Feedback-Based Iterative Learning Design and Synthesis With Output Constraints for Robotic Manipulators

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***Abstract*—Feedback-based iterative learning control (ILC) has been proposed to improve the unacceptable transient performance (either in state or in output) in the iteration-domain. This letter addresses a special performance requirement of output constraints, which are motivated from the safety requirements in robotic manipulators. A barrier-function like Lyapunov function is used to design a new state feedback (or a proportional-derivative controller) to ensure that output constraints are satisfied in the finite time-domain. This state feedback is then combined with the standard feed-forward ILC to track the desired trajectory. With the help of composite energy function, it is shown that, for robotic manipulators, the proposed control method can achieve perfect tracking performance without violating output constraints in any iteration. Simulation results, which are based on the model of recently developed rehabilitation robot EMU, are presented to illustrate the effectiveness of the proposed controller.**

***Index Terms*—Iterative learning control, output constraints, barrier composite energy function, robotic manipulators.**

I. Introduction

ITERATIVE Learning Control (ILC) is a control technique to accurately and precisely track a reference trajectory for a repeated task. Although it has been initially presented to improve the tracking performance of robotic systems in [1], this technique has evolved to a new domain in the control literature and finds wide applications in precision motion control, batch manufacturing, robotic rehabilitation and so on (see survey papers [2]–[4] and references therein).

Two widely used tools in the design and analysis of ILC algorithms are Contraction Mapping (CM) and Composite Energy Function (CEF) [5]. Both techniques ignore the plant dynamics and focus on input-to-output mapping in the convergence analysis [6]. With a suitable ILC algorithm, the tracking error will converge with possibly undesirable transient performance in the first few iterations. If the system state is measurable, a possible solution is to incorporate the state feedback control into the ILC design to improve the transient performance in the iteration-domain [7].

This letter focuses on designing an appropriate state feedback-based ILC algorithm to track reference signals with the consideration of output constraints. To simplify the presentation, the robotic manipulators are considered with output constraints coming from the safety requirement [8].

Constraints on the input, state and output signals are common in engineering applications due to hardware limitations (input constraints) or safety requirements (state and/or output constraints). The input constraints have been addressed in the design of ILC for a certain class of nonlinear continuous-time systems using CEF based design [9]–[11].

When the system of interest is in discrete-time, by using the super vector formulation, the ILC with constraints can be converted into a constrained optimization problem in a high dimensional space, see [12]–[14] for input constraints and [15]–[17] for output constraints.

The state/output constraints in ILC for continuous-time systems are not fully investigated, though they are handled using barrier-function like Lyapunov function (BF-LF) in the stability analysis of nonlinear systems [18]. The BF-LF can be extended to learning control design when the alignment condition is satisfied [19]–[21]. However in those works, though repetition exists, the periodic task is defined over $t \in [0, \infty)$, making it possible to use BF-LF in the convergence analysis. In the standard ILC setting, the alignment condition is no longer satisfied. Adapting the concept of BF-LF in the standard ILC setting is a challenge. Other than using BF-LF as a unified tool, some heuristic methods based on bounded-error have been used in [22] and [23] to handle the output constraints and resulted in a premature termination of tracking in time-domain.

This letter proposes a new state feedback control law that can be incorporated into a standard feed-forward ILC design to achieve a perfect tracking performance without violating the output constraints at any iteration. It is worthwhile to highlight that the role of state feedback in the proposed control structure algorithm is two-fold. On the one hand, it guarantees that output constraints are satisfied. On the other hand, with the state feedback, the uniform boundedness of trajectories in both iteration-domain and time-domain can be ensured. This,

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in turn, can relax some assumptions for ILC design such as the global Lipschitz continuity (GLC) condition. The proposed feedback controller is sensitive to measurement noise, hence carefully designed filters are needed for implementation.

The contributions of this letter are summarized as follows:

1) A new feedback-based ILC scheme is proposed to address output constraints resulting from safety or workspace requirements of robotic manipulators.
2) A novel Barrier function based CEF (BCEF) is utilized to prove the convergence of tracking error, state, input signals, and satisfaction of output constraints.
3) Although the standard induction method is used to show the convergence, by carefully addressing the domain of attraction, it can be shown that a compact invariant set exists in the closed loop system. Such a compact set is used to show the satisfaction of output constraints and the convergence of ILC without GLC condition.

II. Preliminaries and Problem Formulation

The notations $\mathcal{R}$ and $\mathcal{N}$ represent the set of real numbers and natural numbers respectively. For any vector $\mathbf{x} \in \mathcal{R}^n$, $|\mathbf{x}|$ is defined as $|\mathbf{x}| = \sqrt{\mathbf{x}^\intercal \mathbf{x}}$. For a given matrix $A \in \mathcal{R}^{n\times n}$, $|A|$ represents the induced matrix norm. A square matrix $A > (\geq)0$ indicates a positive definite (semi-definite) matrix. For a square matrix A, $\lambda_{min}(A)$ denotes the minimum eigenvalue of A. I_n denotes the identity matrix of dimension n.

The notation $\mathcal{C}^j[0,\ T_f]$ denotes the set of all continuous functions in $[0,\ T_f]$ that are differentiable up to j^{th} order for any $j \in \mathcal{N}$. For any $\mathbf{x}(t) \in \mathcal{C}[0,\ T_f]$, the supremum norm is defined as $\|\mathbf{x}\|_s \triangleq \max_{t\in[0,\ T_f]} |\mathbf{x}(t)|_\infty$,where $|\mathbf{x}|_\infty = \max_{j\in[1,\ \ldots,\ n]} |x^j|$ and x^j denotes j^{th} element of $\mathbf{x} \in \mathcal{R}^n$. The corresponding $\mathcal{L}^2$ norm is defined as $\|\mathbf{x}\|_{\mathcal{L}^2} \triangleq (\int_0^{T_f} |\mathbf{x}(\tau)|^2 d\tau)^{\frac{1}{2}}$.

