Economic Wellbeing Of A Society In A Culture Of Violence Against Women *

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Abstract

We examine how a pervading social culture of violence against women (VAW) undermines economic wellbeing of a society. In a dynamic general equilibrium model of growth with optimising men and women we characterize its non-trivial effects on (1) the women’s participation rate, (2) the gender-gaps in (a) returns to parental investment in education and (b) human capability, arising from a "son-bias" and a nuisance cost of educating girls and (3) the external effect of mother’s human capital on the marginal efficiency of investment in children’s education. We discover an explicit analytical relationship for doing growth empirics, relating the growth rate of GDP, gender-gaps in human capital and productivity and the likelihood of VAW in a society. That analytical restriction helps us to organise the newly developed dataset on crime against women by the UN for conducting a meaningful empirical exercise to shed light on the cross-country disparity of income per capita. Empirical results are weak; but are consistent with the model’s predictions. The welfare analysis characterizes economic conditions under which the culture of VAW significantly reduces economic wellbeing, not just for women but for everyone. Public investment in women’s education fails to close the gender gap as long as the likelihood of VAW exceeds a critical threshold, which, however, varies as per the underlying societal arrangement for marriage and the nuisance cost of educating a girl child.

Keywords: Patriarchy, VAW Likelihood, Gender Gap, Gender Rent from VAW, Endogenous Human Capital, Marital Capital, Marital Goods, Threshold for VAW, Cross-Country Growth Regression.

JEL: E130, E620, O110, O430, O570

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1 Introduction

"Since women form (approximately) half of any national population, an efficient development project must tap their capacities in order to make optimal use of resources." - Amartya Sen, 1998.

"We don't need poverty indices we know who are the poor; the poor are in the rural areas and are landless; and the poorest of the poor are the women" - Mark Rosenweig, 2011.

Does growing-up in a Culture of Socially Acceptable VAW Undermines Economic Well-being of the Whole Society?

A small sticker posted on a number of walls in Delhi states: "... better pay 500 rupees now than 50,000 rupees later." As per an explanation from a local economist, those two numbers describe a trade-off between the medical cost for aborting a female embryo and the dowry that her parents may expect to pay to marry her off to a husband. Both of these acts are illegal. Yet, that trade-off concerns many Indians, affluent or not; and hence those black markets for trading women and their birthrights exist. In a striking contrast Sen’s quote makes the point that untapped capacities of women make up a goldmine for the society that deserves exploration. Adding to this irony, Rosenweig’s quote suggests that social norms and customs leave too few outlets for women to contribute to social welfare; for most women, the present society reserves only a curse, extreme poverty, which bestows little power to women for making a valuable contribution. Beyond the obvious irony a question lies: left behind unrecognized, are those women dragging everyone behind, by slowing down economic growth and welfare?

A Tagore poem poses a similar question by implicitly referring to the Indian caste system; but that idea applies to today’s Indian women as well. In the words of Swami Vivekananda we find a direct reference to the same idea that the fate of a nation is tied to the fate of its women. Absent however, in his words or in any religious scriptures, is a description of how exactly the economic wellbeing of a society and the wellbeing of its women (not the poor and the so-called lower caste) are interlinked, other than the glory of the motherhood, which sounds conspicuously louder than fatherhood, making it quite a burdensome honor for women to carry. Subscribing to the morality of organising resources under patriarchy, most religious scriptures dictate a list of "good deeds" for women. The moral dictum often seems tantamount to a submission of women’s autonomy to men, who typically receive the blessings of the society for undertaking the moral responsibility of making decisions, as providers of "rice and cloth" for women, as their husbands and fathers.

Morality as though it may be, such a gender based division of power and responsibility for economic decision making may not be optimal for a society today. Today’s society has a skill-biased technological foundation. It significantly differs from the muscle-power biased

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1 A noteworthy study by Vijayendra Rao (1993) finds that in rural areas, the average dowry for a daughter’s marriage was six times the parents’ annual income.
primitive technology that upholds moral values for gender based segregation of economic activities. Following the principles of comparative advantage, today’s technology would require a relatively gender neutral organisation of resources. The underlying power imbalance due to a primitive social arrangement endows men with inefficiently large bargaining power over women. Those social and religious customs often lead to an arrangement within the family such that its female members lose autonomy to make decisions over choices that affect their economic wellbeing. Those rituals, morality aside, represents barriers to an efficient matching of talents with suitable educational institutions and occupations. By undermining female autonomy within the family and by exerting "moral" pressure against those who may seek autonomy outside the family, a set of morally rigid gender based principles for permitting access to education and occupation stand as barriers to the wealth of a nation. A large body of research in sociology finds that such a social condition also facilitates violence against women (VAW), both inside and outside the perimeters of the family. If persists, it acts as a potential threat to every woman living in such a society, distorting their decision in a way to lead to inefficient allocation of resources and inappropriate choice of technology for the nation. Consequently, such threat of VAW act as a barrier to economic growth and prosperity of that nation.

The question we ask, therefore, is a straightforward one: how exactly the degree of autonomy women enjoy in a society interacts with the perceived threat of VAW to affect the wealth of a nation? Or, equivalently, how does the women’s empowerment, inside and outside the family, improve economic efficiency and social welfare?

The work of James D. Wolfensohn, the President of the World Bank 1995 - 2005 is noteworthy in that context. His work emphasizes the importance of women’s education in empowering women and elaborates how that, in turn, may boost economic growth via various channels. We accommodate all those channels of growth but work through a standard neoclassical model to consider the marginal costs and benefits of various options. Our analysis however leads to a somewhat different perspective. We identify specific economic conditions when a more direct method of women’s empowerment than education would be the optimal strategy for growth. In particular, we conclude that, without ensuring a minimal level of safety for women against gender specific crimes, additional public expenditure on women’s education and work may simply add to economic waste. Interestingly, in our model economy, a significant improvement in the law the order, which ensures the required minimal level of safety for women against gender specific crimes, turns out to be not just an important prerequisite, but also sufficient for raising education among women and subsequent improvement in economic welfare, in general.

Our hypothetical story goes as follows: Economic growth led by a skill-biased technological evolution creates a conspicuous combination of skill shortage along with an excess supply of unskilled labor. By responding to that shortage of skill in the labour market, women with talents seek more education and better jobs outside home. However, they face strife over rent with men with exclusive social privileges. Such rent exists in a society that gives monopoly rights to men over ancestral resources, education and jobs, based on religious or other grounds. As the VAW subsides, the economy replaces unskilled men, seeking rents

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The common rationality of the patriarchal forms of most religions stems from the physical labour intensive production technology of the past. Following the industrial revolution, technological progress has been
from those rights, with skilled women, possessing entrepreneurial skill and seeking profitable opportunities. As profit seeking skilled women replace rent-seeking unskilled men, the overall economic productivity increases. Economic productivity increases in the same way as in the case when a capital intensive technology replaces manual labor. The political conflict between capitalists’ technology and labor (the so-called working class) parallels here with the conflict between skilled and entrepreneurial women and unskilled men with power due to their social status. The point of the paper is that economic growth alone may not reduce these conflicts, as long as the underlying social and religious foundations of the economy forestall moral values, which often manifest as violent barriers, for various psychological reasons, against women’s endeavour to seek higher education and better occupations. We show in our model how such threat of violence against women endows men with a monopoly power which often lowers economic opportunities for women. Economy may continue to grow despite such violence but it does along a sub-optimal path.

