‘Phased Transition’ to Phase Transition: The Network Consequences of Reconnecting

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EXECUTIVE SUMMARY

As New Zealand, and specifically Auckland, responds to the ongoing outbreak of COVID-19 which began in August, 2021, we seek to explain and explore how changes to Alert Level restrictions may impact on the network of interactions on which COVID-19 is spread. By representing Aotearoa as an interaction network, we can investigate how changes to patterns of interactions between individuals translate into structural network changes that affect potential transmission pathways. We use the Populated Aotearoa Interaction Network (PAIN), a synthetic network representing New Zealand, to illustrate how moves such as changing Alert Levels can affect the number of interactions that individuals will share and the expected size of largest connected component in the network. This combines analysis of the empirical interaction network with some estimates from network science for the level of connectivity in the interaction network for different scenarios of interactions outside the home and workplace.

A key finding of this report is that only a small increase in the number of connections between individuals from different dwellings (an increase from around 10% to around 20% of the number expected at Alert Level 1) is sufficient to increase the size of the largest connected component of the population who could be reached though transmission by a factor of 15; from around 90,000 to over 1.4 million.

1 Introduction

There has been much discussion around the strategy of opening up New Zealand, and specifically Auckland, as it deals with the ongoing impact of the outbreak of the Delta variant of COVID-19 which began in August 2021. On October 4 2021 the New Zealand government announced a phased transition to begin reconnecting Aucklanders. Over the coming weeks, Aucklanders will see restrictions eased, starting with two household bubbles at a time being permitted to gather outdoors with social distancing. While the easing of restrictions will be a large relief for many individuals who are missing friends and family, it is important to consider how this change in strategy could change the nature of transmission pathways across Auckland.

A useful way in which we can evaluate changes in transmission pathways is through analysis of interaction networks, a method commonly used by researchers studying outbreaks of infectious diseases. In simple terms, this network analysis involves representing society as series of nodes and links, where nodes can be used to represent people, and links can be used to represent interactions between individuals. Previous work has used such an interaction network for Aotearoa, combined with a sophisticated model of contagion processes, to directly simulate potential trajectories of outbreaks of COVID-19[1–4].

Representing Aotearoa as a network of connected individuals allows us to estimate the impact different policies may have on transmission of COVID-19. In addition to their role facilitating detailed simulation of contagion, such networks make it possible to investigate changes in network structure as a consequence of guidelines or rules related to how people interact. Over the past year, Auckland has experienced extreme shifts in network structure, as lock downs have been used with the goal of reducing the number of interactions between individuals, hence minimising transmission pathways. With decreases in Alert Levels, these transmission pathways begin to open up once again. The consequences of these changes for the structure of the resulting interaction network when connections are added or removed are not as straight forward as they may seem at first[5].

A policy change which sees the introduction of just one or two new connections between bubbles may result in a city that is much more connected that is initially assumed. A common phenomena with complex networks is *percolation* or *condensation*, where the addition of a small number of additional links results in a *phase transition*, where a network that was previously poorly connected network can quickly turn into a highly connected one. We would like to know when the addition of further links between different families and friend groups is likely to cause a phase transition to the interaction network for Auckland, or Aotearoa.

1.1 The Populated Aotearoa Interaction Network (PAIN) and estimates of community interactions

Developed for use as part of a Contagion Network Model, used to provide direct simulation of COVID scenarios and forecasts, the Populated Aotearoa Interaction Network (PAIN) is an individual based, representative network of Aotearoa containing approximately 4.7 million people[*]. It uses data collected through proprietary (e.g. telecoms, mobility trends) and administrative (e.g. Census, IRD, Ministry of Education) sources to create a ‘synthetic’ version of Aotearoa New Zealand, where every individual has an age, sex and ethnicity, a ‘home’ location, as well as places of employment and/or education. Individuals in the PAIN can interact with one another in the context of homes, schools, and workplaces, as well as through so-called *community events*, much like those we will experience when we begin interacting with family members and friends as restrictions ease[†]. These community events represent both *close* (e.g. picnicking with people from another household or shared caring arrangements between people from different households) and *casual* (e.g. sharing public transport; being co-incident when shopping) contacts.