A. Plant Model

Consider the dynamic model of a revolute, direct-drive robotic manipulator with n rigid links [24]:

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{f}(\dot{\boldsymbol{\theta}}) + \mathbf{g}(\boldsymbol{\theta}) = \mathbf{u}, \tag{1}$$

where $\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\theta}} \in \mathcal{R}^n$ are the vector of joint angles, velocities and accelerations respectively. The control input $\mathbf{u} \in \mathcal{R}^n$ is the joint torque, $M(\cdot) \in \mathcal{R}^{n\times n}$ represents the inertia matrix, $C(\cdot,\cdot) \in \mathcal{R}^{n\times n}$ represents the total Coriolis and Centripetal terms, $\mathbf{f}(\cdot) \in \mathcal{R}^n$ is the friction component and $\mathbf{g}(\cdot) \in \mathcal{R}^n$ is the gravity force vector.

For simplicity of presentation, the model of a robotic manipulator is used as it has some properties to simplify the design of the controller. The same design technique can be extended to a general class of nonlinear dynamic systems under appropriate assumptions.

It is assumed that the model (1) is not completely known to the designer, but it has the following properties [24, Sec. 3.3].

Property 1: The inertia matrix, $M(\cdot)$ is symmetric and positive definite. There exist known parameters μ_1 and μ_2, such that $0 < \mu_1 I_n \leq M \leq \mu_2 I_n$.

Property 2: $(\dot{M} - 2C)$ is a skew symmetric matrix.

Property 3: For a given compact set, B_Δ, there exist three positive constants C_b, F_b, and G_b such that: $\left|C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\right| \leq C_b\left|\dot{\boldsymbol{\theta}}\right|$, $\left|\mathbf{f}(\dot{\boldsymbol{\theta}})\right| \leq F_b\left|\dot{\boldsymbol{\theta}}\right|$ and $|G(\boldsymbol{\theta})| \leq G_b$, for any $\left|\left[\,\boldsymbol{\theta}^\intercal\ \dot{\boldsymbol{\theta}}^\intercal\,\right]^\intercal\right| \leq \Delta$.

Remark 1: Different from Property 1 and Property 2, which hold globally, Property 3 is a local property. That is, three parameters are related to the compact set of the system. It is a global property if three inequalities hold globally with three parameters independent of the size of the compact set.

The system dynamics (1) can be represented by a nonlinear affine multiple-input-multiple-output (MIMO) square[1] system of the following form:

$$\begin{aligned} \dot{\mathbf{x}}_{1,i} &= \mathbf{x}_{2,i} \\ \dot{\mathbf{x}}_{2,i} &= \mathbf{h}(\mathbf{x}_{1,i}, \mathbf{x}_{2,i}) + M^{-1}(\mathbf{x}_{1,i})\mathbf{u}_i \\ \mathbf{y}_i &= \mathbf{x}_{1,i}, \end{aligned} \tag{2}$$

where $(\cdot)_i$ represents the variable at the i^{th} iteration, $\mathbf{x}_{1,i} = \boldsymbol{\theta}_i$, $\mathbf{x}_{2,i} = \dot{\boldsymbol{\theta}}_i$ and $\mathbf{h}(\mathbf{x}_{1,i}, \mathbf{x}_{2,i}) \triangleq -M^{-1}(\mathbf{x}_{1,i})(C(\mathbf{x}_{1,i}, \mathbf{x}_{2,i})\mathbf{x}_{2,i} + \mathbf{f}(\mathbf{x}_{2,i}) + \mathbf{g}(\mathbf{x}_{1,i}))$.

Obviously, the system (2) has a relative degree two [25, p. 407]. This information in needed in the design of ILC [26].

B. Assumptions

The following assumptions are widely used in the convergence analysis of ILC.

Assumption 1: There exists a reference output $\mathbf{y}_r = \mathbf{x}_{1,r} \in \mathcal{C}^2[0,\ T_f]$, $\mathbf{x}_{2,r} \in \mathcal{C}^1[0,\ T_f]$ and a reference input $\mathbf{u}_r \in \mathcal{C}[0,\ T_f]$ that satisfy

$$\begin{aligned} \dot{\mathbf{x}}_{1,r} &= \mathbf{x}_{2,r} \\ \dot{\mathbf{x}}_{2,r} &= \mathbf{h}(\mathbf{x}_{1,r}, \mathbf{x}_{2,r}) + M^{-1}(\mathbf{x}_{1,r})\mathbf{u}_r. \end{aligned} \tag{3}$$

This assumption ensures that there exists a reference input $\mathbf{u}_r$ for any given reference $\mathbf{y}_r$ with a well-behaved internal state. The tracking error is defined as $\mathbf{e}_i(t) \triangleq \mathbf{y}_r(t) - \mathbf{y}_i(t),\quad \forall t \in [0,\ T_f]$.

Assumption 2: It is assumed that the identical initial condition is satisfied in every iteration, *i.e.,* $\mathbf{x}_{1,i}(0) = \mathbf{x}_{1,r}(0)$, $\mathbf{x}_{2,i}(0) = \mathbf{x}_{2,r}(0)$, and $\dot{\mathbf{x}}_{2,i}(0) = \dot{\mathbf{x}}_{2,r}(0)$, $\forall i \in \mathcal{N}$.