To sum up, we note that our story puts together the idea of an inherent conflict between a society and technology that gives rise to VAW, which potentially has a detrimental effect on economic growth and welfare of a nation. We develop the above idea, in a unified theoretical framework, which we explicitly describe in the model section of this paper. The model’s mathematical consistency helps us to organise the newly available data on the crime against women, in a systematic way for conducting empirical experiments afterwards. Our model also helps to shed lights on a few policy questions, regarding the merit of education and other forms of empowerment of women for promoting economic growth. In particular, it provides a tool for analysing how a policy maker can intervene to make a difference in women’s experience in a society and thereby to improve economic welfare, not only for women but also for everyone.

Section 2 presents related literature, followed by a short sub-section on motivating growth empirics, based on which we develop our model in Section 3. Section 4 includes a discussion on our empirical analysis, followed by the welfare and policy analyses in Section 5. We add a few concluding remarks in Section 6, summarising our contribution and a list of unfinished tasks for future research. A list of references, followed by the technical appendix is included at the end.

2 Related Literature

A vast amount of literature on the societal cost from the violence against women (VAW) exists. The focus of that literature is primarily in measuring cost of rehabilitation and various types of costs incurred by the women as victims. However, how a pervading culture of VAW in a society may affect economic choices of every man and woman, living in such a society, and how their perceived risk of VAW may affect the wealth of a nation seems to be an area in economics, yet to be explored. So, we briefly review the existing literature on various closely related issues.

primarily skill-biased. The accumulation of human capital fuels that change and runs the engine of economic growth in a modern economy. Men alone could not accumulate so much of human capital and by the laws of nature, which we call the Bell curve, only a fraction could acquire sufficient skill to do innovative activities with a skill-biased technology. Yet, religions give the moral authority to men for making decision.
2.1 Poverty Amidst Plenty

There are at least four different ways a society may live in poverty despite carrying within a plenty of potential avenues for overcoming it.

2.1.1 Low Participation and Occupational Mismatch

Despite the wealth of economic resources with which women are endowed, their participation in the labour market has been significantly lower than men in most developing countries. Such a large \textit{gender gap in the participation rates} could be a barrier to growth for the fast growing developing countries like India. Bandiera and Natraj (2013) explore empirical evidence on the efficiency loss due to employers' use of unskilled men facing a shortage of skill, even when skilled women are around in surplus. Those skilled women cannot supply their labour due to the lack of assurance for their safety. Consequently, employers have no choice but to fill in jobs that require skill with unskilled men. The resulting \textit{mismatch in occupation} causes efficiency loss for the society.

2.1.2 Parental Investment In Girls

The economic loss does not end there. The lack of a minimum assurance to women's safety decreases the set of feasible place and time for women to work and that lowers the return to women's education. The prospect of expected harassment at the work place, on the way, or even at home where the choice of work outside may not be appreciated, increases expected disutility from work. Parents who care about their girl child's welfare, may rationally under-invest in their daughters' education, if they perceive \textit{a gender gap in returns to education} and a larger disutility for the daughters who plan to work in a society with a high degree of VAW. Nancy Qian (2010) writes about the sensitivity of the parental investment in daughters to how the society values girls. In her study of the Chinese experiment with the introduction of Household Responsibility System in the cultivation of tea, she notes that a better work opportunity for the girls in growing tea, because of the shape of their fingers, raised the value of girls in the society and that, in turn, raised parental investment in girls as well. In particular, the ratio of girls to boys in the population increased soon after the introduction of that new system for tea production. Duflo (2012), however, reports evidence to suggest that such gender discrimination by the parents are not typical in normal economic times. She does note, however, that in an economic crisis situation such as a natural disaster, during an epidemic or in a sudden economic hardship, girls and women are likely to suffer more than the boys from parental discrimination.

2.1.3 Knowledge Diffusion: The Curse and Blessings Of Social Networks

Access to a social network increases returns to education and a rent for those who enjoy a privileged status in the society. Absent typically are the women from the status-quo networks and especially from positions with status. Religious practices often require men; and the positions of the high priests are typically reserved for men. Women even with their costly acquired education in traditional customs and religions benefit little from those networks.
Consequently, a strong influence of traditional social and religious networks plays a critical role in creating a gender gap in returns to education. However, there is a silver lining to the gender-based discrimination by social networks. Those very pro-men networks sometimes trap men, ironically with privileges, into the traditional vintage of technology. Women with little to benefit from those traditional networks enjoy a comparative advantage in investing in new technology and, thereby, act as catalyst to facilitate the diffusion of modern technology. Munshi and Rosenweig (2006) report that, following economic reforms in the 1990s, despite the sudden growth spur in returns to non-traditional white-collar occupations which require ability to speak English, male working-class—lower-caste—networks continued to channel boys into local language schools that led to the traditional occupation. In contrast, lower-caste girls, who benefit little from the social network and hence historically have low labor market participation rates, took full advantage of the opportunities that became available in the new economy, by switching rapidly to English training schools. This example illustrates how such potential entrepreneurial instincts of the underprivileged women can be the usher of economic development by facilitating knowledge diffusion. It also illustrates how social networks could be a curse for those who enjoy its privileges to block economic progress, while it’s a blessing for the outcast, who write the story of development with diffusion of new knowledge.

2.1.4 Policy Actors

The comparative advantage of being an underprivileged in a society stems from the fact that opportunity cost of change that necessarily comes with adopting a new technology is relatively less than those who enjoy a privileged status in a social network. However, willingness to being a change as a catalyst doesn’t necessarily bring that change. Asim Khwaja (2007) argues that understanding the mechanics of the policy actors for manipulating the bureaucracy and the political system is important for implementing a change. How politicians work, how bureaucrats work, we simply don’t have the data on this. Yet, we know that very women play as "policy actors" and that’s the most important factor in determining the path of development a country would choose. Thus there is a plenty of evidence for us to recognise the potential role of women in changing the course of economic development and the socio-economic evolution of a society.

2.1.5 Social Psyche

There is an important reason for us to be a bit skeptical about the feasibility and significance of the economic gain from the investment in women’s potential, however. Duuo develops a hypothesis of "stereotype traits" from psychology. Women conform to stereotypes so as to minimise the punishments they would receive otherwise from the society and thus reinforcing those stereotypes. Her explanation of gender gap crucially depends on the social norms and stereotypes and not economic development. These stereotypes benefit men; so to break these stereotypes the policies that we need would reduce rents that men earn. Why would they go along with that? That may explain why men typically avoid any discussion along these lines. Given that most economists are men it is not surprising that there have been little discussion on this issue. On the other hand, there is weak evidence that economic
development, however achieved, may empower women in due course of time.

2.2 The Family

If different societies display alternative shapes and sounds of human evolution then the family values of each society would be the DNA governing those shapes and the sounds they make.

2.2.1 Insufficient Insurance

A family that ensures little financial autonomy for women typically lacks an efficient structure for insuring its members against shocks. The weak position of women, combined with their weak property rights lead to a lot of money being wasted and poorer condition of the house, drinking water and nourishment of the children. Typically, the bargaining power of women within a family matters for the final outcome. So giving more political power to the women would change economic consequences within and outside family. Duflo (2012) notes from the state level data in the US that a drop in infant mortality typically follows the period after women get their right to vote. DT (2009) argue that economic growth may encourage men to give up more bargaining power to women because they are better able to raise children and men care about the children. However, it is likely that those incentives might not be sufficient to offset the initial loss of rent that men will experience because of the policies favouring women against men. It means that economic gains will not be enough and we will need political courage to stand for gender equality for its own sake. There may be a pecuniary benefit to men from policies that reduce violence against women in the form of a better quality of human experience. However, the family values could locally trap them into a suboptimal world. We demonstrate such possibility in our theoretical model.

