Series data and growth constant, amplitude and exponent estimates

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In this appendix we have gathered together the series expansions for self-avoiding polygons on square, honeycomb and triangular lattices enumerated by either perimeter or area and the counts for the number of polyominoes on the same lattices. In addition we provide data for the number of SAP on three-dimensional lattices and the number of three-dimensional polyominoes (or polycubes).

Below we also provide a listing for the estimated growth constants[1], critical amplitudes and critical exponents for these problems. For any lattice, the growth constant for SAP and SAW is the same. The amplitude B for polygons is defined through $p_n \sim B\mu^n n^{\alpha-3}$. For polyominoes if the growth constant is τ, the amplitude B is defined by assuming the number of n-celled polyominoes grows as $B\tau^n/n$, while for polycubes the correponding expression is $B\tau^n/n^{1.5}$. In estimating the amplitudes of polygons, we used the value of the growth constant μ in the table below and assumed $\alpha = 0.5$ for two-dimensional lattices, and $\alpha = 0.23721$ for three-dimensional lattices. The analysis of the amplitudes assumed only analytic correction-to-scaling terms. For the two-dimensional problems we believe this to be appropriate, while for the three-dimensional problems it is generally believed that there are non-analytic corrections to scaling, but the data we have is so limited that incorporating this refinement into the analysis is probably not justified.

[1] The estimate for cubic SAP exponents has been obtained by Nathan Clisby using as yet unpublished Monte Carlo data.

Table 1: Growth constants and amplitudes for various problems and lattices. For any lattice, the growth constant for SAP and SAW is the same.

Problem	Growth constant	Amplitude
Honeycomb SAP by perimeter	$\sqrt{2+\sqrt{2}} = 1.84775\ldots$	1.2719299(1)
Square SAP by perimeter	2.63815853031(3)	0.56230130(2)
Triangular SAP by perimeter	4.150797226(26)	0.2639393(1)
Diamond SAP by perimeter	2.87905(12)	0.3057(5)
Simple cubic SAP by perimeter	4.684043(12)	0.2625(5)
Body-centred cubic SAP by perimeter	6.5304(4)	0.2403(5)
Face-centred cubic SAP by perimeter	10.0363(6)	0.1173(5)
Honeycomb SAP by area	5.161930154(8)	0.2808499(1)
Square SAP by area	3.97094397(9)	0.408105(2)
Triangular SAP by area	2.9446600(8)	1.33652(6)
Polycubes	8.344(10)	0.184(3)
Honeycomb polyominoes	5.1831453(4)	0.273525(5)
Square polyominoes	4.0625696(5)	0.316915(5)
Triangular polyominoes	3.0359688(3)	0.81243(3)

References

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[2] P. Butera and M. Comi, 1999 Enumeration of the self-avoiding polygons on a lattice by the Schwinger-Dyson equations *Ann. Comb*. **3** 277

[3] Clisby N, Liang R and Slade G 2007 Self-avoiding walk enumeration via the lace expansion *J. Phys. A: Math, Theor.* **40** 10973–11017

[4] Clisby N 2008 Private communication. A calculation for this appendix performed in 29.5 hours on the Victorian Partnership for Advanced Computing (VPAC) machine 'tango', which consists of AMD Opteron processors.

[5] Jensen I 2003 Counting Polyominoes: A Parallel Implementation for Cluster Computing *Lect. Notes Comp. Science* **2659** 203–212

[6] Jensen I 2003 A parallel algorithm for the enumeration of self-avoiding polygons on the square lattice *J. Phys. A: Math. Gen.* **36** 5731–5745

[7] Jensen I 2004 Self-avoiding walks and polygons on the triangular lattice *J. Stat. Mech: Theor. Exp.* P10008

Table 2: Critical exponents. Note the hyperscaling relation $d\nu = 2 - \alpha$, which has been used in estimating α from estimates of μ. For polygons, the crtical exponent is α, while for SAW it is γ. The size exponent, for both SAW and SAP, is ν.