Remark 2: For system (2), Assumption 2 indicates that $\mathbf{e}_i(0) = \mathbf{0}$, $\dot{\mathbf{e}}_i(0) = \mathbf{0}$ and $\ddot{\mathbf{e}}_i(0) = \mathbf{0}$ $\forall i \in \mathcal{N}$. Although this assumption can be relaxed at the cost of sacrificing a perfect tracking performance, it plays an important role in convergence analysis of the proposed control law. ∘

Consider the following feed-forward ILC law:

$$\mathbf{u}^{ff}_{i+1}(t) = \mathbf{u}^{ff}_i(t) + q\Gamma(t)\ddot{\mathbf{e}}_i(t),\ \ \mathbf{u}^{ff}_1(t) = 0, \tag{4}$$

where $\mathbf{u}^{ff}$ is the ILC input, $\Gamma(t) = \Gamma^T(t) > 0$ and $q > 0$ is the learning rate.

The following lemma shows that a standard feed-forward ILC can achieve perfect tracking when used with system (2).

Lemma 1: Assume that the system (2) satisfies Assumptions 1, 2 and Property 3 holds **globally**. If the control law (4) satisfies the convergence condition

$$\left|I_n - q\Gamma(t)M^{-1}(\mathbf{x}_1(t))\right| < 1,\ \ \forall t \in [0,\ T_f], \tag{5}$$

the output tracking error converges.

Proof: The proof of Lemma 1 can be found in [26, Th. 2] for system (2) with relative degree two. ■

[1] A square system has same the dimension for input and output vectors.

Remark 3: Lemma 1 needs the global Property 3 to ensure the global Lipschitz continuity of nonlinear mappings in (2). ILC algorithm (4) cannot be directly applied when only Property 3 holds. Moreover, even if the proposed ILC can work with convergent tracking error, it cannot ensure the satisfaction of output constraints.

Remark 4: If Property 3 holds globally, the convergence condition only needs the information of μ_1 and μ_2 with a simpler convergence condition (5), i.e., $\Gamma(t) = I_n$. If the nominal model of the inertia matrix M_0 is known, $\Gamma(t)$ can be selected such that $\Gamma(t)M_0^{-1} = I_n$, while q is selected to tune the convergence speed of the ILC.

C. Control Objective

The control objective is to find a control input sequence $\{\mathbf{u}_i\}_{i\in\mathcal{N}}$ such that the tracking error $\mathbf{e}_i$ converges to zero uniformly and the output at each iteration satisfies the constraint, i.e., $(\mathbf{y}_i(t))^\mathsf{T}\mathbf{y}_i(t) \le k_b^2, \forall t \in [0, T_f], \forall i \in \mathcal{N}$ for some $k_b > 0$. It is noted that the bound on output trajectories k_b depends on region of operation for the robotic manipulator in the joint space. For any given k_b, there always exists an $\varepsilon_b > 0$ such that if $\mathbf{e}_i^\mathsf{T}(t)\mathbf{e}_i(t) \le \varepsilon_b^2$ is satisfied, output constraints will be satisfied. Hence the control objective is to achieve the convergence of tracking error and satisfy the constraints $\mathbf{e}_i^\mathsf{T}(t)\mathbf{e}_i(t) \le \varepsilon_b^2$ in every iteration.

Remark 5: Note that the bound of any given reference $\mathbf{y}_r(t)$ ($\|\mathbf{y}_r\|_s \triangleq k_r$) is known. Usually the output will stay within an ε_b- neighborhood of this reference. It follows $|\mathbf{y}(t)| \le |\mathbf{y}_r(t)| + |\mathbf{e}(t)| \le |\mathbf{y}_r(t)| + \varepsilon_b$. If ε_b is selected as $\varepsilon_b = k_b - k_r$, $\mathbf{e}_i^\mathsf{T}(t)\mathbf{e}_i(t) \le \varepsilon_b^2$ will indicate that the output constraints are satisfied.

Remark 6: It is noted that the original output constraints can be converted into error-related constraints. This can simplify the convergence analysis at the cost of unnecessary tight constraints for each element of the error signals. Techniques such as adding an appropriate weighting in each element of the error could solve this problem. How to choose this weighting is application dependent. Due to space limitation, this issue will not be addressed in this letter.

III. The Proposed Feedback-Based ILC Design

The proposed control architecture consists of a state feedback and a feed-forward ILC. The role of feedback is to ensure that output constraints are not violated in any iteration whereas the ILC learns the desired control input $\mathbf{u}_r$.

The overall control input takes the following form:

$$\mathbf{u}_i(t) = \mathbf{u}_i^{ff}(t) + \mathbf{u}_i^{fb}(t), \ \forall t \in [0, \ T_f], \ i \in \mathcal{N}, \tag{6}$$

where $\mathbf{u}_i^{ff}(t)$ represents ILC input from (4) and $\mathbf{u}_i^{fb}(t)$ is a stabilizing feedback control law to ensure that the tracking error constraints are satisfied.

A. Design of Feedback Control via a Barrier-Function Like Lyapunov Function

This section introduces a BF-LF to design the feedback controller. If the output (or the tracking error) approaches the constraints, this Lyapunov function approaches infinity. The role of the state feedback is to ensure the boundedness of the BF-LF, indicating that output constraints are satisfied.

To assist the boundedness analysis of the tracking error, this section introduces a fictitious tracking error dynamics that are linked to the tracking error constraints. Then a feedback controller is presented. If the initial condition of BF-LF is bounded, the proposed state feedback will ensure that output constraints are satisfied by showing the boundedness of the BF-LF.