2.2.2 Importance Of Mother’s Education

Rosenweig and Wolpin (1994) provide evidence to show that the marginal efficiency of expenditure on education increases with mother’s educational attainment. As a result, women’s education and health multiplies intergenerational transmission of knowledge which sometimes has a much deeper beneficial effect on the children than men (Doepke & Tertilt, 2011). Thus to the extent the threat of VAW discourages women from pursuing education, the overall rate of accumulation of human capital and the associated growth in the economy’s total factor productivity decreases, undermining its overall economic growth (Day et al, 2005). Thus undertaking the task of understanding the economic roots of VAW and estimating the direct and indirect cost of VAW (and its threat) that affects women’s choice of education and occupation would be an economically vital aspect of addressing the issues concerning growth in developing countries, where gender based family values significantly influences the allocation of resources in the economy.

2.3 VAW: At Home, Way To Work and At Workplace

Violence within the household could be due to the effect of the "male backlash" where men assert their dominance over their wives by threatening or subjecting her to violence as a
result of the independence she would gain from working (Aizer, 2010). This will dissuade women from joining the labour force due to the probability of violence (POV) women face from their husbands. A study done by Shepard and Pence (1988) show women who are subject to domestic violence will be less productive at the workplace and they are more likely to quit (Day et al, 2005). The POV women face outside their homes could be any type of physical or sexual violence women face when travelling to and from work. The existence of this form of violence such as what they face in a developing country, when they use public transport or travel at night (Buvinic & Morrison, 1999). Such constraints due to women’s lack of safety reduce their work capability, causing a gender inequality in earnings. At the same time, the economy incurs net loss from such gender inequality.

2.3.1 Institutional Discrimination Against Women

Workplace discrimination is also another source of gender inequality. Most women in developing countries face discrimination in the workplace as they are often assigned to low paying jobs and other high paying jobs get branded as male jobs, sometimes with trivial or no excuses. Those justifications often receive religious and social justification under various rules of patriarchy. Another common occurrence in developing countries is men facing greater opportunities to climb the hierarchical ladder than women (Budhwar et al., 2005). In certain cases the level of discrimination acts to completely deprive women of higher paying jobs due to those particular jobs not being "fit for feminine labour". The existence of harassment at the workplace also deepens the gap between men and women as it creates an unsafe environment for women causing them to be less productive or leave the labour force altogether. In India before 1997 women experiencing sexual harassment at the workplace had to lodge a complaint under the section that deals with the "criminal assault of women to outrage women’s modesty" the definition of modesty was left to the discretion of police officers (Patel, 2005). Therefore workplace discrimination in the form of harassment or through male preference increases the gender gap.

2.3.2 Barriers To Women’s Education

Violence faced by women in the form of barriers to education also increases the gender gap. In some developing countries there is a strong cultural idea of protecting the virtue of the daughter. As a result parents may be less willing to send their daughters to higher education due to the social unacceptability of having prolonged interactions with men (Chanana, 2000). Women also face socially constructed barriers to education due to societal disapproval of women’s education. Such was the case of Malala, a 14 year old girl who was shot in the head for promoting female participation in education. A study done in India showed that many parents in rural areas believed educating girls is not necessary because they are only expected to marry and look after the household (Duflo, 2011). These barriers to education cause the human capital of women to be lower than men creating gender inequality.

2.3.3 Declining Female To Male Ratio

There is a noted biasness towards sons rather than daughters in developing countries. This is evident by the observation of "missing women" which is a term coined by Amartya Sen
The World Development Report (2012) estimated that more than six million women are missing each year globally. Although it includes violence against women (VAW) we mentioned above, there is also the presence of unequal investment in human capital in the form of medical care and nutrition between a son and a daughter leading to a higher level of mortality for girls. There have been a number of studies which promote this observation. Jensen (2010) shows that the mortality rate of girls in infancy and childhood is 40-50% greater than boys due to the relatively low level investment in human capital they receive.

Jensen (2010) argues that the reason for the unequal investment in human capital is due to the societal idea that the return from investment will be lower as the daughter is expected to be married off. Duflo (2011) also confirmed this by presenting a study done in the poor neighbourhoods of India where girls are more than twice as likely to die of diarrhoea as boys, due to a lack of medical attention and nutrition. Parish & Willis (1993) looked at the same phenomenon in Taiwan to observe that, due to the job opportunities of sons being greater, daughters received a relatively lower investment in human capital due to the rate of return from daughters being lower.

Moreover, as perceiving a higher likelihood of violence, skilled women will withdraw from the labour force their returns to schooling decreases. As parents perceive a lower rate of return from investment in a girl child due to higher POV against women in the society, they would reduce resources available for educating their daughters. Such underinvestment in human capital would create a human capability gap between men and women.

To the extent children benefit from the mother’s human capital, the underinvestment in female children in one generation can lead to under accumulation of human capital for all children in subsequent periods (Breierova & Duflo, 2004, Doepke & Tertilt, 2011). Consequently growing violence against women could generate an endogenously adverse, dynamic effect on the growth rate of GDP, bringing down the long-run growth rate to a sub-optimal rate.

### 2.3.4 Technology and the Value of Women

Galor and Weil (1996) assume that in the production technology, there are two types of labour: mental and physical labour. Men are assumed to be capable of supplying both types of labour to the market whereas women can supply only mental labour. They effectively assume that both men and women have brains but women do not have brawn. Initially the economy runs on a primitive technology which does not value mental labour as much as physical labour. As a result women first and foremost raise children at home, depending on the time cost of raising children. Overtime as the technology in the economy improves; mental labour becomes relatively more desired than physical labour. Thus increasing women’s wages and reducing the fertility rate, as women substitute away from childrearing towards joining the labour force. When human capital accumulation is possible in the economy, it will magnify the mental labour input, thus strengthening the above results. However, note that the accumulation of human capital only affects the parents not the children. This is because they were interested in the quantity rather than the quality of the children. We assume a constant population growth and endogenous human capital. We do this to analyse how VAW affects the rate of accumulation of human capital and, how that, in turn, affects the long run economic growth of the society as well as the wellbeing of its members.


2.3.5 Choices Of Married Couples

Becker (1973) uses the unitary approach to model consumption choices of a married couple. The couple maximises a single utility function subject to a single budget constraint in which all income is pooled. Such unitary model does not capture any gender specific differences in the decision making of a married household. Presumably, family consumption and investment in their children’s education and health are like public goods from which both partners derive identical utility.

Doepke and Tertilt (2009) develop a model of the patriarchy regime, where the husband and the father care about the utility of his wife and the human capital of his children. The degree of his care for his wife varies from zero to one, under different regimes of patriarchy; but the men always have complete decision making power.

In the model, they show how an improvement to the legal rights of women occurred despite women not being able to vote for these changes. They argue that when the significance of human capital increases in the economy, due to technological progress, men become more willing to empower women. This is because although men do not care about empowering his own wife, he does care about his daughters and grandchildren, and would prefer them not to be exploited by their husband. Doepke and Tertilt show how the economy shifts from the patriarchy regime, where men have complete bargaining power to the empowerment regime where men and women have equal bargaining power. The key to their result was the assumption that the women care more for their children than men; and men have altruistic concerns for their children’s future spouse and hence favour women’s empowerment. In other words, they argue that “glorification of motherhood” in the 19th century actually helped advancing the cause of women’s right in the earlier phases.