Under Alert Levels 3 and 4, schools are closed and measures such as masking are in place to reduce or minimise the risk of transmission through those workplaces which are operating on site. In this situation, the main remaining interaction context where there is risk of spread between dwellings is through ‘close community’ interactions, such as those that result from shared or extended bubbles.

In order to quantify how Alert Levels are likely to affect the number of interactions that people have with contacts outside their household, we used data from a longitudinal contact survey that asked New Zealanders about the number and type of contacts they had during both Alert Level 4 and Alert Level 3 lockdown periods in April and May of 2020. From these data we can get some idea of the sizes of ‘bubbles’ and “extended bubbles”. Figure 1 shows the number of contacts that New Zealanders share with individuals who are outside of their household or workplace at Alert Levels 3 and 4. From this we are able to get some idea of how decreases in Alert Levels may increase the number of connections individuals have within the network. At Alert Level 4, the majority (over 70%) of survey respondents had no contact with other individuals outside their household or place of work. These types of contacts increased for Alert Level 3 with only 60% of respondents indicating zero non-work, non-dwelling contacts. While this survey data has its limitations (including that the sample was not representative of New Zealand as a whole), it gives a first approximation of the extent to which New Zealand, and Auckland specifically, will see an increase in households reconnecting due to deescalating Alert Levels.

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[*] This was the population at the time of Census 2018.

[†] The PAIN uses a *bipartite network* structure to explicitly represent individuals linked to interaction contexts, from which the person-to-person links can be inferred.

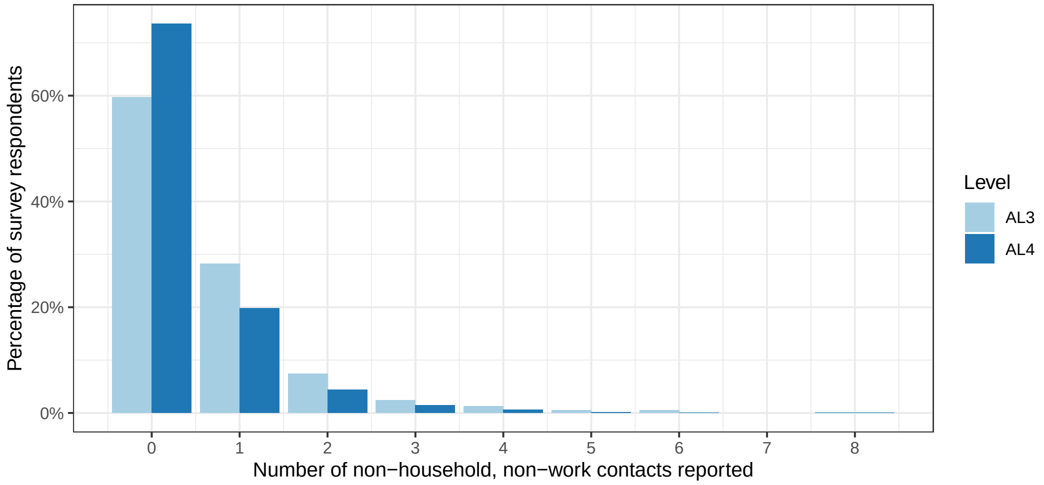

Figure 1. Percentage of survey respondents with non-household, non-work contacts at Alert Level 4 and Alert Level 3. Derived from the contact survey question asking 'What is [NAME]'s relationship to you?'.

2 Gradual Reopening or Phase Transition?

2.1 Hypothetical network example

In order to illustrate how small changes in the number of links can lead to large changes in the connectivity of an interaction network we first demonstrate this effect with a toy model. Figure 2 provides an example of a network of 100 individuals and where we gradually increase the number of links between these individuals between these individuals[‡]. From this, we can see how connected components in the network start to form and to connect to other components. At a certain point, addition of only a few more links is sufficient to merge together the existing connected components to create a connected component that is far larger than any of the others. This largest connected component, or LCC, represents the set of individuals who are directly or indirectly connected to one another, all of whom could be reached by an infection spreading within the LCC. If we think of this toy model as representing our individual household bubbles, we can see how a small increase in the number of interactions can result in bubbles connecting, gradually at first, until at some point, all it takes is a few more links to put most people within the same chain of potential transmission.

To quantify the degree of 'connectedness' in a network, we look at the number of *connected components* of various sizes in the network. Intuitively, a connected component in the network is a set of nodes where there is a pathway via the links of the network between any pair of nodes in the set.