1) Feedback Controller: A fictitious velocity error $\boldsymbol{\sigma}$ is defined as:

$$\boldsymbol{\sigma} \triangleq \dot{\mathbf{e}} + \cos^2\left(\frac{\pi \mathbf{e}^\mathsf{T}\mathbf{e}}{2\varepsilon_b^2}\right)K_1\mathbf{e}, \tag{7}$$

where $K_1 = K_1^\mathsf{T} > 0$. The proposed stabilizing feedback control $\mathbf{u}^{fb}$ is given by

$$\mathbf{u}^{fb} = K_2\boldsymbol{\sigma} + \sec^2\left(\frac{\pi \mathbf{e}^\mathsf{T}\mathbf{e}}{2\varepsilon_b^2}\right)\mathbf{e}, \tag{8}$$

where $K_2 = K_2^\mathsf{T}$ is selected such that $\lambda_{min}(K_2) > 2$.

2) BF-LF: The design of feedback is based on the choice of BF-LF. This letter adapts the *tan*-type BF-LF proposed in [19] as:

$$V(\mathbf{e}, \boldsymbol{\sigma}) \triangleq \frac{\varepsilon_b^2}{\pi}\tan\left(\frac{\pi \mathbf{e}^\mathsf{T}\mathbf{e}}{2\varepsilon_b^2}\right) + \frac{1}{2}\boldsymbol{\sigma}^\mathsf{T}M(\mathbf{x}_1)\boldsymbol{\sigma}, \tag{9}$$

for all $|\mathbf{e}(0)| \le \varepsilon_b$. As $|\mathbf{e}(0)|$ is bounded, the boundedness of the BF-LF can ensure the satisfaction of the constraints as the BF-LF will diverge if the output approaches the constraint boundary.

Remark 7: By using L'Hospital's rule, the barrier function in BF-LF has the form $\lim_{\varepsilon_b\to\infty}\frac{\varepsilon_b^2}{\pi}\tan\left(\frac{\pi \mathbf{e}^\mathsf{T}\mathbf{e}}{2\varepsilon_b^2}\right) = \frac{1}{2}\mathbf{e}^\mathsf{T}\mathbf{e}$, when the constraints are not present. Consequently, the feedback control (8) becomes a standard PD type control law when $\varepsilon_b \to \infty$: $\lim_{\varepsilon_b\to\infty}\mathbf{u}^{fb} = (K_2K_1 + I_n)\mathbf{e} + K_2\dot{\mathbf{e}}$, which is a special form of the state feedback.

Proposition 1 shows that tracking error constraints are satisfied.

Proposition 1: Assume that the system (2) satisfies Assumption 2. With the feedback control law (8), if the feedforward control input is in $\mathcal{L}^2[0, T_f]$, the output of the closed loop system will satisfy $|\mathbf{y}(t)| \le k_b, \forall t \in [0, T_f]$.

Proof: A fictitious reference signal is defined as

$$\bar{\mathbf{u}}_r = M(\mathbf{x}_1)M^{-1}(\mathbf{x}_{1,r})\mathbf{u}_r. \tag{10}$$

Multiplying $M(\mathbf{x}_1)$ with the time derivative of (7) yields:

$$M(\mathbf{x}_1)\dot{\boldsymbol{\sigma}} = \delta\mathbf{u} - C(\mathbf{x}_1, \mathbf{x}_2)\boldsymbol{\sigma} + \boldsymbol{\omega}, \tag{11}$$

where $\delta\mathbf{u} \triangleq \bar{\mathbf{u}}_r - \mathbf{u}$, $\boldsymbol{\omega} \triangleq M(\mathbf{x}_1)\delta\mathbf{h}(\mathbf{x}_1, \mathbf{x}_2) + C(\mathbf{x}_1, \mathbf{x}_2)\boldsymbol{\sigma} + M(\mathbf{x}_1)\boldsymbol{\zeta}$, $\delta\mathbf{h}(\mathbf{x}_1, \mathbf{x}_2)) \triangleq \mathbf{h}(\mathbf{x}_{1,r}, \mathbf{x}_{2,r}) - \mathbf{h}(\mathbf{x}_1, \mathbf{x}_2)$, and

$$\boldsymbol{\zeta} \triangleq \cos^2\left(\frac{\pi \mathbf{e}^\mathsf{T}\mathbf{e}}{2\varepsilon_b^2}\right)K_1\dot{\mathbf{e}} - \frac{\pi}{\varepsilon_b^2}\sin\left(\frac{\pi \mathbf{e}^\mathsf{T}\mathbf{e}}{\varepsilon_b^2}\right)K_1\mathbf{e}^\mathsf{T}\dot{\mathbf{e}}. \tag{12}$$

Taking the time derivative of V in (9) results in

$$\begin{aligned}\dot{V} &= \sec^2\left(\frac{\pi \mathbf{e}^\mathsf{T}\mathbf{e}}{2\varepsilon_b^2}\right)\mathbf{e}^\mathsf{T}\dot{\mathbf{e}} + \frac{1}{2}\boldsymbol{\sigma}^\mathsf{T}\dot{M}(\mathbf{x}_1)\boldsymbol{\sigma} + \boldsymbol{\sigma}^\mathsf{T}M(\mathbf{x}_1)\dot{\boldsymbol{\sigma}} \\ &= \sec^2\left(\frac{\pi \mathbf{e}^\mathsf{T}\mathbf{e}}{2\varepsilon_b^2}\right)\mathbf{e}^\mathsf{T}\boldsymbol{\sigma} - \mathbf{e}^\mathsf{T}K_1\mathbf{e} - \boldsymbol{\sigma}^\mathsf{T}\mathbf{u}^{fb} + \boldsymbol{\sigma}^\mathsf{T}\delta\mathbf{u}^{ff} \\ &\quad + \boldsymbol{\sigma}^\mathsf{T}\boldsymbol{\omega} + \frac{1}{2}\boldsymbol{\sigma}^\mathsf{T}\big(\dot{M}(\mathbf{x}_1) - 2C(\mathbf{x}_1, \mathbf{x}_2)\big)\boldsymbol{\sigma},\end{aligned} \tag{13}$$

where $\delta\mathbf{u}^{ff} = \bar{\mathbf{u}}_r - \mathbf{u}^{ff}$.