Lattice dimension and model	Exponent α	Exponent γ	Exponent ν
2 dimensional SAP, SAW	1/2	43/32	3/4
2 dimensional polyominoes	0	n/a	3/4
3 dimensional SAP, SAW	0.237209(5)	1.156594(10)	0.587597(5)

[8] Jensen I 2006 Honeycomb lattice polygons and walks as a test of series analysis techniques *J. Phys.: Conf. Ser.* **42** 163–178

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[14] Vöge M and Guttmann A J 2003 On the Number of Hexagonal Polyominoes *Theor. Comp. Science* **307** 433–453

Table 3: Honeycomb lattice SAP by perimeter [8]

n	p_n	n	p_n
6	1	84	491178083710637922
8	0	86	1581253032639606603
10	3	88	5097396231311136576
12	2	90	16453443639137280047
14	12	92	53174059422032707866
16	18	94	172050510835696160358
18	65	96	557317380072692456032
20	138	98	1807258331991309982446
22	432	100	5866638786295311037590
24	1074	102	19063052682366555372695
26	3231	104	62002958760017830656522
28	8718	106	201852240235414126847460
30	25999	108	657719622379254733608348
32	73650	110	2144964109006762422233748
34	220215	112	7000963262791176379323774
36	643546	114	22868699709042371811249463
38	1937877	116	74758226305954241461635666
40	5783700	118	244567626180741238468116552
42	17564727	120	800664473720027754818907952
44	53222094	122	2623031329578131961697097700
46	163009086	124	8598999467519861534628992430
48	499634508	126	28208148022365305050992600420
50	1542392088	128	92592230245440655263097731630
52	4770925446	130	304116068616798314381111483673
54	14832934031	132	999449733343511867195340557200
56	46227584010	134	3286484546424040737658726637502
58	144632622552	136	10812942780599284505769165580800
60	453628244950	138	35595150650752109542282463008318
62	1427228330481	140	117237253821144147623389790232954
64	4500947210772	142	386333037219041809402422116617575
66	14231512500103	144	1273717978234721619312805589240114
68	45095972401236	146	4201399640625869267335427358479958
70	143219294049399	148	13864949790803209919972696473332510
72	455745199043542	150	45776305345419919371285850329676756
74	1453111646955645	152	151201484848510390617214210630381542
76	4641449091849300	154	499642320580635896564744348617472658
78	14851454597198009	156	1651754024778342247238567655133162198
80	47598148798881660	158	5462731645121972078039977740308145672
82	152789607567089925		

Table 4: Square lattice SAP by perimeter [6]

n	p_n	n	p_n
4	1	58	59270905595010696944
6	2	60	379108737793289505364
8	7	62	2431560774079622817356
10	28	64	15636142410456687798584
12	124	66	100792521026456246096640
14	588	68	651206027727607425003232
16	2938	70	4216407618470423070733556
18	15268	72	27355731801639756123505014
20	81826	74	177822806050324126648352460
22	449572	76	1158018792676190545425711414
24	2521270	78	7554259214694896127239818088
26	14385376	80	49360379260931646965916677280
28	83290424	82	323028185951187646733521902740
30	488384528	84	2117118644744425875029583096670
32	2895432660	86	13895130612692826326409919713700
34	17332874364	88	91319729650588816198004801698400
36	104653427012	90	600931442757555468862970353941700
38	636737003384	92	3959306049439766117380237943449096
40	3900770002646	94	26117050944268596220897591868398452
42	24045500114388	96	172472018113289556124895798382016316
44	149059814328236	98	1140203722938033441542255979068861816
46	928782423033008	100	7545649677448506970646886033356862162
48	5814401613289290	102	49985425311177130573540712929060556804
50	36556766640745936	104	331440783010043009106782321492277936522
52	230757492737449636	106	2199725502650970871182263620080571090156
54	1461972662850874916	108	14612216410979678692651320184958285074180
56	9293993428791901042	110	97148177367657853074723038687712338567772

Table 5: Triangular lattice SAP by perimeter [7]

n	p_n	n	p_n
3	2	32	2692047018699717
4	3	33	10352576717684506
5	6	34	39902392511347329
6	15	35	154126451419554156
7	42	36	596528356905096920
8	123	37	2313198287784319026
9	380	38	8986249863419780682
10	1212	39	34969337454759091232
11	3966	40	136301962040079085257
12	13265	41	532093404471021533628
13	45144	42	2080235431107538787148
14	155955	43	8144154378525048003270
15	545690	44	31927176350778729318192
16	1930635	45	125322778845662829008494
17	6897210	46	492527188641409773340797
18	24852576	47	1937931188484341585677962
19	90237582	48	7633665703654150673637363
20	329896569	49	30101946001283232799847562
21	1213528736	50	118823919397444557546535851
22	4489041219	51	469508402822449711313115200
23	16690581534	52	1856933773092076293566747007
24	62346895571	53	7351015093472721439659392448
25	233893503330	54	29126027071450640626653986531
26	880918093866	55	115500592701344029351721102550
27	3329949535934	56	458398255374927436357237021173
28	12630175810968	57	1820727406941365079260306390484
29	48056019569718	58	7237327695683743010999188700157
30	183383553173255	59	28789332223533619621001538109842
31	701719913717994	60	114602547490254934327469368968190