Doepke and Tertilt (2011) examine the association between female empowerment and economic development by evaluating different economic theories of household decision making. They argue that there is a limitation in using a unitary model because spouses within a marriage do not value the allocation of private and public consumption goods equally. In order to take into account conflicts of interests between spouses they consider cooperative and non-cooperative bargaining models of marriage. In cooperative bargaining models, the existence of conflicts of interests results in spouses having different utility functions. Yet, the spouses arrive at a Pareto optimal solution with the solution being a function of their relative bargaining power. Doepke and Tertilt also considers, Nash bargaining with divorce as a threat point, following the idea of Manser and Brown (1980) and McElroy and Horney (1981). Although we are not interested in modelling divorce we will explore the outside option of the couple by identifying their respective steady state indirect utility as singles.

2.3.6 Social Culture and Attitude

Fernández (2009) emphasises the crucial role of a society’s attitude toward the supply of work by married women. Arguably, a hostile attitude toward women’s work and women’s education in the society contribute to the violence against women, which act as a barrier to economic growth in our model. Fernandez (2009) asks why the progress of women’s economic and political rights overlap with economic development, by focusing on property rights for women. She explores this question using a similar approach to Doepke and Tertilt (2009),
where, men prefer not to yield property rights to his own wife, but wants property rights for his daughter. As the economy develops by achieving greater accumulation of human capital and a drop in fertility, men switch from the patriarchy regime to a regime where women have equal rights to property. Unlike Doepke and Tertilt (2009) this change occurs due to the drop in the fertility rate rather than the growth in human capital accumulation. She provides indicative evidence from the United States by showing that states with a faster reduction in fertility were more likely to expand women’s rights to property. From the above literature we see that the economic conditions of women and the violence they face, in terms of unequal marital bargaining power and societal and legal rights, is an important concept to model and further interpret.

2.3.7 Women’s Bargaining Power

Bina Agarwal (1997) calls our attention to the fact that women derive bargaining power from having resources such as income and assets. Noting also that compared to men, traditionally women enjoy limited property rights, Pradeep Panda and Bina Agarwal (2005) conclude that new laws for joint property ownership of land and houses improve women’s bargaining power and reduces domestic violence. A study of women and men in the US confirms this conjecture. John Knowles (2005) finds evidence that a greater bargaining power of empowered women causes married men to work 2.1 hours per week longer than unmarried men and, married women to work about 2.7 hours per week less than unmarried women. Kaushik Basu (2006) argues, however, that bargaining power and bargaining outcomes affect family partners in subtle ways such that household bargaining is a complex endogenous process going well beyond the assumption that limits individual bargaining power to a parameter. As a starting point for our macroeconomic analysis, however, we limit our analysis to such parametric representation.

2.3.8 Inheritance Rights For Women and VAW

Amaral (2012)’s work parallels the ideas of Fernández (2009) as she explores the effect of legal changes on violence against women in India. She examines the effect the amendment of the Hindu Succession Act, which was heavily biased towards men prior to 1956 (Bala, 2013), on the subsequent incidence of violence against women. By analysing reported violence as well as surveyed violence, she finds a huge improvement in the rate of violence against women after the amendments to the bill had passed. She notes that this could be due to the improved inheritance rights of women increasing the bridal value which could reduce the dowry payments and thus violence relating to dowry by the husband and in-laws. The increase in bridal value could also increase the groom value giving her the ability to attain a better quality partner, which will increase her welfare. This piece of literature also outlines the son bias that is present in some developing countries. We will use that fact when forming our models for the family. Therefore, we can see that there is a gap in the literature where the overall violence women face is not taken into account. We try to fill this gap by creating a model which has different forms of VAW, aspects such as the wage gap (workplace discrimination), son bias, domestic violence in the form of bargaining and the probability of violence women face outside the home.
2.3.9 The Women Experience

For women the welfare loss goes deeper than the mere loss of GDP. It diminishes the quality of her life by forcing them to accept less than equal right as men. Violence against women is unique in many ways. A girl child achieves her womanhood by accepting the gender-biased line drawn around her access to privileges in a patriarchy. If she attempts to cross them, she risks trouble and violence of various kinds. She makes mistakes because she is not born with such knowledge; but in an effort to avoid violence, she learns to be good by acquiring womanhood.

"One is not born a woman, but becomes one." - Woman: Myth and Reality of The Second Sex, Simone de Beauvoir

2.4 Data and Motivating Growth Empirics

Prior to describing the model, we present a summary of an empirical analysis with an objective to identify key economic determinants of the growth rate of GDP per capita. We use a modern econometric technique that can pick the most relevant and most fundamental set of explanatory variables from a large number of plausible variables. We pool data from a combination of two reputable and popular data sources as well as a new database that report data on a large number of variables on women.

After selecting the most relevant variables, we run regressions and select model based on three alternative criteria. A new set of variables emerged as important determinants of growth; most of which concerns the status of women in a society. In particular, we discover a set of robust observations on the significance of new variables such as (i) early marriage and (ii) restriction against women’s ownership of property, both of which are likely to determine women’s autonomy in a family and, (iii) civil liberty which facilitates women’s freedom, and especially the freedom of movement to places of education and work to be three important determinants of the economy’s growth rate of GDP per capita.

The next two pages describe our new findings which motivate the design of our model in the following section:
Table 1: Lasso model selection

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<td>Growth rate of employment</td>
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<tr>
<td>Inheritance</td>
<td></td>
</tr>
<tr>
<td>Early marriage</td>
<td>-0.029</td>
</tr>
<tr>
<td>Polygamy</td>
<td></td>
</tr>
<tr>
<td>Violence against women</td>
<td></td>
</tr>
<tr>
<td>Average years of schooling</td>
<td></td>
</tr>
<tr>
<td>Tertiary qualified female</td>
<td></td>
</tr>
<tr>
<td>Tertiary qualified male</td>
<td></td>
</tr>
<tr>
<td>Tertiary qualified gender ratio</td>
<td></td>
</tr>
</tbody>
</table>

2.4.1 Violence against women - Observations from Lasso regressions

In our regression model, we apply Lasso-type regression on observations of a 69 country sample with growth rate of GDP 1980-2010 as dependent variable and 19 explanatory variables including GDP 1980, growth rates of population and employment 1980-2010 investment rate, total factor productivity (Penn World Table 8.0), categories and sub-categories of social institutions and gender index (SIGI 2009) and human capital variables (Barro & Lee 2010). Out of all explanatory variables, Lasso regression selected 8 meaningful variables (Table 1). In the regressions of table 2, we present estimates of the meaningful variables selected in Lasso-type regression above (regression 1) and estimates of the variables selected using forward stepwise regressions with both AIC (regression 2) and BIC (regression 3) methods. The impact of women’s civil liberties and ownership restrictions is significant across the regressions with opposite signs. While women’s civil liberties have positive impact on economic growth, women’s ownership restrictions and early marriage bring in negative impact.