Substituting $\mathbf{u}^{fb}$ from (8), leads to cancelling of the term $\sec^2(\frac{\pi \mathbf{e}^\intercal \mathbf{e}}{2\varepsilon_b^2})\mathbf{e}^\intercal \boldsymbol{\sigma}$. Because of the Property 2, the last term in (13) is zero. Therefore (13) can be written as

$$\dot{V} = -\mathbf{e}^\intercal K_1 \mathbf{e} - \boldsymbol{\sigma}^\intercal K_2 \boldsymbol{\sigma} + \boldsymbol{\sigma}^\intercal \delta \mathbf{u}^{ff} + \boldsymbol{\sigma}^\intercal \boldsymbol{\omega}. \tag{14}$$

Let $\eta_1 = \lambda_{min}(K_1)$, $\eta_2 = \lambda_{min}(K_2)$ and $V(t) = V(\mathbf{e}(t), \boldsymbol{\sigma}(t))$. Noticing that $V(0) = V(\mathbf{e}(0), \boldsymbol{\sigma}(0)) = 0$, it follows that

$$\begin{aligned} V(t) \leq & \int_0^t -\eta_1 |\mathbf{e}(\tau)|^2 d\tau - \int_0^t \eta_2 |\boldsymbol{\sigma}(\tau)|^2 d\tau \\ & + \int_0^t \frac{1}{2}\Big[|\boldsymbol{\sigma}(\tau)|^2 + \left|\delta \mathbf{u}^{ff}(\tau)\right|^2\Big] d\tau \\ & + \int_0^t \frac{1}{2}\Big[|\boldsymbol{\sigma}(\tau)|^2 + |\boldsymbol{\omega}(\tau)|^2\Big] d\tau. \\ \leq & \int_0^t -\eta_1 |\mathbf{e}(\tau)|^2 d\tau - \int_0^t |\boldsymbol{\sigma}(\tau)|^2 d\tau \\ & + \int_0^t \frac{1}{2}\Big[|\boldsymbol{\omega}(\tau)|^2 + \left|\delta \mathbf{u}^{ff}(\tau)\right|^2\Big] d\tau, \end{aligned}$$

as $\eta_2 \geq 2$. It is not hard to show that for $(\mathbf{e}, \boldsymbol{\sigma})$ in a given compact set, $\boldsymbol{\omega}$ is bounded. Moreover, as $\mathbf{u}^{ff} \in \mathcal{L}^2[0, T_f]$, $V(t)$ is uniformly bounded for any bounded $V(0)$. Thus, it is not hard to find an invariant compact set of the closed loop. As the initial condition $V(0) = 0$ is always inside this invariant compact set, this completes the proof. ■

In the proof of the main result, Proposition 1 is used to show that the output constraints are satisfied at the first iteration. Then a CEF is used to show that in each iteration, the feed-forward controller is still in $\mathcal{L}^2[0, T_f]$ with a uniform bound. By using induction as well as selecting appropriate compact sets, we can show that a perfect tracking performance is achieved and output constraints are satisfied.

IV. Convergence Analysis

The convergence of tracking error as well as satisfaction of output constraints are stated in Theorem 1.

Theorem 1: If Assumptions 1 and 2 are satisfied for the system (2), then the control laws (6), (8) and (4) satisfying the convergence condition (5) can

1) ensure that the output constraints are satisfied for any iteration;
2) achieve the convergence of tracking error, *i.e.*, $\lim_{i\to\infty} \mathbf{e}_i \to \mathbf{0}$ uniformly;
3) ensure that $\mathbf{u}_i^{ff}$ converges to the reference input $\mathbf{u}_r$ in $\mathcal{L}^2$ norm sense.

Proof: For simplification of notation, the variable t is omitted wherever appropriate.

Let $\boldsymbol{\psi} \triangleq [\mathbf{e}^\intercal \ \dot{\mathbf{e}}^\intercal]^\intercal$, then $\boldsymbol{\sigma} = \Omega \boldsymbol{\psi}$, where $\Omega = \Big[\cos^2(\frac{\pi \mathbf{e}^\intercal \mathbf{e}}{2\varepsilon_b^2})K_1 \ I_n\Big]$.

The following BCEF is considered for the analysis of convergence in the iteration-domain,

$$\begin{aligned} E_i = e^{-\lambda t} V_{i-1} + \int_0^t e^{-\lambda \tau} \delta \mathbf{u}_i^{ff\intercal} \delta \mathbf{u}_i^{ff} d\tau \\ \forall t \in [0, T_f], i \in \mathcal{N}, \lambda > 0, \ V_0(t) = 0 \end{aligned} \tag{15}$$

where $\delta \mathbf{u}_i^{ff} = \bar{\mathbf{u}}_r - \mathbf{u}_i^{ff}$.

The key idea of this BCEF is to carefully analyze the domain of attraction of the closed loop system to show the existence of an invariant set.

For any given $\lambda > 0$ and any interval $[0, \ T_f]$, there exists a positive constant $\Delta_0 > 0$ such that if $E \leq \Delta_0$, then $\boldsymbol{\psi}$ belongs to a compact set $\mathcal{D}$ and there exists $\Delta_1 > 0$ satisfying $\|\boldsymbol{\psi}\|_s^2 \leq \Delta_1$. If E is non-increasing along the iteration-domain, the compact set of $\boldsymbol{\psi}$ will be the same. Moreover, if E_i is bounded, it indicates that $\mathbf{u}_i^{ff}$ is in $\mathcal{L}^2[0, \ T_f]$. Proposition 1 shows that $\boldsymbol{\psi}_i$ is in an invariant compact set. Note that the resetting condition in Assumption 2 ensures that the initial condition will be always in this invariant compact set.

