

A Multi-Unit Dominant Strategy Double Auction

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Motivation

- Market design is a vibrant field, with a host of theoretical challenges and practical applications.
- Focus of literature on market design with money transfers has been on problem with one-sided private information, e.g. the sale of radio spectrum licenses.
- Yet, many markets involve privately informed traders on both sides of the exchange.
- Recently the US Congress mandated the FCC to organize an *incentive auction* enabling trade between current owners of spectrum licenses like radio and TV broadcasters and potentially higher-value users like mobile telephony providers.

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- A major impediment to theoretical progress, and practical implementation, has been the lack of satisfactory allocation mechanisms that do not run a deficit with private information on both sides for the empirically important case of multi-unit buyers and sellers.
- Vickrey (1961) put forth an early and forceful argument against efficient but deficit-generating mechanisms, noting that they would be “inordinately expensive in terms of their demands on the fiscal resources of the state relative to the net benefits to be realized” and further reasoned as follows:

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Vickrey (1961, p.13/4)

*It is tempting to try to modify this scheme in various ways that would reduce or eliminate this cost of operation while still preserving the tendency to optimum resource allocation. However, it seems that all modifications that do diminish the cost of the scheme either imply the **use of some external information** as to the true equilibrium price or reintroduce a direct incentive for **misrepresentation** of the marginal-cost and marginal-value curves. To be sure, in some cases the impairment of optimum allocation would be small relative to the reduction in cost, but, unfortunately, the analysis of such variations is extremely difficult; ... [emphasis added].*

Summary

- We develop a detail-free double-auction for environments with homogenous goods and multi-unit sellers.
- Our double-auction endows agents with
 - dominant strategies,
 - respects participation constraints ex post,
 - never runs a deficits,
 - and sacrifices efficiency only by eliminating marginal trades.

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- Intuitively, we split the two-sided problem into two one-sided auctions with appropriately chosen reserve prices, in which the quantities traded are weakly less than the Walrasian quantity.
- These one-sided auctions generate a surplus to the market maker.
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Related Literature

- Double-auctions: Chatterjee and Samuelson (1983), Cripps and Swinkels (2006), Gresik and Satterthwaite (1989), Rustichini, Satterthwaite and Williams (1994), Satterthwaite and Williams, (1989, 2002) and Wilson (1985).
- Dominant strategy mechanisms in the tradition of Vickrey (1961), Clarke (1971) and Groves (1973).
- Important precursor with unit demands and supplies: McAfee (1992).

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Setup

- There is a homogenous good, N buyers and N sellers
- $v_k^i \in [0, 1)$ is buyer i 's marginal value for the k -th unit of the good
- $c_k^j \in (0, 1]$ is seller j 's cost for (either producing, or giving up the use of) the k -th unit
- $k = 1, \dots, K$

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- Diminishing marginal values and increasing marginal costs: $v_k^i \geq v_{k+1}^i$ and $c_k^j \leq c_{k+1}^j$
- Quasilinear preferences:
 - Buyer i 's payoff from receiving q units at prices p_1, \dots, p_q :
$$\sum_{k=1}^q (v_k^i - p_k)$$
 - Seller j 's payoff from selling q units at prices p_1, \dots, p_q :
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Direct Mechanism

- Our double-auction is a direct mechanism, asking all buyers and sellers to report their types (\mathbf{v} , \mathbf{c}).
- These types are reported only once, and as a function of these reports, the mechanism determines payments and quantities traded according to the rules we lay out next (and that are determined ex ante).

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“As if” Two-Stage Mechanism

- For the purpose of explaining and illustrating these rules, it is nevertheless useful to think of it as a two-stage mechanism:
 - 1 Agents submit reports (\mathbf{v}, \mathbf{c}) .
 - 2 As a function of these reports, the designer selects reserve prices r^B and r^S and the quantity to be traded q ; agents on each side of the market participate in a VCG-auction.
- Despite this way of telling the story, agents act only once (when they submit their bids).
- Everything that follows is done by the algorithm.

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U-active and U-inactive Traders

- A **U-active** agent (**U-inactive**) is an agent that trades (does not trade) under the efficient allocation when traders are restricted to demand and supply at most one unit.
- The **weakest U-active** traders: the buyer i^A with the lowest marginal value and the seller j^A with the highest marginal cost for the first unit, among the U-active traders.
- The **strongest U-inactive** traders: the buyer i^I with the highest valuation and the seller j^I with the lowest cost for the first unit, among the U-inactive traders.