Table 6: Honeycomb lattice SAP by area [13, 9]

n	p_n	n	p_n
1	1	26	36138633393334038
2	3	27	179768675964165939
3	11	28	895425672624735867
4	44	29	4465589678921947602
5	186	30	22295966620155816954
6	813	31	111439693993112940196
7	3640	32	557558620919353655115
8	16590	33	2792233438943251452902
9	76663	34	13995852369729891369431
10	358195	35	70212003186716473817832
11	1688784	36	352506828543839738006802
12	8022273	37	1771125269041561567830953
13	38351973	38	8905113919188230264955009
14	184353219	39	44804571829235959198699855
15	890371070	40	225570974088699920561748746
16	4318095442	41	1136340745302289809680018862
17	21018564402	42	5727773558054438208070950886
18	102642526470	43	28887056504374868913302241736
19	502709028125	44	145763914212751560334802981991
20	2468566918644	45	735894997233174457602406978869
21	12150769362815	46	3716988842355112053567240722854
22	59937663454017	47	18783102592560998779533576292617
23	296245438278258	48	94958908613774943408509332060260
24	1466858366128911	49	480273434248924455452231252618009
25	7275229222292218	50	2430068453031180290203185942420933

Table 7: Square lattice SAP by area [11]

n	p_n	n	p_n
1	1	22	261803388854
2	2	23	996971935098
3	6	24	3802944302442
4	19	25	14528816598358
5	63	26	55585800967658
6	216	27	212949334034600
7	756	28	816822217132804
8	2684	29	3136762752545213
9	9638	30	12058858335360206
10	34930	31	46405735929935474
11	127560	32	178752169549746269
12	468837	33	689161111033801080
13	1732702	34	2659240868309971570
14	6434322	35	10269318260428629674
15	23993874	36	39687503569859369443
16	89805691	37	153488864908550236363
17	337237337	38	594011587420226879158
18	1270123530	39	2300345838908310537296
19	4796310672	40	8913696266990663512620
20	18155586993	41	34560203892113934050327
21	68874803609	42	134071571821918373415776

Table 8: Triangular lattice SAP by area [10]

n	p_n	n	p_n
1	2	31	13768900283696
2	3	32	39381761647878
3	6	33	112731209513148
4	14	34	322944141486223
5	36	35	925821793030182
6	94	36	2655999419889775
7	250	37	7624549165478464
8	675	38	21901410528537396
9	1832	39	62948996221716186
10	5005	40	181030734048561330
11	13746	41	520896427277498160
12	37901	42	1499600346000360661
13	104902	43	4319315951924817740
14	291312	44	12446880627889433646
15	811346	45	35884225522293806438
16	2265905	46	103498974852276615147
17	6343854	47	298641621862752294144
18	17801383	48	862063552257379673111
19	50057400	49	2489408387765856393710
20	141034248	50	7191418561913160942210
21	398070362	51	20782056815997725229126
22	1125426581	52	60077568702764010825658
23	3186725646	53	173732332629110214974028
24	9036406687	54	502560484888013883133120
25	25658313188	55	1454221557880565649765344
26	72946289247	56	4209231246688674394442949
27	207628101578	57	12187106400969184313465204
28	591622990214	58	35295544624608480713053597
29	1687527542874	59	102248441850332810905160592
30	4818113792640	60	296283374352751571959397999

Table 9: Honeycomb lattice polyominoes [14, 10]

n	p_n	n	p_n
1	1	24	1570540515980274
2	3	25	7821755377244303
3	11	26	39014584984477092
4	44	27	194880246951838595
5	186	28	974725768600891269
6	814	29	4881251640514912341
7	3652	30	24472502362094874818
8	16689	31	122826412768568196148
9	77359	32	617080993446201431307
10	362671	33	3103152024451536273288
11	1716033	34	15618892303340118758816
12	8182213	35	78679501136505611375745
13	39267086	36	396658618080234793950206
14	189492795	37	2001232317628022658203349
15	918837374	38	10103836183314489605735070
16	4474080844	39	51046672861235124190631667
17	21866153748	40	258063337786459258279344114
18	107217298977	41	1305417245856690662912152269
19	527266673134	42	6607298985024639624903163419
20	2599804551168	43	33460963467529713458350419245
21	12849503756579	44	169543788582768431534598929547
22	63646233127758	45	859496482176765849253640160036
23	315876691291677	46	4359288232974777294574313228655