The proof using the induction involves two steps.

The first step is to show that for the first iteration E_1 is uniformly bounded such that the system (2) satisfies the output constraint as in the first iteration $\mathbf{u}_1^{ff} = 0$ and $V_0 = 0$. Because of the uniform continuity and the boundedness of $\mathbf{u}_r$, E_1 is uniformly continuous. By applying Proposition 1, the output trajectory satisfies the constraint and $\boldsymbol{\psi}_1$ belongs to a compact set $\mathcal{D}$.

Assume that V_{j-1} is bounded and E_j is bounded and positive. This indicates that $\mathbf{u}_j^{ff} \in \mathcal{L}^2[0, \ T_f]$ as $\delta \mathbf{u}_j^{ff} \in \mathcal{L}^2[0, \ T_f]$ and $\mathbf{u}_r$ is uniformly bounded. The induction is to show that $E_{j+1} \leq E_j$ is bounded and $\mathbf{u}_{j+1}^{ff} \in \mathcal{L}^2[0, \ T_f]$ is finite.

At the $(j+1)^{\text{st}}$ iteration, the difference of the CEF between two consecutive iterations is given by $\Delta E_{j+1} = E_{j+1} - E_j$, therefore

$$\begin{aligned} \Delta E_{j+1} = & e^{-\lambda t}\big(V_j - V_{j-1}\big) \\ & + \int_0^t e^{-\lambda \tau} \left(\left|\delta \mathbf{u}_{j+1}^{ff}\right|^2 - \left|\delta \mathbf{u}_j^{ff}\right|^2\right) d\tau. \end{aligned} \tag{16}$$

Firstly, (16) can be expanded to an integral form. Substitution of (9) yields

$$\begin{aligned} e^{-\lambda t} V_j = & \int_0^t e^{-\lambda \tau} \dot{V}_j d\tau - \lambda \int_0^t e^{-\lambda \tau} V_j d\tau \\ \leq & \int_0^t e^{-\lambda \tau} \dot{V}_j d\tau - \frac{\lambda \varepsilon_b^2}{\pi} \int_0^t e^{-\lambda \tau} \tan\left(\frac{\pi \mathbf{e}_j^\intercal \mathbf{e}_j}{2\varepsilon_b^2}\right) d\tau \\ & - \frac{\lambda}{2} \int_0^t e^{-\lambda \tau} \boldsymbol{\sigma}_j^\intercal M(\mathbf{x}_j) \boldsymbol{\sigma}_j d\tau. \end{aligned} \tag{17}$$

It can be shown that, for the given compact set $\mathcal{D}$, there exist positive constants C_s and C_t such that $|\boldsymbol{\omega}| \leq C_t |\boldsymbol{\psi}| + C_s |\boldsymbol{\psi}|^2$ and $|\boldsymbol{\sigma}| \leq C_s |\boldsymbol{\psi}|$.

By using completion of squares, there exists a $\beta > 0$ such that:

$$\left|\boldsymbol{\psi}_j\right|\left|\delta \mathbf{u}_j^{ff}\right| \leq \frac{\beta}{2}\left|\boldsymbol{\psi}_j\right|^2 + \frac{1}{2\beta}\left|\delta \mathbf{u}_j^{ff}\right|^2. \tag{18}$$

For the proof by induction, it is assumed that constraints are satisfied for j^{th} iteration. By considering the positiveness of $\tan(\frac{\pi \mathbf{e}_j^\intercal \mathbf{e}_j}{2\varepsilon_b^2})$, followed by substituting for $\dot{V}_j$ from (14) to (17) and using (18), it can be shown that

$$\begin{aligned} e^{-\lambda t} V_j \leq & -\lambda \int_0^t e^{-\lambda \tau} \mu \left|\boldsymbol{\psi}_j\right|^2 d\tau + \int_0^t e^{-\lambda \tau} N(\left|\boldsymbol{\psi}_j\right|) d\tau \\ & + \frac{C_s}{2\beta} \int_0^t e^{-\lambda \tau} \left|\delta \mathbf{u}_j^{ff}\right|^2 d\tau, \end{aligned} \tag{19}$$

where $\mu(t) = \frac{1}{2}\lambda_{min}(\Omega_j{}^\intercal M(\mathbf{x}_j)\Omega_j) \geq \delta > 0$. Here $N(\left|\boldsymbol{\psi}_j\right|) = (-\lambda_{min}(\Omega_j^\intercal K_2 \Omega_j) + C_s C_t + \frac{C_s \beta}{2})\left|\boldsymbol{\psi}_j\right|^2 + C_s^2 \left|\boldsymbol{\psi}_j\right|^3$ is a polynomial function in $|\boldsymbol{\psi}|$.

Secondly, using (4) shows that

$$\delta \mathbf{u}_{j+1}^{ff} = \mathrm{P}(\mathbf{x}_{1,j})\delta \mathbf{u}_j^{ff} + \mathbf{w}_j, \tag{20}$$

where $\mathrm{P}(\mathbf{x}_{1,j}) = I_n - q\Gamma M^{-1}(\mathbf{x}_{1,j})$ and $\mathbf{w}_j = -q\Gamma\delta\mathbf{h}(\mathbf{x}_{1,j}, \mathbf{x}_{2,j}) + q\Gamma M^{-1}(\mathbf{x}_{1,j})\mathbf{u}_j^{fb}$.