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Reserve Prices in Our Double Auction

- r^P is the preliminary reserve price, a function of the lower and upper bound of the U-inactive-traders price gap $[v_1^{j^I}, c_1^{j^I}]$.
- if r^P belongs to the U-active-traders price gap, $r^P \in [c_1^{j^A}, v_1^{j^A}]$, then the buyer and seller reserve prices equal r^P ; that is, $r^B = r^S = r^P$.
- if r^P does not belong to the U-active-traders price gap, $r^P \notin [c_1^{j^A}, v_1^{j^A}]$, then the buyer and seller reserve prices are $r^B = v_1^{j^A}$ and $r^S = c_1^{j^A}$.

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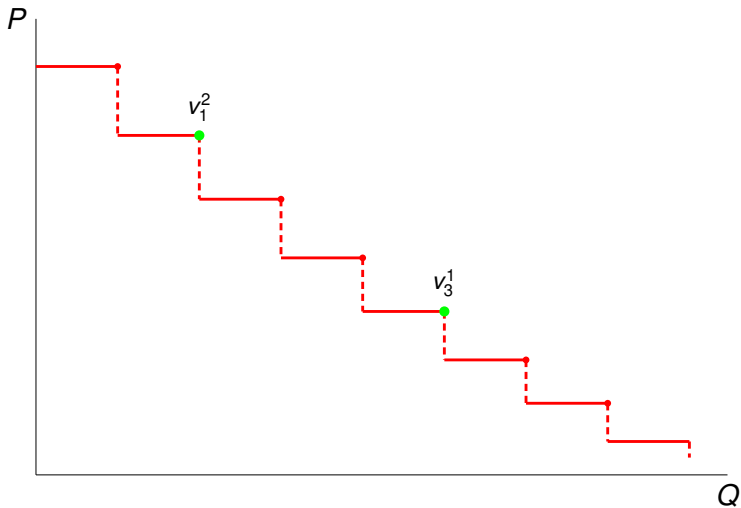
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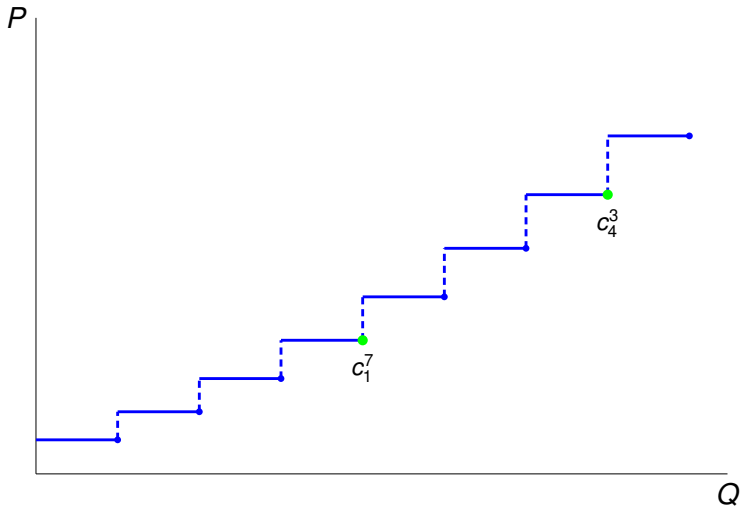
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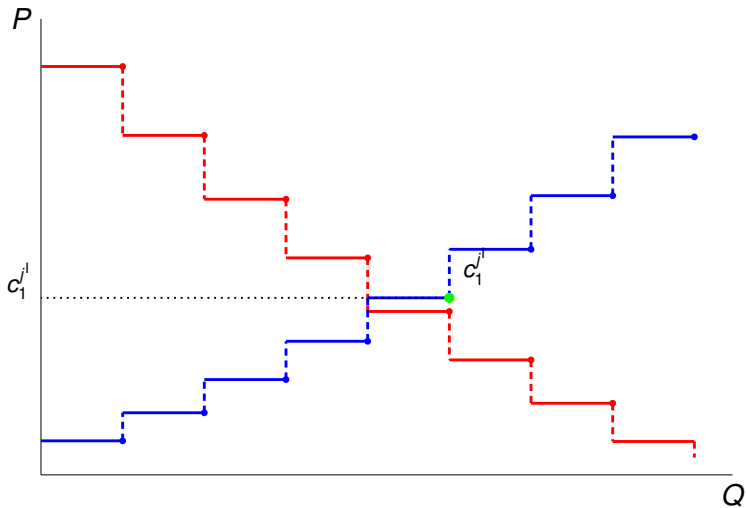
The Multi-Unit Dominant Strategy DA