Figure 3 shows the abrupt change, or phase transition in the size of the LCC as the addition of a single extra link to the toy network causes an approximate doubling in the size of the LCC at around the addition of the 67th link.

Extending this idea to the case of Aotearoa during the August 2021 Delta outbreak, we would hope to have many different connected components in the interaction network for Auckland during Alert Level 4 as we seek to keep bubbles separate and, hence to reduce transmission pathways. In contrast, for other areas such as the South Island, once they were at Alert Level 2 we would expect to a connected component that spans a much greater fraction of the relevant region. This could, in theory, stretch across all of the South Island, as individuals connect to individuals within their community who in turn interact with their friends and so on.

The largest connected component (LCC) of a network can hence provide a first approximation of how far it would be possible for disease transmission to travel on links within an interaction network. Going back to our hypothetical example, we can quantify the change in size of the LCC with each additional edge added.

While our hypothetical case does a good job of demonstrating how the structure of a network changes when we add more connections, what does this really mean when we consider New Zealand's response to COVID-19? We use this same approach on the PAIN to try and explore how individuals across New Zealand may begin to connect to each other.

[‡] This type of random network is known as an Erdős–Rényi model, and many of the properties including percolation thresholds are known theoretically.

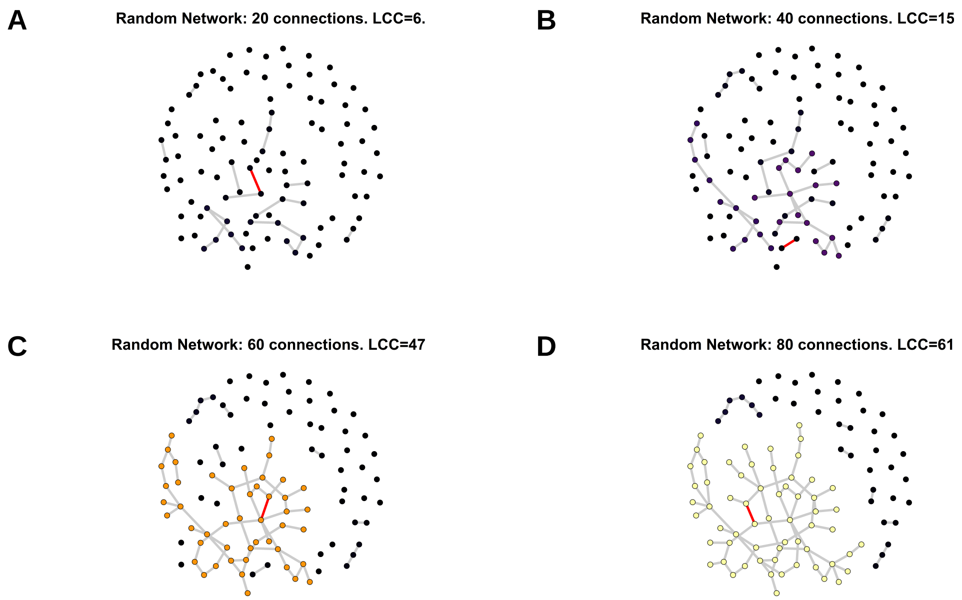

Figure 2. Snapshots of a toy network with 100 individuals and random connections added in sequence. These illustrations show the network after 20 (**A**), 40 (**B**), 60 (**C**), and 80 (**D**) connections added. Brighter node colours indicate that a node is directly/indirectly connected to more individuals in the network. Sections of the network where individuals are linked are known as *connected components*. These tend to be high in number and low in size when few connections are present, but change suddenly to being low in number and high in size beyond a certain threshold — a phenomenon known as a *phase transition*. In each snapshot, we give the size of the Largest Connected Component (LCC); this has size 6 for 20 links present, 15 with 40 links, 47 with 60 links, and contains 61 of the 100 nodes in the network, once 80 random links have been added. Red links indicate the most recent link to be added in each snapshot of the growing network.