If the convergence condition (5) is satisfied, it is possible to find $\Gamma > 0$ and $q > 0$ such that $|\mathrm{P}(\mathbf{x}_{1,j})| < \rho < 1$. For j^{th} iteration, $|\mathbf{u}_j^{fb}|$ is bounded. Using the Lipschitz continuity of $q\Gamma\mathbf{h}(\cdot,\cdot)$ and the Property 1, it can be shown that there exist two positive constants C_u and C_h such that

$$\begin{aligned}\delta \mathbf{u}_{j+1}^{ff\,\mathsf{T}}\delta \mathbf{u}_{j+1}^{ff} - \delta \mathbf{u}_j^{ff\,\mathsf{T}}\delta \mathbf{u}_j^{ff} &\le -(I_n - |\mathrm{P}(\mathbf{x}_{1,j})|^2)|\delta \mathbf{u}_j^{ff}|^2 + |\mathbf{w_j}|^2 + 2\rho|\mathbf{w_j}||\delta \mathbf{u}_j^{ff}| \\ &\le -(\lambda_p - C_u)|\delta \mathbf{u}_j^{ff}|^2 + C_h|\boldsymbol{\psi}_j|^2, \end{aligned}\tag{21}$$

where $\lambda_p = \left|I_n - |\mathrm{P}(\mathbf{x}_{1,j})|^2\right| > 0$.

Finally, substituting (19) and (21) into (16) and considering the positiveness of V_{j-1} yields

$$\begin{aligned}\Delta E_{j+1} \le& -\lambda\int_0^t e^{-\lambda\tau}\mu|\boldsymbol{\psi}_j|^2 d\tau + \int_0^t e^{-\lambda\tau}R(|\boldsymbol{\psi}_j|)d\tau \\ &- (\lambda_p - C_{lu})\int_0^t e^{-\lambda\tau}|\delta\tilde{\mathbf{u}}_j^{ff}|^2 d\tau, \end{aligned}\tag{22}$$

where $R(|\boldsymbol{\psi}_j|) = N(|\boldsymbol{\psi}_j|) + C_h|\psi_j|^2$, $C_{lu} = C_u + \frac{C_s}{2\beta}$. Hence, there exists a $r_0 > 0$ such that $R(|\boldsymbol{\psi}_j|) = r_0|\boldsymbol{\psi}_j|^2 + C_s^2|\boldsymbol{\psi}_j|^3$ when $\boldsymbol{\psi}_j$ is in a compact set.

For $\lambda > \frac{1}{\mu}(r_0 + C_s^2\Delta_1)$ and $\lambda_p > C_{lu}$, $\Delta E_{j+1} \le 0$. Hence it is possible to find $N_\lambda > 0$ and $N_\beta > 0$ such that

$$\Delta E_{j+1} \le -\int_0^t e^{-\lambda\tau}\left[N_\lambda|\boldsymbol{\psi}_j|^2 + N_\beta|\delta\mathbf{u}_j^{ff}|^2\right]d\tau. \tag{23}$$

This leads to $E_{j+1} \le \Delta_0$ and $\boldsymbol{\psi}_j$ belongs to the compact set $\mathcal{D}$. Hence by using induction, for all $j = 1, 2, 3, \ldots,$ all the trajectories $\boldsymbol{\psi}_{j+1}$ satisfy the constraints.

Moreover E_{j+1} is non-increasing along the iteration axis and satisfies (23). Hence $\lim_{j\to\infty} V_{j-1} = 0$ and $\lim_{j\to\infty}\|\delta\mathbf{u}_j^{ff}\|_{\mathcal{L}^2} = 0$. Thus point-wise convergence is achieved, leading to the uniform convergence of $\boldsymbol{\psi}_i$ [9]. Therefore, we can conclude the convergence of control input in $\mathcal{L}^2$ norm. This completes the proof. ∎

V. An Illustrative Example

In order to demonstrate the effectiveness of the proposed method, it is applied to the robot manipulator, EMU, which has three degrees of freedom and was built in the Robotics Lab at The University of Melbourne [27].

Let $\boldsymbol{\theta} = [\theta_1,\ \theta_2,\ \theta_3]^\mathsf{T}$. The model is given by

$$M(\boldsymbol{\theta}) = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & P_5 & P_4\sin(\theta_2-\theta_3) \\ 0 & P_4\sin(\theta_2-\theta_3) & P_6 \end{bmatrix},$$

$$C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} = \begin{bmatrix} 2\dot\theta_1\dot\theta_2\cos(\theta_2)(P_4\cos(\theta_3) - P_3\sin(\theta_2)) \\ -2\dot\theta_1\dot\theta_3\sin(\theta_3)(P_2\cos(\theta_3) + P_4\sin(\theta_2)) \\ \frac{P_3}{2}\dot\theta_1^2\sin(2\theta_2) - P_4\dot\theta_3^2\cos(\theta_2-\theta_3) \\ -P_4\dot\theta_1^2\cos(\theta_2)\cos(\theta_3) \\ P_4\dot\theta_2^2\cos(\theta_2-\theta_3) \\ +\dot\theta_1^2\sin(\theta_3)(P_2\cos(\theta_3) + P_4\sin(\theta_2)) \end{bmatrix},$$

$$\begin{aligned}\mathbf{g}(\boldsymbol{\theta}) &= [0,\quad P_7\sin(\theta_2),\quad P_8\cos(\theta_3)]^\mathsf{T}, \\ \mathbf{f}(\dot{\boldsymbol{\theta}}) &= [P_9\dot\theta_1,\quad P_{10}\dot\theta_2,\quad P_{11}\dot\theta_3]^\mathsf{T}, \end{aligned}\tag{24}$$

where $m_1 = P_1 + P_2\cos^2(\theta_3) + P_3\cos^2(\theta_2) + 2P_4\cos(\theta_3)\sin(\theta_2)$. The model parameters which are identified from experiments are given in Table I.