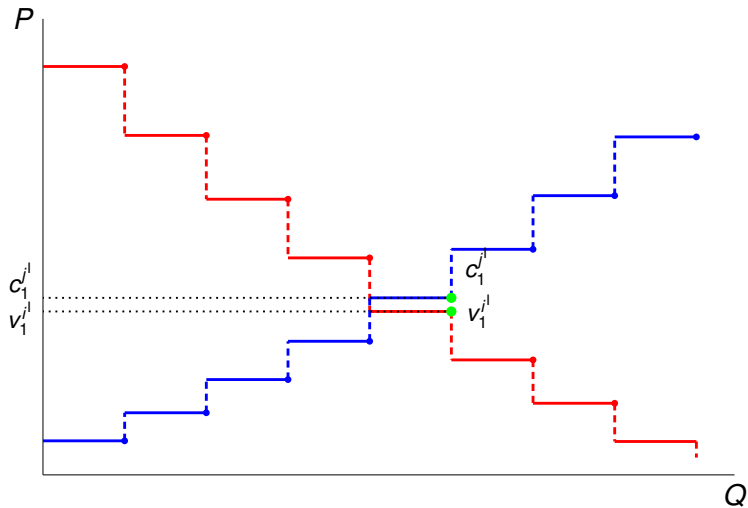
Clock Implementation

Discussion

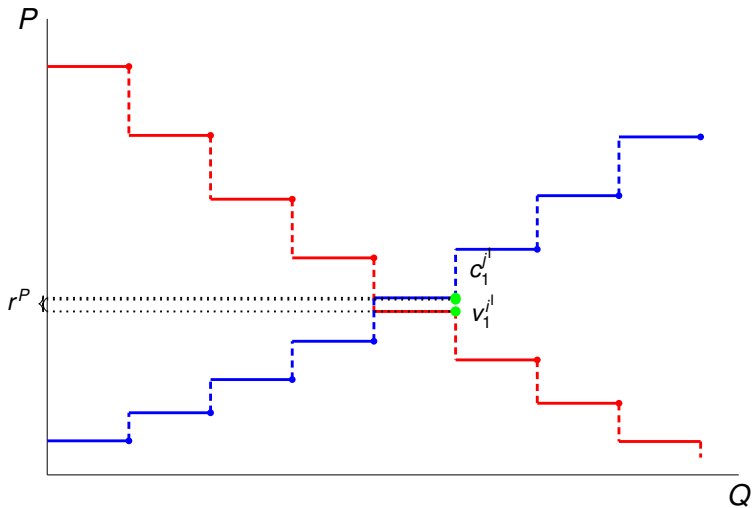








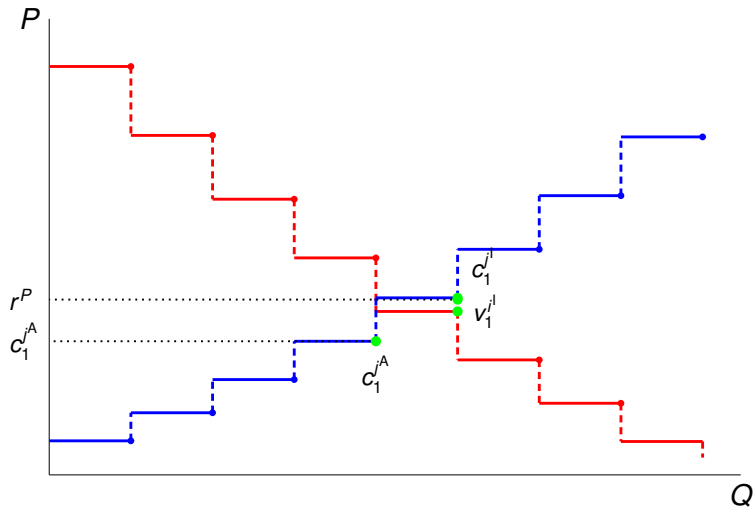
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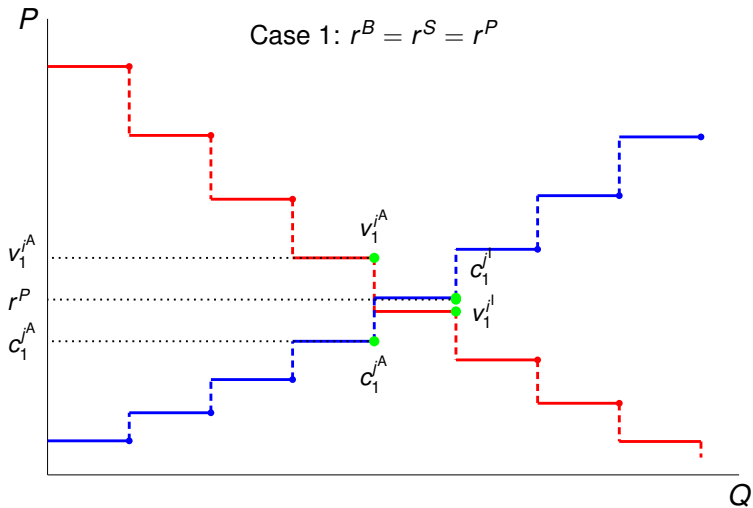