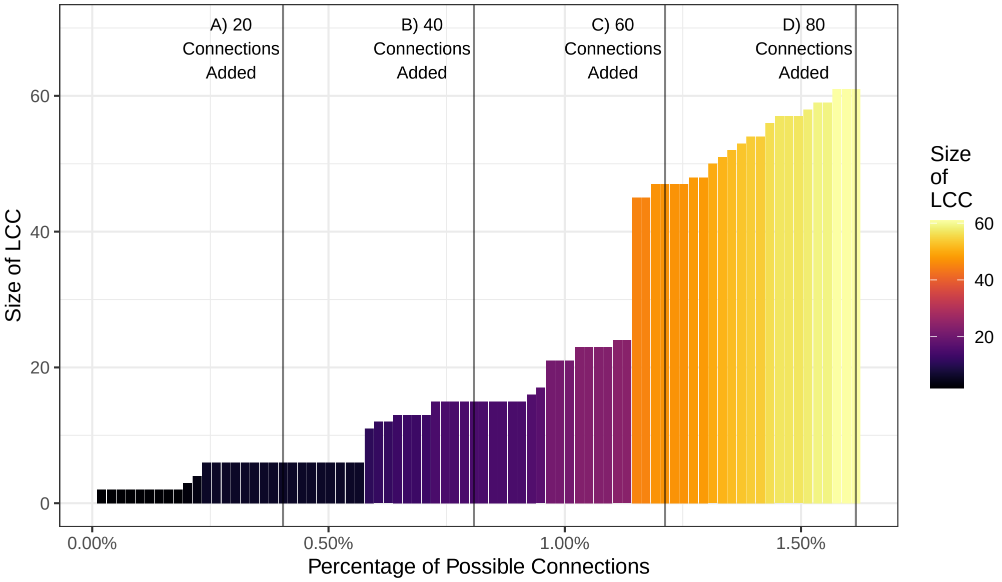

Figure 3. Size of the Largest Connected Competent (LCC) for the toy network, as a function of the percentage of possible links, or connections that are present. The results here are for the same hypothetical network presented in Figure 2. Vertical black lines represent the times corresponding to the snapshots in Figure 2. The percentage of possible connections is the number of links int he network as a fraction of the total possible number of links. In our hypothetical example with 100 nodes, there are $100 \times (100-1)/2$ possible connections. We can see that the size of the LCC increases slowly when we first begin adding connections, and then quickly accelerates to cover over 50% of the nodes in the network, around the addition of the 67th link.

Percentage of people with at least one close community contact	Percentage of people in a connected component (bubble) over size 10	Size of Largest Connected Component
2%	6%	2,496 [1,845, 2,821]
4%	9%	3,814 [3,574,4,059]
9%	22%	86,199 [80,164, 95,133]
17%	40%	1,404,179 [1,399,973, 1,407,761]
32%	64%	2,924,978 [2,924,148, 2,926,275]
45%	78%	3,636,990 [3,636,200,3,638,351]

Table 1. Proportions of people in a connected component of size greater than 10, and median size of the LCC [LQ, UQ], for various values of the number of individuals with a connection outside their household bubble. There is clear evidence of a phase transaction between 9% and 17% of people having links beyond their bubble. This is shown by the size of the LCC increasing from $\sim 90,000$ to over 1.4M. the accompanying component size distributions are shown in Figure 4

2.2 A phase transition from community interactions in the PAIN?

To investigate the impact of Alert Level changes on connectivity in the PAIN, we created a series of networks, each representing New Zealand under different levels of community interaction. To do so, we first constructed a restricted version of the PAIN consisting of individuals connected only via dwellings and those 'close community' interaction contexts. We then investigate what happens to the distribution of connected components in the PAIN as the links associated with different numbers of these community interaction contexts are added or removed. The network of community contacts in the PAIN is built as a form of bipartite random graph[§], which captures the main properties of real-world complex networks[6].

Using data from the contact survey in section 1.1 we can inform choices of the parameters for proportions of 'community' interactions removed that at different Alert Levels. By looking at the fraction of community interactions that are present when a phase transition occurs for the connectivity of the network we can attempt to answer the question of whether a the gradual reopening of Auckland result in only a gradual increase in potential transmission pathways? In short, the answer is probably no.