TABLE I
Model Parameters

P_1	P_2	P_3	P_4	P_5	P_6
0.66	0.42	−0.63	0.34	0.44	0.38

P_7	P_8	P_9	P_{10}	P_{11}
−6.36	−3.66	4.67	1.63	3.75

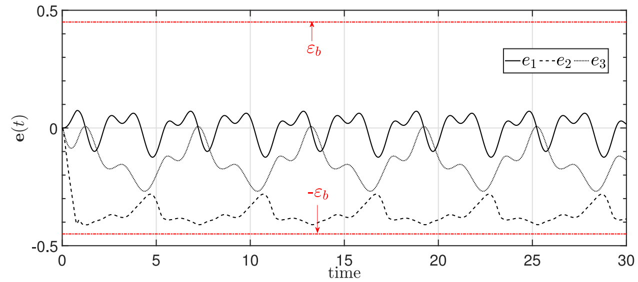

Fig. 1. Tracking error in time-domain for $T_f = 30$ when $\mathbf{u}^{ff} = 0$.

Two simulations are performed to illustrate the effectiveness of this controller. For both the simulations, the output trajectory is chosen as, $\mathbf{y}_r(t) = [y_{r_1}(t),\ y_{r_2}(t),\ y_{r_3}(t)]^\mathsf{T}$, where $y_{r_1}(t) = 0.5\sin^3(\frac{\pi t}{3})\cos(\frac{\pi t}{3})$, $y_{r_2}(t) = 0.4\sin^3(\frac{\pi t}{3}) + 0.7$, $y_{r_3}(t) = 0.5\sin^3(\frac{\pi t}{3}) + 0.8$. For the given reference trajectory, the desired region of operation for the output trajectory is given by the error bound, $\varepsilon_b = 0.45$.

Firstly, the performance of proposed feedback control (8) is demonstrated when the feed-forward input is zero. This represents the first iteration of the proposed control structure. The feedback gains K_1 and K_2 are chosen as diagonal matrices with element [3, 3.6, 3.6] and [3.6, 3, 2.4] respectively. The simulation is performed for a finite time $T_f = 30$. When there is no feed-forward input, the error trajectories, $\mathbf{e} = [e_1,\ e_2,\ e_3]^\mathsf{T}$, satisfy constraints as shown in Fig. 1. But a perfect tracking in a finite-time interval cannot be achieved.

The second simulation is performed with the proposed feedback controller (8) and the ILC (4) with a finite time interval $T_f = 3$ for 200 iterations satisfying the Assumption 2. The learning parameters are selected as $q = 0.7$, and $\Gamma = \begin{bmatrix} 0.504 & 0 & 0 \\ 0 & 0.394 & -0.124 \\ 0 & -0.124 & 0.342 \end{bmatrix}$, which satisfies the convergence condition (5). The convergence of output tracking error

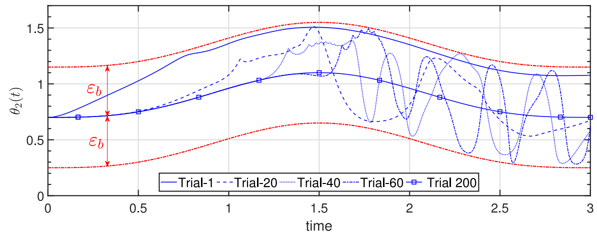

Fig. 2. Output trajectory, $\theta_2(t)$ for trials: 1, 20, 40, 60, and 200.

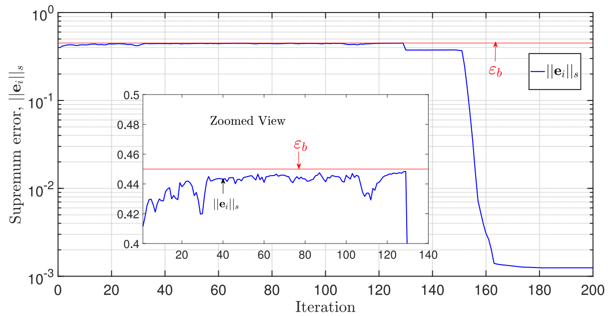

Fig. 3. Output tracking error in the iteration-domain with $T_f = 3$.

in terms of supremum norm, $\|\mathbf{e}\|_s$ in the iteration-domain is shown in Fig. 3. An output trajectory, θ_2, in time-domain for iteration 1, 20, 40, 60 and 200 are shown in Fig. 2. It can be seen that the supremum norm of the tracking error is less than the ε_b bound in any iteration, i.e., output constraints are satisfied. Hence, the proposed controller achieves the control objective. It is noted that Theorem 1 shows the convergence, but the supremum norm does not converge monotonically. In Fig. 3, the supremum norm of error shows the convergence as well as the satisfaction of constraints.

Experiments for different choices of ε_b have been obtained. It is observed that when ε_b is getting smaller, the feedback controller dominates, resulting in more iterations for convergence. On the other hand, a large ε_b leads to a faster convergence in the iteration domain. The influence of feedback gain matrices K_1 and K_2 in the convergence speed of ILC will be investigated in our future work.

VI. Conclusion

This letter proposes a feedback-based ILC scheme to handle output constraints in robotic manipulators. The feed-forward ILC design is based on the Contraction Mapping design to track the desired trajectory whereas the feedback design is based on a barrier-like Lyapunov function to satisfy output constraints. A novel barrier composite energy function is used to prove the convergence of tracking error. A new induction based proof technique is incorporated into the analysis to show that the output constraints are not transgressed in any iteration while the ILC learns the desired control input.

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