Mixed Sharing Rules

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The problem

- Each of n players supplies labour to a joint enterprise under a predetermined rule for sharing the resulting output.
- What are the implications of the chosen sharing rule for (a) efficiency, and (b) distribution, in the resulting noncooperative labour supply game?
- Can a mechanism be devised that leads to an efficient noncooperative equilibrium?

Assumptions Throughout

•n players

•Technology: neoclassical, diminishing returns to ‘labour’ $Y = F(L):\ F(0) = 0,\ F'(L) > 0,\ F''(L) < 0$

•Preferences: well-behaved in every way – labour supply irksome

$$u_i = u_i(c_i, \ell_i):$$

$$u_{ic}(c_i, \ell_i) > 0,\ u_{i\ell}(c_i, \ell_i) < 0,\ i = 1, \ldots, n$$

Preferences exhibit convexity and **normality**

Proportional surplus sharing

Assume output shared in proportion to input supplied: $c_i = \frac{\ell_i}{L} Y$

$$\Rightarrow u_i = u_i\left(\frac{\ell_i}{L} F(L), \ell_i\right) = u_i\left(\frac{\ell_i}{\ell_i + L_{-i}} F(\ell_i + L_{-i}), \ell_i\right)$$

Player i chooses ℓ_i to maximise utility: $MRS_i = MRT_i$

or

$$-\frac{\partial u_i(.)/\partial l_i}{\partial u_i(.)/\partial c_i} = \left[\frac{l_i}{L}\right] F'(L) + \left[\frac{L - l_i}{L}\right] \frac{F(L)}{L} = MRT$$

$\Rightarrow$ Overexploitation: l_i 'too high' [Tragedy of the commons]

Exogenous surplus sharing

$$c_i = \theta_i Y,\ \sum_{j=1}^{n} \theta_j = 1,\ \theta_j\text{'s exogenous}$$

$$\Rightarrow u_i = u_i\left(\theta_i F\left(L\right), \ell_i\right) = u_i\left(\theta_i F\left(\ell_i + L_{-i}\right), \ell_i\right)$$

Player i chooses ℓ_i to maximise utility: $MRS_i = MRT_i$

or

$$-\frac{\partial u_i\left(.\right)/\partial l_i}{\partial u_i\left(.\right)/\partial c_i} = \theta F'\left(L\right) < MRT$$

$\Rightarrow$ Under-exploitation: l_i ‘too low’ [Public good provision]

A mixed sharing rule

- A sharing rule is initially agreed.
- The rule divides total output into two piles: $\lambda Y = \lambda F(L)$ is allocated to pile P, and $(1 - \lambda)Y = [1 - \lambda]F(L)$ to pile E.
- We call the parameter λ the **mixing parameter.**
- The pile P will be divided up proportionally, and the pile E exogenously:

$$c_i = \frac{\ell_i}{L}\lambda F(L) + \theta_i [1 - \lambda] F(L)$$

- Players, knowing the rule, choose their labour inputs noncooperatively

Questions

1. Does a pure strategy equilibrium exist?
2. If so, is it unique, or are there many?
3. Is there a value of λ which implies a Pareto efficient equilibrium outcome?
4. Can any Pareto efficient outcome be an eq outcome given suitable choices of λ and the θ_i's?
5. Is there a mechanism for selecting a λ that produces a Pareto efficient outcome?

Method of Analysis

- Define for every player a share function: $$\frac{\hat{\ell}_i}{L} = s_i(L)$$
- Observe that, at a Nash equilibrium,

