

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