---

3 The technique of lasso (for “least absolute shrinkage and selection operator”) was introduced by Tibshirani (1996) for estimation in linear models. Basically it minimizes the usual sum of squared errors, with a bound on the sum of the absolute values of the coefficients. The technique is often used in data mining where other statistical techniques such as OLS and t-test cannot be applied due to huge numbers in the number of variables and/or in the number of observations.
Table 2: Impact of barriers against women on economic growth 1980-2010

<table>
<thead>
<tr>
<th>Dependent variable: growth rate of GDP per capita</th>
<th>(1) (lasso)</th>
<th>(2) (AIC)</th>
<th>(3) (BIC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.089 (**)</td>
<td>0.107 (***)</td>
<td>0.110 (***)</td>
</tr>
<tr>
<td>GDP per capita 1980</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth rate of population</td>
<td>-0.409</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth rate of employment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>-0.010 (*)</td>
<td>-0.015 (***)</td>
<td>-0.013 (***)</td>
</tr>
<tr>
<td>Investment rate</td>
<td>0.043</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family code</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Civil liberties</td>
<td>0.029 (**)</td>
<td>0.036 (***)</td>
<td>0.033 (***)</td>
</tr>
<tr>
<td>Physical integrity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Son preference</td>
<td>0.008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ownership restrictions</td>
<td>-0.028 (**)</td>
<td>-0.032 (***)</td>
<td>-0.034 (***)</td>
</tr>
<tr>
<td>Parental authority</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inheritance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Early marriage</td>
<td>-0.048 (**)</td>
<td>-0.043 (*)</td>
<td>-0.058 (***)</td>
</tr>
<tr>
<td>Polygamy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Violence against women</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average years of schooling</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tertiary qualified female</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tertiary qualified male</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tertiary qualified gender ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj Rsq</td>
<td>0.406</td>
<td>0.4016</td>
<td>0.3852</td>
</tr>
<tr>
<td>AIC/BIC</td>
<td></td>
<td>570.01</td>
<td>557.88</td>
</tr>
<tr>
<td>F-statistic</td>
<td>6.819</td>
<td>10.13</td>
<td>11.65</td>
</tr>
<tr>
<td>Number of obs</td>
<td>69</td>
<td>69</td>
<td>69</td>
</tr>
</tbody>
</table>

Significant level of: 0 '****' 0.001 '***' 0.01 '**' 0.05 '*' 0.1 '.' 1
3 The Model

Economic vocabulary that dominates the model’s focus consist of three specific gender gaps: a gender gap in earning due to the state of technology modelled parametrically as in Galor and Weil (1996), an endogenous gender gap in human capability (or, human capital) arising from economic conditions to which various works of Amartya Sen draw our attention and endogenous gap in the participation rates between men and women, which is well established in the data. Two key variables that we model parametrically as our key explanatory factors are: (i) the perceived likelihood or probability of violence (POV) against women, measured by its relative frequency in the overall crime statistics, and, (ii) the relative strength ($\sigma$) of patriarchal control of men over women, measured by data on civil liberty and restrictions against women’s ownership of properties.

Galor and Weil (1996) model (GW) considers the decision making framework of a unitary family made up of a man and a woman and n couples of children\(^4\). The joint utility function for the family depends on the number of children and the consumption in the next period (old age). Because the main focus of GW in this paper was to show the relationship between output growth and fertility they concentrated more on the quantity rather than the quality of children. In our model we assume that each family receives two children (a son and a daughter) and that parents care about the quality (measured by human capital) of their children. We introduce this additional feature because we are interested in modelling gender bias even in the capability of the children. GW assumes that the man and woman pool their income within a family, we will take this into account in our model by assuming that the man and woman will combine their individual saving to form the family savings which is used to spend on the expenses of the family unitarily.

Following GW we build an overlapping generation model where people live for three periods as children, young adults and married parents or single with no children. Children receive human capital from their parents, which could include nutrition and medical care, as well as education through personal interaction and schooling.

In GW model men and women differ only in adulthood due to the different types of labour they are able to supply. Men can supply both physical and mental labour, but women can only supply mental labour due to the unequal endowment of inherent abilities between the genders. Therefore the opportunity cost of men looking after the children is greater than for women, resulting in women primarily looking after the children and working part time if at all. Thus a wage gap is created between men and women. This implies that the ratio of men’s wages to women’s wages will always be greater than or equal to one (it will only equal one if physical labour is deemed completely redundant). We will use this idea in our model to capture the gender gap due to the male bias $m$ among the employers which causes the wage rate of men ($z^m$) relative to women ($z^w$), ($m = \frac{z^m}{z^w}$) to be greater than or equal to one. The ratio of the wages also captures the relative productivity of men and women, as modelled in GW.\(^5\) GW assumes that physical capital rewards mental labour more than physical labour. Therefore an exogenous improvement in technology favouring mental labour raises the wage

\(^4\)GW assumes the unit of measure is a couple.

\(^5\)Such gender gap in earning could also arise from gender related discrimination and harassment at the work place that may also negatively affect their productivity resulting in the relative wage women receive to be lower than men’s (Nath, 2000).
rate of women relative to men, causing the opportunity cost of raising children to increase. Consequently, women substitute away from child rearing towards joining the labour force. Thus, in the GW model, a reduction in the gender gap in earning leads to a reduction in the gender gap in the participation rates. Our model abstracts from the fertility choice. Yet, it accommodates the above link between the two gender gaps by examining the decision making of a family which takes into account a positive probability of violence against women (POV) in the underlying social culture.

3.1 Societal Structure: Patriarchy versus Empowerment

Following Doepke and Tertilt (2009), henceforth DT model, we expand the framework of our analysis to consider a model where men and women negotiate the allocation of resources. Similar to GW it is an overlapping generation model where women and men form a couple through marriage.

3.1.1 The Economic Unit: Family

Each married couple decides together on the allocation of consumption between the spouses, and how much to allocate to the education of the children, following the underlying rules of the society. Each spouse gain utility from their own consumption, the consumption of their partner, the number of children they have and the average utilities of their children. Importantly, following DT, we assume that each spouse places a weight ($\sigma < 1$) lower than their own on the consumption of their partner. Each family makes decisions under the following constraints: (i) earning depends on education, employment and productivity of both parents; (ii) budgeting requires that income must equal the sum of consumption, saving and investment in children; (iii) growth of family wealth relies on the intergenerational transfer of human capital involving parental contacts, education and inborn talents. To avoid irrelevant details we assume that each couple receives one son and one daughter and abstract from population growth. We assume an "warm-glow" type preference for the altruistic parents such that each parent gains utility from the human capital their children and from the joint consumption of the couple. The human capital is the state variable of the model. It determines the maximum value of the utility of a person. Thus by choosing human capital of their children, the parents determine the maximum value of their children’s utility as grown-up adults in the next period.

We allow for a gender based asymmetric preference of son over daughter to capture the well-known "son-bias" in developing countries (Duflo, 2011). In particular the elasticity of parents’ utility with respect to the human capital of the son is denoted $\gamma^m$ and that of the daughter $\gamma^w$ and the "son-bias" is denoted by the ratio ($\omega = \gamma^m/\gamma^w$) of the two elasticities. Note that the data reported in http://genderindex.org/data, used for our motivating data analysis in the previous section, offer ample evidence of $\omega > 1$ in many developing countries. However, the model does not require $\omega > 1$ to uphold its main hypothesis that the culture of VAW undermines economic growth and economic wellbeing.

Following the DT model, we assume men work full time ($n^m = 1$) and women’s work bring to the family some disutility associated with the probability of violence (POV) women face. Thus, the model accommodates two commonly known adverse effects of the perceived
threat of VAW on the family’s welfare. A positive POV lowers family income by discouraging women to work; and it also negatively affects the happiness and wellbeing of women directly.

Thus, following DT, the utility functions for men and women satisfy:

\[ U^m(t) = \ln[c^m_t] + \sigma(\ln[c^w_t] - (1 + p)n^w_t) + \gamma^m\ln[h^m_{t+1}] + \gamma^w\ln[h^w_{t+1}] \]  

(1)

and,

\[ U^w(t) = \ln[c^w_t] + \sigma\ln[c^m_t] - (1 + p)n^w_t + \gamma^m\ln[h^m_{t+1}] + \gamma^w\ln[h^w_{t+1}], \]  

(2)

where \( 0 < \sigma < 1 \) denotes the weight given to the economic choices specific to the individual’s spouse, \( \gamma^m \) is the weight given to the human capital of the son, and \( \gamma^w \) is the weight given to the human capital of the daughter. As mentioned above we have placed two different weights on the son and the daughter to accommodate potential parental gender bias that occurs in some developing countries, where sons are preferred to daughters.