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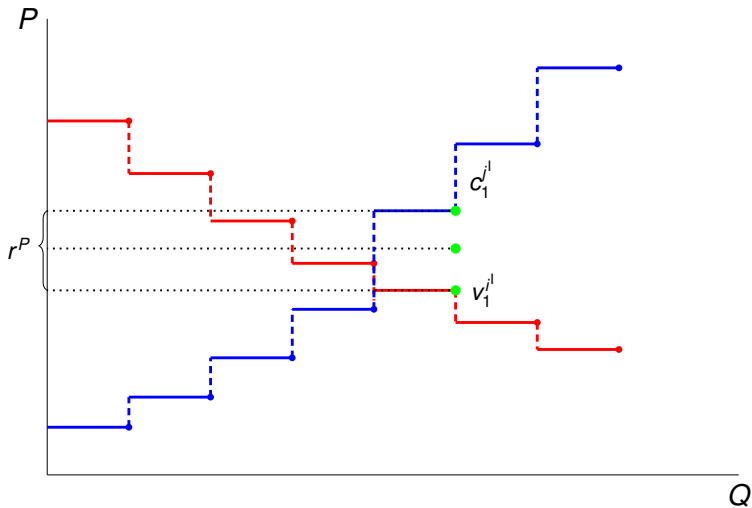




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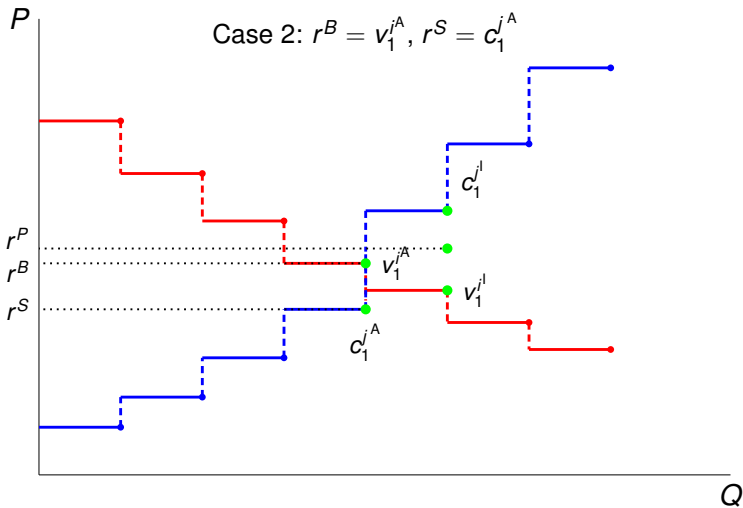
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Quantity Traded in Our Double Auction

- $D(r^B; \mathbf{v})$ is the quantity demanded if the price is equal to the buyer reserve price r^B
- $S(r^S; \mathbf{c})$ is the quantity supplied if the price is equal to seller reserve price r^S
- The quantity traded in the double auction is given by

$$q(\mathbf{v}, \mathbf{c}) = \min\{D(r^B; \mathbf{v}), S(r^S; \mathbf{c})\}.$$

- $q(\mathbf{v}, \mathbf{c})$ is the quantity demanded or supplied by the short side of the market at reserve prices r^B and r^S .