$$\hat{\ell}_1 + \hat{\ell}_2 + \ldots + \hat{\ell}_n = L$$

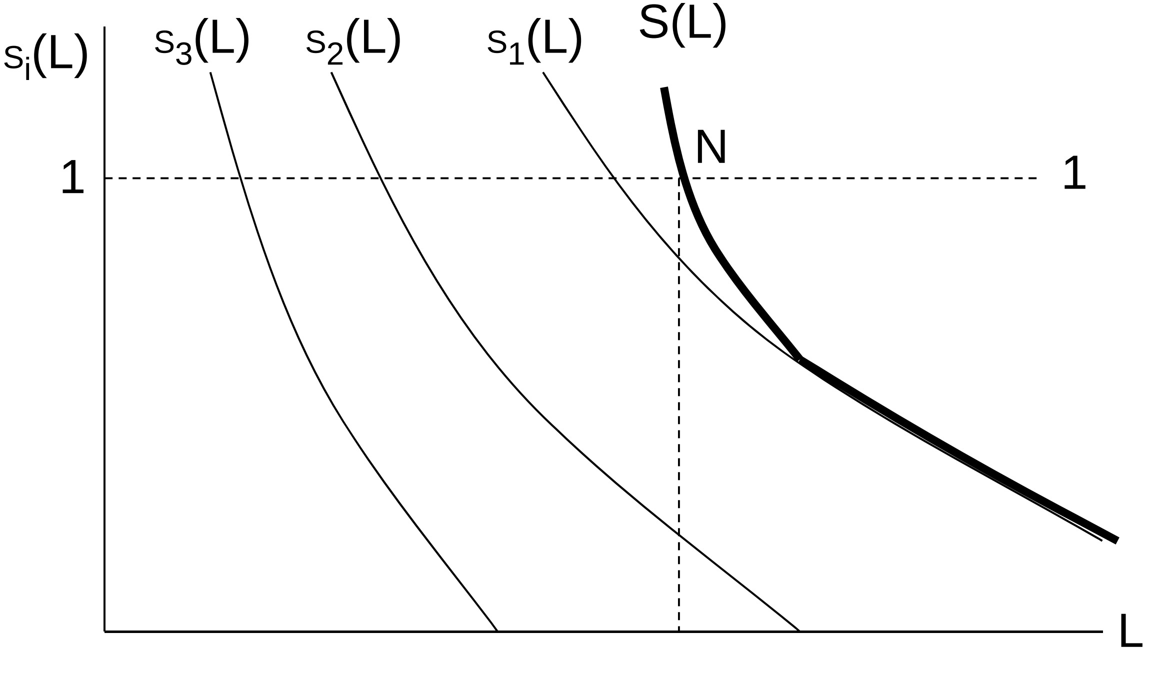
$$\Rightarrow \frac{\hat{\ell}_1}{L} + \frac{\hat{\ell}_2}{L} + \ldots + \frac{\hat{\ell}_n}{L} = 1$$

$$\Rightarrow s_1(L) + s_2(L) + \ldots + s_n(L) = 1, \text{ or } S(L) = 1$$

- Analysis of Nash equilibrium now involves the solution of one equation in one unknown. [cf the best response function approach, involving n equations.]

Example

- 3 players, λ=1/2 , $\theta_1 = \theta_2 = \theta_3$=1/3
- $U_i() = a_i x_i - \ell_i$, $(a_1, a_2, a_3) = (30, 20, 15)$
- $F(L) = L^{1/2}$



Some answers

- For a given set of values $(\lambda, \theta_1, \ldots, \theta_n)$, there exists a unique noncooperative equilibrium.
- Assume identical preferences and equal shares [θ_i=1/n for all i]. If the mixing parameter λ equals the equilibrium elasticity of production, $\eta(L)$, the equilibrium allocation is efficient.

Another result

- Assume preferences are quasilinear in income. Consider an efficient allocation in which player i receives output x_i^e and aggregate input is L^e. Then the exogenous shares can be chosen so that the equilibrium of the surplus sharing game with $\lambda=\eta(L^e)$ satisfies $x_i = x_i^e$ for all i and $\Sigma_i \ell_i^e = L^e$.

Freeness from (average) envy

- If the mixing parameter is chosen to equal the elasticity of production, every player prefers her equilibrium bundle to the bundle consisting of $[F(L^N)/n, L^N/n]$.

Unanimity Test

- If F(L) = L$^{\alpha}$ and the mixing parameter $\lambda = \alpha$, then for every player ,

$$u_i\left(c_i^N, \ell_i^{\,N}\right) \geq \underset{L \geq 0}{Max}\ u_i\left(\frac{F(L)}{n}, \frac{L}{n}\right)$$

Furthermore

$$u_i\left(c_i^N, \ell_i^{\,N}\right) \geq \underset{L \geq 0}{Max}\ u_i\left(\theta_i F(L), \theta_i L\right)$$

The stand-alone test

The stand-alone test formalizes the idea that no player should benefit from the negative externality they impose on others.

Call i a net contributor if, in equilibrium,

$$\ell_i / L \geq \theta_i$$

- Then at equilibrium all net contributors pass the stand-alone test.

Results for large games

Our results may be strengthened for large games – that is, games with large n.

For example:

- The sets of efficient allocations respecting voluntary participation and of equilibrium allocations with optimal mixing are identical.
- By varying the exogenous weights, the whole set of efficient allocations can be mapped out.

Results for large games (II)

- If exogenous weights are equal, asymptotic equilibria are envy free and pass the unanimity test.
- If exogenous weights are equal, a stronger stand-alone test is satisfied.
- Eq. payoffs for each type are proportional to the same function of the mixing parameter.
- Consequently, all types of player prefer the same value of the mixing parameter.
- **Voting for the optimal value is a dominant strategy for every player in the first stage.**