### 3.1.2 Social Welfare - Patriarchy

Under a patriarchal societal structure, women’s primary role is limited to that of a mother and a daughter with no outside options unlike men, who make all the decisions for the family. Also educating a daughter incurs additional cost as it increases the length of her unmarried life as an adult and that coupled with her education decreases the chance of finding a husband and hence increases her dowry. The socially prohibitive cost of a divorce, customary denial of inheritance rights and arranged marriages which typically serve the interests of parents, reduce women’s autonomy inside the family under patriarchy. The morality that sustains such a system with no overt violence against women appeals to the man’s enlightenment with religious ground for loving his wife and daughters. To capture that reality we model the man of the family as the decision making husband and father, who cares about his wife and children, and the degree of his care increases with his enlightenment.

In the patriarchy regime there is no marital bargaining between the spouses and men make all the decisions so the utility of the family is equal to the utility of men.\(^6\) The utility function for the family under the patriarchy regime \( (U^P) \) is, by definition, the same as the utility of the representative Patriarch male as given by (1) such that at date \( t \), \( U^P(t) \) is given by:

\[ U^P(t) = \ln[c^m_t] + \sigma(\ln[c^w_t] - (1 + p)n^w_t) + \gamma^m\ln[h^m_{t+1}] + \gamma^w\ln[h^w_{t+1}]. \]  

(3)

\(^6\)Note that when the wife has less than proportionate bargaining power within a marriage, over the various economic choices made for the family, it could represent the existence of exploitation within the relationship, which may include the threat of domestic violence (Pollak, 2005).
3.1.3 Social Welfare - Empowerment

Following DT, as an alternative societal structure to patriarchy, we also consider an "empowerment" regime which upholds women's empowerment by assigning equal weights to the utility of men and women in the family welfare function. In particular, the utility of the family is equal to the average utility of men and women and, the social utility under the empowerment regime \( U^E \) is given by:

\[
U^E(t) = \frac{1 + \sigma}{2} \left( \ln[c^m_t] + \ln[c^w_t] - (1 + p)n^w_t \right) + \gamma^m \ln[h^m_{t+1}] + \gamma^w \ln[h^w_{t+1}],
\]

(4)

where \( h^i_{t+1} \), as mentioned above, denotes the human capital of the child \((i = m, w)\) when he or she is an adult in the next period \((t + 1)\).

In the DT model, the cost of educating the children as a time cost only affects women. The GW model does not consider family investment in education for the children and their human capital. Both models treat children as babies and infants that require the care and nurture and they take up a significant chunk of productive time endowment of the mothers. As a result the number of hours women work is determined in excess of the time cost of raising children. In our model, we deviate from the idea that children are babies. We assume that they are at an age where both the mother and father can share their capital to invest in the children's future human capital stock while allowing for some heterogeneity in the technology for transferring this human capital. Also, we focus on the significant part of a woman's life in which she does not have to play the role of a mother, at least not by assumption. Women do enjoy a higher life-expectancy than men and hence on average they do not spend a large portion of their career nurturing babies.

Also, we assume that the cost of primary schooling is approximately zero because it is publically funded. Consequently, the date \( t \) investment \( c^i_t \) in the child i's education represents the amount that is saved by the parents for the secondary and tertiary education of their children. The budget constraint for a typical family takes the following form:

\[
c^m_t + c^w_t + e^m_t + e^w_t(1 + q) = y_t,
\]

(5)

where,

\[
y_t = z^m h^m_t + z^w h^w_t n^w_t
\]

(6)

As mentioned above we assume that \( z^m \) is the productivity or the wage rate for men and \( z^w \) is the productivity or the wage rate for women. We assume \( n^m_t = 1 \), following DT.

The budget constraint for the family contains consumption expenditure for the man \((c^m_t)\) and woman \((c^w_t)\) and education investment expenditure for the son \((e^m_t)\) and the daughter \((e^w_t)\). We have also included an additional cost that will only affect the education investment of the daughter \((q)\). The additional cost \((q)\) represents the additional parental investment necessary to ensure a safe environment for the daughter to carry out her education (Buvinic & Morrison, 1999). We will refer to \( q \) as the additional cost of educating women (ACEW).
3.1.4 Intergenerational Evolution of Human Capital

The human capital of the adult \(i\) in period \(t+1\), \(h_{t+1}^{i}\) depends on the portion of human capital they receive, due to a family environment externality, from their close association with the mother \((\delta_{t}^{w}h_{t}^{w})\) and the father \((\delta_{t}^{m}h_{t}^{m})\)\(^7\), the parental investment in education \((e_{t}^{i})\) for the child and the inherent ability of the child \((\tau)\)\(^8\) such that:

\[
h_{t+1}^{i} = \delta_{t}^{m}h_{t}^{m} + \delta_{t}^{w}h_{t}^{w} + e_{t}^{i} + \tau
\]  

(7)

The human capital function (5) above which we use in our model is a close approximation of the logarithmic transformation of the human capital function used by DT and corresponds to one typically used in large body of literature that follows empirical growth models of Mankiw, Romer and Weil (1992). Similar formulation can also be seen in the context of endogenous growth models such as one in Bandyopadhyay and Basu (2005).

3.1.5 Endogenous Gender Gap in Consumption

Each family maximises the utility specific to each regime, i.e., (3) under patriarchy and (4) under empowerment, by choosing men’s consumption \((c_{t}^{m})\), women’s consumption \((c_{t}^{w})\), education for the son \((e_{t}^{m})\) and daughter \((e_{t}^{w})\), and the number of hours women work \((n_{t}^{w})\) subject to the budget constraint expressed in equation (6). Where \(h_{t+1}^{i}\) is given by (5) and \(y_{t}\) is given by (7).

**Patriarchy**  Under patriarchy, we obtain the following optimal choices for consumption:

\[
c_{t}^{m} = (h_{t}^{w}z^{w})/\sigma(1 + p)
\]  

(8)

\[
c_{t}^{w} = (h_{t}^{w}z^{w})/(1 + p)
\]  

(9)

Note that \(c_{t}^{w} < c_{t}^{m}\) and the gender gap in consumption decreases in \(\sigma\), the strength of the patriarchal control of men over women; and the model provides an empirical estimate such that \(\sigma = c_{t}^{m}/c_{t}^{w}\). This result matches DT. In addition, our model reveals how a rise in POV decreases consumption for both men and women.

**Empowerment**  Under the empowerment regime, the gender gap in consumption no longer exists such that

\[
c_{t}^{m} = c_{t}^{w} = \frac{h_{t}^{w}z^{w}}{(1 + p)}.
\]  

(10)

\(^7\)We allow for heterogeneity arising from inter gender interaction between parents and children. This is to capture the home environment externality where the father and mother have different relationships with their son and daughter. However we allow this heterogeneity keeping in mind the empirical identification of the model in future research. The key findings of this dissertation do not rely on this heterogeneity.

\(^8\)If a child receives no human capital or education from his or her parents the child will still have inherent ability to participate in low skilled work. We assume that the inherent abilities are equal between the genders.
The above result also coincides with DT. Note that the consumption for women does not change from the patriarchy regime. However the consumption choices of men are lower in the empowerment regime compared to the patriarchy regime. Interestingly, the impact of POV on consumption of either men or women does not vary between the two regimes.