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Double-Auction Rules

- Given a vector (\mathbf{v}, \mathbf{c}) of reported values and costs, the market maker determines the reserve prices r^B , r^S and the quantity $q(\mathbf{v}, \mathbf{c})$ auctioned on each side of the market.
- The market maker runs a VCG auction for buyers with reserve price r^B and a reverse VCG auction for sellers with reserve price r^S

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Note:

- All traders on the short side of the market at reserve prices r^B and r^S trade at the short-side reserve price, and obtain what they demand or supply
- All traders on the long side of the market at reserve prices r^B and r^S trade as in a VCG auction with the long-side reserve price and the quantity $q(\mathbf{v}, \mathbf{c})$ determined by the short side; hence they obtain VCG quantities and pay VCG prices

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Proposition

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The multi-unit double-auction never runs a deficit and makes reporting truthfully a dominant strategy for every buyer and every seller.

Privacy Concerns

- Privacy preservation (for winning bidders) is a major concern for direct mechanisms such as VCG.
- Fearing that the information they provide to the mechanism may be used against them further down the track, bidders may refrain from providing this information.
- Dynamic (“clock-auction”) implementation can mitigate or eliminate this problem.

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The Clock Double Auction 1

- **First stage:** the state at t is an ask price $\alpha(t)$, a bid price $\beta(t)$, and the vectors of individual exit decisions by buyers and sellers. At $t = 0$ all traders are in.
- A strategy for each buyer i and each seller j is a time t^i and t^j at which to exit irrevocably.
- $X(t)$ is the total number of buyers and $Y(t)$ is the total number of sellers, who are still in at time t .
- τ is the first time when $X(\tau) = Y(\tau)$ and $\beta(\tau) \geq \alpha(\tau)$.
 - 1 If $X(\tau) = Y(\tau) = 0$, then the clock double auction (not just its first stage) ends at τ and no trade takes place.
 - 2 If $X(\tau) = Y(\tau) > 0$, then the first stage of the clock double auctions ends at τ with the selection of the second-stage reserve prices $r^B = \beta(\tau)$ and $r^S = \alpha(\tau)$.

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- **First stage:** the state at t is an ask price $\alpha(t)$, a bid price $\beta(t)$, and the vectors of individual exit decisions by buyers and sellers. At $t = 0$ all traders are in.
- A strategy for each buyer i and each seller j is a time t^i and t^j at which to exit irrevocably.
- $X(t)$ is the total number of buyers and $Y(t)$ is the total number of sellers, who are still in at time t .
- τ is the first time when $X(\tau) = Y(\tau)$ and $\beta(\tau) \geq \alpha(\tau)$.
 - 1 If $X(\tau) = Y(\tau) = 0$, then the clock double auction (not just its first stage) ends at τ and no trade takes place.
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The Clock Double Auction 2

- **Second stage:** The active traders are all the traders that in the first stage did not drop out at, or before, time τ .
- Each active buyer i reports the quantity he demands at price $\beta(\tau)$ and each active seller j reports the quantity supplied at price $\alpha(\tau)$.
- $D(\beta(\tau))$ is the total reported quantity by the active buyers; $S(\alpha(\tau))$ is the total reported quantity by active sellers. The market maker selects the quantity $q(\mathbf{v}, \mathbf{c}) = \min\{D(\beta(\tau)), S(\alpha(\tau))\}$.
- The clock restarts and the market maker runs an Ausubel auction with quantity $q(\mathbf{v}, \mathbf{c})$ and reserve price $\beta(\tau)$ for buyers and a reverse Ausubel auction with quantity $q(\mathbf{v}, \mathbf{c})$ and reserve price $\alpha(\tau)$ for sellers.

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Proposition

In the dominant strategy equilibrium of the clock double auction traders bid truthfully and prices paid and quantities traded are the same as in the multi-unit double auction.