When fewer than 10%[¶] of individuals in the PAIN have a non-work connection outside their dwelling (top three facets of Figure 4) we see that the majority of individuals in the the PAIN are in a connected component of size 10 or smaller with significant largest connected component or LCC only beginning to emerge for the upper limit of this range with an LCC of around 90,000. For this level of connectivity, which can be interpreted as approximately corresponding to Alert Level 4, a little over a fifth of the population of the PAIN are part of a connected component of size greater than 10 (see Table 1). These connected components larger than ten but under 100 will consist of a mixture of large single dwellings, acting as single bubbles, along with linked dwellings forming extended bubbles.

In contrast, by the time that 17%[‖] of individuals in the PAIN have at least one connection outside of their bubble, the size of the largest connected component in the resulting interaction network has grown to over 1.4 million — increasing by a factor of over 15 when the number of links in the network has increased by less than a factor of two. This sudden emergence of a single so-called *giant connected component* — a single LCC that is much larger than the size of the next-largest connected component — is a hallmark of a percolation or condensation phase transition in a network. At this point, approximately 30% of all individuals in the PAIN are now part of a single connected component. This is compared with under 2% of individuals in the LCC when the fraction of individuals with one or more non-work link, external to their dwelling, is less than 10%. Increasing the proportion of people who have a non-work link outside of their dwelling further accentuates this trend — the size of the LCC continues to grow and the fraction of the population in a connected component of size under 10 continues to fall; once 45% of people have a link outside their dwelling, only 12% of people are in a connected component of size under 10 with almost all of the remaining 78% of the population part of a single giant connected component.

Figure 4 and the accompanying Table 1 illustrate this trend of the rapid transition of the interaction network from a large number of small connected components where transmission between components would be difficult, to a single giant connected component where a single infection, left unchecked could eventually reach most of the population.

3 Summary

The current report has described the potential consequences of seemingly small increases in the number of links in an interaction network that is representative of Aotearoa New Zealand. We see that the introduction of relatively small numbers of

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[§] Specifically following a configuration model approach with prescribed degree distributions (event sizes and numbers of events per person)

[¶] This corresponds to a reduction of 95% or more in the number of community interaction events, since interaction events are assumed to involve more than one other person on average.

[‖] The value of 17% is chosen not because it corresponds to a sharp bound on the phase transition, but because it corresponds approximately to the estimates from the contact survey for AL3.