3.1.6 Effect of POV and Son-Bias in Children’s Education and Human Capital Growth

The POV has a negative impact on the parental investment in children as well as on the number of hours women work under both regimes:

**Patriarchy**

\[
e_t^m = \gamma^m h_t^w z^w \frac{h_t^m}{(1 + p)\sigma} - \delta^m h_t^m - \delta^w h_t^w - \tau
\]

\[
e_t^w = \gamma^w h_t^w z^w \frac{h_t^m}{(1 + q)(1 + p)\sigma} - \delta^w h_t^m - \delta^w h_t^w - \tau
\]

The investment in education for the son and daughter increases as the weight given to the son \((\gamma^m)\) and daughter \((\gamma^w)\) increases respectively, due to parents caring more about their children. The greater the strength \((\sigma)\) of apatriarchal regime is, i.e., the lower the men’s weight on the woman’s utility is, \((\sigma)\) the investment in education for the children will increase. Effectively this represents men substituting the utility they gain away from the wife to the children. In addition, as the higher the POV is in a society, the lower the number of hours women work. Such violence induced reduction in women’s work decreases the family income, resulting in a lower educational investment for the children.

Predictably the investment in education decreases with the human capital each child receives from their mother and father as well as with their inherent abilities. This captures the idea that a child who received a large amount of human capital from his or her parents or has a high level of inherent ability could fund their own education. For example the child can earn a scholarship to fund for their education or use their inherent ability and work to save money for their education.

We can see that, holding all else constant, the choice of educational investment for the daughter is less than that of the son due to the additional cost of educating a daughter which does not affect the son. It is possible that because of this extra cost, parents may not want to send their daughter to higher education or a better quality school due to its location or safety (Chanaana, 2000).

The resulting human capital of the son and the daughter after the education investment has been taken into account is as follows.

\[
h_{t+1}^m = \frac{(\gamma^m h_t^w z^w)}{(1 + p)\sigma}
\]

\[
h_{t+1}^w = \frac{(\gamma^w h_t^w z^w)}{(1 + q)(1 + p)\sigma}
\]
The parental contribution to their children’s human capital is expressed by the mothers’ capabilities and contributions because we assumed that men work full time. This is why $h_{t+1}$ as well as $h_{t+1}^m$ is negatively affected by POV. Also we can see that the human capital of the daughter is lower than that of the son due to the daughter being affected by ACEW.

With endogenous human capital accumulation the model produces endogenous growth.

$$\gamma = \gamma_w z_w \frac{\gamma^m (1 + q)}{\sigma (1 + p)(1 + q)} - 1$$  \hspace{1cm} (15)

We find that due to the reason above regarding $h_{t+1}$ the growth rate of human capital is found to have a significant influence from the human capital of women rather than men. Therefore the growth rate of human capital is negatively affected by POV and ACEW and positively affected by the productivity of women and the weight allocated to daughters’ human capital.

The gender ratio ($R$) of human capital between men and women is equal to:

$$R = \frac{\gamma^m (1 + q)}{\gamma^w}$$  \hspace{1cm} (16)

The ratio of human capital of men to women captures the built-in gender gap ($R - 1$) in earning capabilities. This is due to the parental bias against the welfare of the girl child, indicated by the ratio $R$ and the additional cost $q$ of educating a girl child in a society which disapproves or discourages women’s education. Such cost may include the cost of deterring violence against girls in and out of school, such as popularly seen in case of Malala in Pakistan. Therefore in a society with these two characteristics we are likely to find a high gender gap in human capital or in other words an imbalance of power favouring men. However, the human capital gap between men and women is not affected by POV. Therefore, even in societies where POV is low a gender gap in human capital gap can still emerge if parents are son biased and barriers to educating women are present.

Typically gender gap refers to the earnings gap between men and women. In our model earnings is determined by $y_i = f(h_i, n_{it}, z_i)$. Therefore although the POV does not affect the ratio of human capital, it does affect the ratio of men’s number of units worked compared to women’s. Because this model assumes men work full time, we can see the clear inverse relationship between the POV and women’s labour force participation.

The equation (15) for the balanced growth rate indicates that factors that expands the gender gap in human capability also contribute to lowering the rate of growth of GDP. Two of those factors have a close link to the regime of patriarchy: (i) social cost ($q$) of educating a girl child in a system where women are only glorified as mothers and dowry cost of marriage increases with women’s age and education; (ii) parental preference for son over daughter ($\gamma^m > \gamma^w$), presumably, to avoid having to incur the nuisance cost $q$ that the society imposes upon them.

**Empowerment**

$$e_t^m = \frac{\gamma^m h_t^w z_w}{(1+p)(1+\sigma)/2} - \delta^m h_t^m - \delta^w h_t^w - \tau$$  \hspace{1cm} (17)
\[ e_t^w = \frac{\gamma^m h_t^w z^w}{(1 + q)(1 + p)(1 + \sigma)/2} - \delta^w m_t - \delta^w h_t^w - \tau \] (18)

As long as \( \sigma < 1 \) the investment in education choices for the son and daughter are lower in the empowerment regime compared to the patriarchy regime. This is because in the patriarchy regime the weight given to women’s economic choices is less than in the empowerment regime \( \sigma < ((1 + \sigma))/2 \). By allocating a relatively lower weight to women, in order to achieve the same level of utility, men place a higher weight on the human capital of the children \((\gamma^i)\). Resulting in a higher level of educational investment for the children in the patriarchy regime compared to the empowerment regime. Note that this result is exactly the opposite of what is found by DT. They found that children’s education is greater in empowerment compared to patriarchy. The reason for this significant difference is because DT assumed women care more about the children than the men, thus raising the bargaining power of women effectively improved conditions for the children. Because we assumed men and women place the same individual weights on the son and daughter we do not get the same result.

The resulting human capital of the son and the daughter after the education investment has been taken into account is as following.

\[ h_{t+1}^m = \frac{\gamma^m h_t^m z^w}{(1 + p)(1 + \sigma)/2} \] (19)

\[ h_{t+1}^w = \frac{\gamma^m h_t^w z^w}{(1 + p)(1 + q)(1 + \sigma)/2} \] (20)

Due to the education investment of the children being higher in the patriarchy regime compared to the empowerment regime, the human capital also has the same effect. As noted previously, the human capital of the daughter is less than that of the son due to ACEW.

The growth rate of human capital is given by:

\[ \gamma = \frac{\gamma^w z^w}{(1 + p)(1 + q)(1 + \sigma)/2} - 1 \] (21)

The growth rate of human capital in the empowerment regime is lower than the growth rate in the patriarchy case. This effect is directly due to the human capital of women being greater in patriarchy than empowerment. In other comparative literature that modelled the difference in the growth rate between a patriarchy and empowerment regime received the opposite result. We find that because we assumed both parents care about the wellbeing of their children equally and women experience an additional disutility of working due to violence, when more weight is placed on the economic choices of the women, it will result in reduction in the growth rate because of violence. We find that the ratio of human capital \((R)\) between men and women does not change between the regimes.
3.1.7 Declining Gender Gap in the Participation Rate

Next, we consider how the growth of human capital increases women’s participation rate over time and how a society with a higher POV, or equivalently, a higher threat of VAW undermines that process directly and indirectly by decelerating the growth of human capital in the society.

**Women’s Participation Rate under Patriarchy**

\[
n_t^w = \frac{(1 + \gamma^m + \gamma^w + \sigma)}{(1 + p)\sigma} - \frac{(h_t^m (\delta^m_1 + \delta^w_1 (1 + q))) + h_t^w (\delta^m_2 + \delta^w_2 (1 + q)) + \tau (2 + q)}{h_t^w z^w} - Rm
\] (22)

The number of hours women work predictably decreases as POV increases, because the disutility they gain from working is magnified by POV. An increase in the women’s wage rate \(z^w\) will increase the number of hours women work. This is similar to the result GW found. When the value of mental labour rises relative to physical labour, women’s labour force participation will increase. However adding to GW’s insight we note that it is not only the evolution of skill-biased technology but also but the decrease in POV that increases women’s labour force participation rate over the course of economic development.