Efficiency

- 1 How much efficiency is sacrificed?
 - In thin markets?
Use simulations.
 - Does equilibrium converge to efficient outcome as N goes to infinity?
We think that yes.
- 2 What is the appropriate measure for efficiency (within domain of prior-free mechanisms)? What is an appropriate benchmark?
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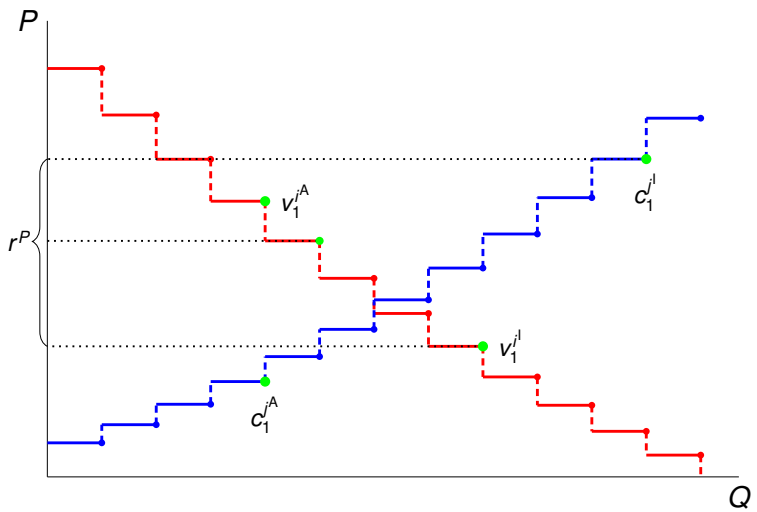
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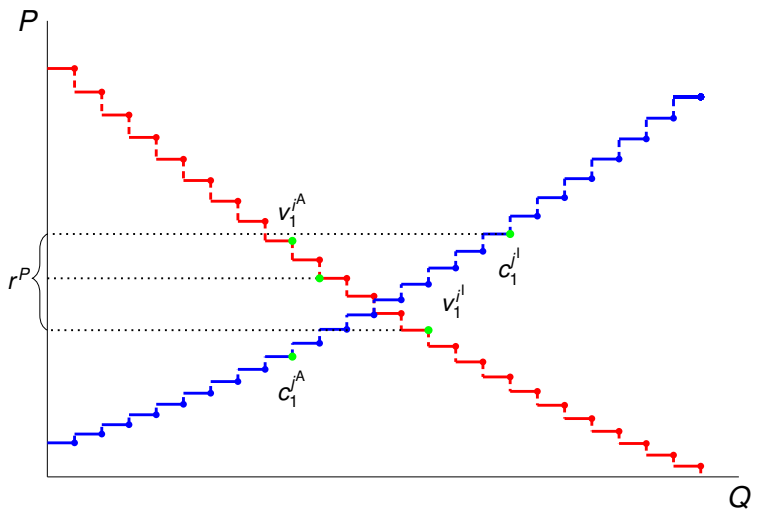
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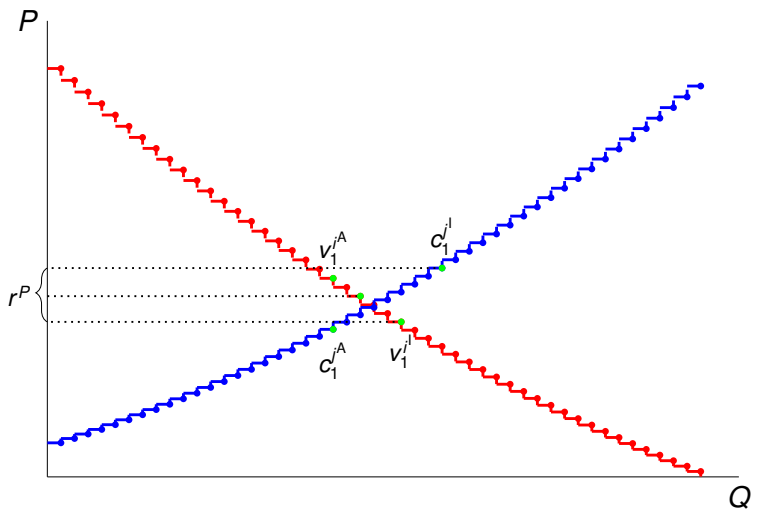
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Open Questions

- Common Value: Does the clock double-auction outperform the direct mechanism when there is a common value component (as observed by Ausubel 2004 in a one-sided setup)?
- Revenue: Achieve a minimum revenue R per trade by choosing a reserve spread $r^B - r^S > 0$?

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Further Research

Extend analysis to

- Shapley-Shubik's assignment model (matching is one-to-one, but types are multi-dimensional as buyers perceive sellers as heterogenous).
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- heterogenous objects.

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