Table 10: Square lattice polyominoes [5]

n	p_n	n	p_n
1	1	29	4820975409710116
2	2	30	18946775782611174
3	6	31	74541651404935148
4	19	32	293560133910477776
5	63	33	1157186142148293638
6	216	34	4565553929115769162
7	760	35	18027932215016128134
8	2725	36	71242712815411950635
9	9910	37	281746550485032531911
10	36446	38	1115021869572604692100
11	135268	39	4415695134978868448596
12	505861	40	17498111172838312982542
13	1903890	41	69381900728932743048483
14	7204874	42	275265412856343074274146
15	27394666	43	1092687308874612006972082
16	104592937	44	4339784013643393384603906
17	400795844	45	17244800728846724289191074
18	1540820542	46	68557762666345165410168738
19	5940738676	47	272680844424943840614538634
20	22964779660	48	1085035285182087705685323738
21	88983512783	49	4319331509344565487555270660
22	345532572678	50	17201460881287871798942420736
23	1344372335524	51	68530413174845561618160604928
24	5239988770268	52	273126660016519143293320026256
25	20457802016011	53	1088933685559350300820095990030
26	79992676367108	54	4342997469623933155942753899000
27	313224032098244	55	17326987021737904384935434351490
28	1228088671826973	56	69150714562532896936574425480218

Table 11: Triangular lattice polyominoes [10]

n	p_n	n	p_n
1	2	39	131764274746623618
2	3	40	390209282091660817
3	6	41	1156271319511222890
4	14	42	3428243851059071792
5	36	43	10170021606617062092
6	94	44	30185576357912854854
7	250	45	89638467588131276054
8	675	46	266316031025897652002
9	1838	47	791588201780520478260
10	5053	48	2353922513181100648048
11	14016	49	7002741498223502133792
12	39169	50	20841060277596144244446
13	110194	51	62049806988299870226456
14	311751	52	184809160446574540356778
15	886160	53	550633812416956110450696
16	2529260	54	1641167126237394780804458
17	7244862	55	4893142168882883602047972
18	20818498	56	14593611643638701475828219
19	59994514	57	43538430128312213641221102
20	173338962	58	129931105423136465757345880
21	501994070	59	387864007832776437943416162
22	1456891547	60	1158157489920023082651029625
23	4236446214	61	3459183249840776065090197424
24	12341035217	62	10334596819468361754858559890
25	36009329450	63	30883315424482772009364074195
26	105229462401	64	92312659826727115613777214819
27	307942754342	65	275995688697147821120388899585
28	902338712971	66	825360885842983560010493834969
29	2647263986022	67	2468783621745427137367974848117
30	7775314024683	68	7386128647683127584488035530328
31	22861250676074	69	22102564476279407636273464326490
32	67284446545605	70	66154257908909010874896059091279
33	198214729430994	71	198043122493458453529791815751245
34	584439943107748	72	592988333797578245954808147666097
35	1724665203979836	73	1775884216384559876692792048399568
36	5093434042872294	74	5319404853729116558903334777867316
37	15053558945238166	75	15936363137225733301433441827683823
38	44521869233046747		

Table 12: Diamond lattice SAP by perimeter [4]

n	p_n	n	p_n
6	2	22	759846
8	3	24	4930656
10	24	26	32852424
12	94	28	221672022
14	582	30	1519813822
16	3126	32	10538532360
18	19402	34	73902970188
20	118110		

Table 13: Simple cubic lattice SAP by perimeter [3]

n	p_n	n	p_n
4	3	20	1768560270
6	22	22	29764630632
8	207	24	512705615350
10	2412	26	9005206632672
12	31754	28	160810554015408
14	452640	30	2912940755956084
16	6840774	32	53424552150523386
18	108088232		

Table 14: Body-centred cubic lattice SAP by perimeter [2]

n	p_n	n	p_n
4	12	14	43702920
6	148	16	1282524918
8	2736	18	39354507576
10	61896	20	1250685059616
12	1759324	22	40887160690224

Table 15: Face-centred cubic lattice SAP by perimeter [12]

n	p_n	n	p_n
3	8	9	301376
4	33	10	2241420
5	168	11	17173224
6	970	12	134806948
7	6168	13	1079802216
8	42069	14	8798329080

Table 16: Number of polycubes [1]

n	p_n	n	p_n
1	1	10	8294738
2	3	11	60494549
3	15	12	446205905
4	86	13	3322769321
5	534	14	24946773111
6	3481	15	188625900446
7	23502	16	1435074454755
8	162913	17	10977812452428
9	1152870	18	84384157287999