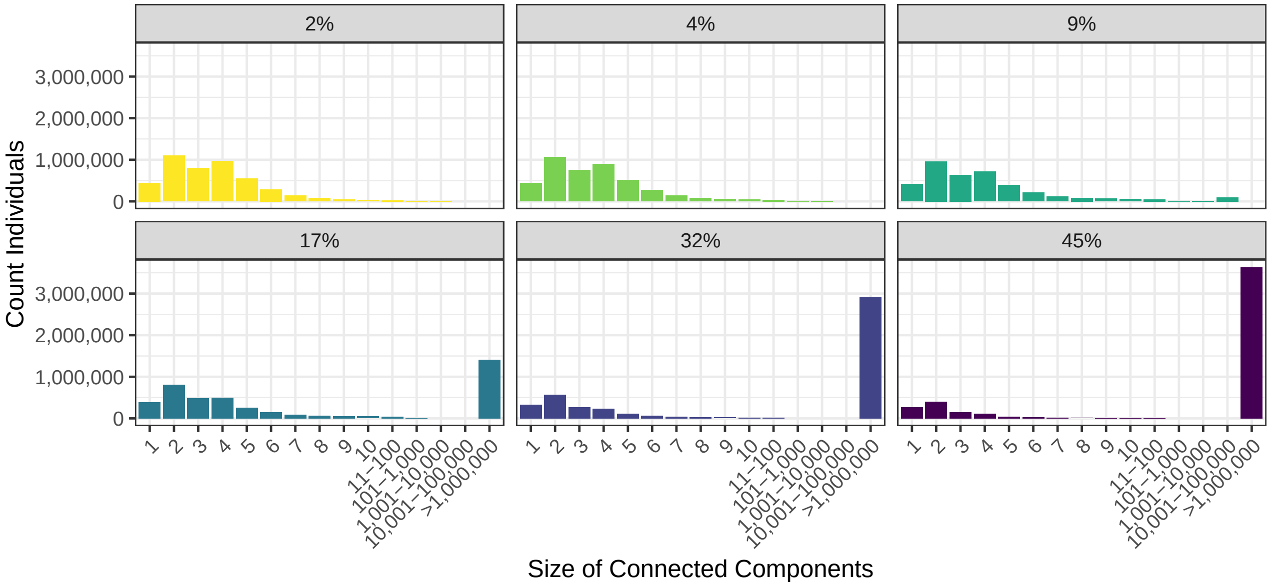

Figure 4. Number of individuals who are in a component of each size for a range of different Alert Level restrictions — characterised as the percentage of the population with at least one close community (non-dwelling, non-workplace) contact. Between 9% and 17% the size of the LCC increases by a factor of over 15 even though the fraction of people with a close community link has less than double. People in a connected component of size one are people who live in a dwelling by themselves and have no close community contacts.

new links between bubbles leads to a phase transition, where a once highly disconnected system quickly turns into a highly connected one. To illustrate this, we have used a hypothetical example of a random network to demonstrate the phenomenon of a percolation or condensation phase transition. We then applied this approach to the Populated Aotearoa Interaction Network (PAIN), a representative, synthetic network representing New Zealand society. We observed that even with gradual easing of restrictions on interactions within the community, and the increased connections that occur as a consequence, there is a dramatic increase the size of the Largest Connected Component (LCC) — the largest number of individuals in the PAIN who share a direct or indirect (i.e, via other individuals) connection over which infection could be spread. While the LCC is not a measure of potential outbreak size, it does highlight the reach of potential transmission pathways that are available, should there be uncontrolled spread of infectious disease across the network.

We find that with restrictions on community interactions in place that were estimated to correspond to the interaction under Alert Level 4 (i.e., when only 9% of people have a close community contact outside their home or work), the network is highly disconnected with the vast majority of people in a relatively isolated bubble of size ten or less. Easing these restrictions, even to the point of allowing just twice as many community interactions to occur (i.e. conditions that are estimated to correspond approximately to interactions under Alert Level 3), results in a dramatic increase in the size of the LCC: from $\sim 90,000$ to over 1.4 million individuals.

A limitation of the current community interaction network is that it assumes that everyone is equally likely to interact with anyone else in the same Territorial Authority (with some age-contact matrix structure), and the real contact network is likely to be much more spatially and demographically assortative. However, the existence of these sharp transitions (percolation thresholds) is a general finding in complex networks, including random bipartite graphs (like PAIN) and real-world networks[7–9].

3.1 Why is This Important?

These example show that only a small number of additional connections are necessary to sufficiently connect existing bubbles such that a large fraction of the population would be reachable through transmission by undetected cases. This highlights the fact that New Zealand is a complex and highly connected system, where individuals are typically not too far removed from each other. This point is something that permeates our national consciousness; New Zealand is a tightly-knitted country where the degrees of separation between people are low. Alert Levels, and specifically lock downs, work because they reduce the vast majority of interactions within the community and limit chains of potential transmission. Our findings here show is that the effect of these reduced interactions from Alert Level interventions can involve a delicate balance that is susceptible to a percolation or condensation phase transition that dramatically alters the connectivity of the interaction network once we reach a threshold in the number of interactions. At some point, the 'phased transition' of reconnecting becomes its own phase transition and a gradual reopening leads suddenly to a densely connected network of potential transmission pathways.

It is worth noting that while this phase transition in the size of the largest connected component in the network tells us that there has been a large increase in the reach of potential infection, it does not tell us about the speed at which infections could propagate over the LCC. This will depend upon, for example, the number of 'hops' between people (path lengths) and the transmission reduction measures introduced in the remaining connections. These higher order aspects of network structure are essential to understanding the consequences of altering Alert Level restrictions for disease spread on the PAIN. These effects are better captured by using the PAIN in conjunction with a contagion model such as in[4].

We do not comment here on the additional strategies that may be in play to mitigate the transmission of COVID-19; we only seek to highlight some potentially non-intuitive consequences of the fact that Aotearoa is a highly connected society. Even without the large number of the community interactions that are typical in our lives, the vast majority of New Zealanders will still be connected to each other through the chains of independent interactions that we share.

We suggest that this is an important, and easy to overlook factor in the design of contagion mitigation strategies that may be put in place, going forward. Creating large chains of previously disconnected components by linking together sequences of different bubbles will very quickly result in one large connected component making extensive disease spread possible and making contact tracing more difficult.

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