The first negative term in the \(n_t^w\) expression represents the sum of human capital of the son and daughter excluding the education investment. The terms specific to the daughter is multiplied by \((1 + q)\) to show ACEW. As explained above, an increase in the human capital children receive from their parents and their inherent abilities will reduce the educational investment of their parents. Due to the expenses of the family being lower, women can decrease the number of hours they work. Similarly as ACEW increases, the educational investment in the daughter decrease, causing the family expenditure to decrease resulting in women needing to work less.

The second negative term indicates that as the ratio of human capital \((R)\) and productivity \((m)\) between men and women increases, women’s income relative to men will decrease. This will cause the opportunity cost of staying home to decrease, resulting in women lowering the number of hours they work.

**Threshold Tolerance of VAW under Patriarchy:** The threshold POV for women under the patriarchy regime with \(\tau = 0\) and with endogenous human capital is as follows. We will compare this with the threshold POV under the empowerment regime.

\[
p^*_P = \frac{\gamma^w z^w (1 + \gamma^m + \gamma^w + \sigma)}{(\gamma^m (1 + q) (\delta^m_1 + \delta^w_1 (1 + q) + z^m) + \gamma^w (\delta^m_2 + \delta^w_2 (1 + q) + \sigma))} - 1
\] (23)

Similar to the case of patriarchy with exogenous human capital (done in the appendix), as men place a higher weight \((\sigma)\) on women’s economic choices the threshold POV will decrease. An increase in the ACEW causes women’s education to decrease which will decrease their human capital and income. Therefore, due to women’s income being relatively lower, the opportunity cost of women not working is also lower, resulting in a low POV threshold.
Women’s Participation Rate under Empowerment

\[ n^w_t = \frac{(1 + \gamma^m + \gamma^w + \sigma)}{(1 + p)(1 + \sigma)/2} - \frac{h^m_t(\delta^m_m + \delta^m_w(1 + q) + h^w_t(\delta^m_w + \delta^w_w(1 + q)) + \tau(2 + q))}{h^w_t z^w} Rm \quad (24) \]

The interpretation of \( n^w_t \) is the same as in the patriarchy regime with endogenous human capital. However we can see that the number of hours women work under the empowerment regime is less than in the patriarchy regime. This is because, as explained above, under the empowerment regime economic choices for women receive a higher weight than in patriarchy. So the disutility women receive from working due to POV receives a higher weight causing women to decrease the number of hours worked.

**Threshold Tolerance of VAW under Empowerment:** The threshold POV for women under the empowerment regime with endogenous human capital is as follows.

\[ p^*_E = \frac{(\gamma^w z^w(1 + \gamma^m + \gamma^w + \sigma))}{(\gamma^m(1 + q)(\delta^m_m + \delta^m_w(1 + q) + z^m) + \gamma^w(\delta^m_w + \delta^w_w(1 + q)))\sigma} - 1 \quad (25) \]

The interpretation of \( p^*_E \) is similar to \( p^*_P \). By comparing the two POV thresholds we can see that \( p^*_P > p^*_E \). The reason why this is so can be explained by the above argument. Because the weight allocated to women’s disutility is higher under empowerment than in patriarchy, women under empowerment will be more sensitive to POV regarding the number of hours they work. This is similar to the result obtained in the exogenous human capital case (refer to appendix).

### 3.2 A Modern Society With Singles and Married People

We now extend our framework to consider a modern family problem which allows life as a single woman or a single man as an option to a marital relationship. We no longer assume that men work full time. In our economy we have two tracks for each person: marriage (Track 1) and singlehood (Track 2).

**3.2.1 Economic Environment**

People choosing Track 1 earn as single adults and accumulate marital capital prior to marrying a partner and raising children together. To be consistent with the earlier cases we will assume that each family receives one son and one daughter. However, unlike the previous cases, we allow men’s participation to be endogenous. We do it to understand how men’s incentive to work is affected by the likelihood of VAW in a society and to check if married men work more than the singles, following an interesting question that Knowel (2005) poses.
Personal Earning Technology and Budget  Also, similar to the previous cases, an adult person $i$’s date $t$ income $y_i^t$ depends on $i$’s human capital $h_i^t$, number $n_i^t$ of working hours and productivity or wage $z_i^t$, where $i = m, w$ indicates men or women, respectively. In other words, we continue with our assumption of a linear earning technology for analytical tractability. Thus, for each adult $i$:

$$y_i^t = z_i^t h_i^t n_i^t.$$  \hfill (26)

At each date $t$, each adult $i$’s choice of private consumption $c_i^t$, number $n_i^t$ of working hours and saving $s_i^t$ satisfies the private budget constraint as follows:

$$c_i^t + s_i^t = y_i^t, \quad i = w, m,$$

where (26) determines $y_i^t$. Note that in our model, private consumptions for men and women, respectively, $c_i^m$ and $c_i^w$ come from their personal earnings and do not come out of any family budget. We believe that, in the real world, every decision on consumption is not determined by the family as a whole, unlike what is typically assumed in the unitary models of the family, such as Pollak (2005) and Doepke and Tertilt (2011). Essentially, we separate the use of personal income into two non-substituable components: private consumption which benefits each person in isolation and public goods which benefit both the husband and wife jointly and equally.

Utility  At each period $t$, an adult $i$ gets utility $U_i^t$ from private consumption $c_i^t$, disutility from work $n_i^t$, which increases with the likelihood $p_i^j$ of harassment and violence at work or on the way to work, and from the state contingent use value $V_{T=1,2}^i(.)$ of the private saving $s_i^t$ such that

$$U_i^t = \ln[c_i^t] + \beta V_{T=1,2}^i(s_i^t) - (1 + p_i^j)n_i^t, \quad 0 < \beta < 1,$$

where, in Track 1, $T = 1$, for the married adult, $V_{T=1}^i(s_i^t) = V_\alpha(s_t)$, the utility value of the family, which we derive in next, as a function of the sum $s_t$ of personal saving of the marital partners and a marriage contract $\alpha$:

$$s_t = \sum_{i=m,w} s_i^t; \quad s_i^m = \alpha s_t, \quad 0 < \alpha < 1.$$  \hfill (29)

and, in Track 2, $T = 2$, for the single, $V_{T=2}^i(s_i^t) = \ln c_i^{t+1}$, the utility value of consumption next period $c_i^{t+1}$ related to a single life: $c_i^{t+1} = (1 + r)s_i^t$, where $r$ denotes the real rate of return on saving from the global financial market.
Marital Capital  We define the endowment of saving $s_t$ as per (29) that partners gather, prior to committing to a marital relationship together, to be the *marital capital* necessary for that commitment. We assume that spouses agree to a contract that specifies the fraction of the total budget for the family specific public goods each partner will be committed to contribute. Equal division is a possibility; but we do not assume it. Depending on the variations of culture and societal structure, we allow that the marriage contract, indexed by the fraction $\alpha$ of total saving, the man commits to contribute, to vary with an assumption that there is no commitment problem. First, we treat this fraction parametrically and then determine it as a solution to a cooperative Nash bargaining exercise.

3.2.2 Computation of the Family Value Function:

We model the organisation of a modern family in a way similar to the formation of a company partnership. Each partner contributes to a pool of capital which we call "marital capital" which along with the returns it earns from the financial market constitutes the budget for the family. However, the partners follows a cooperative Nash bargaining procedure to determine the share of their individual contribution.

Family Budget  The family budget is limited by the marital capital $s_t$ and the real rate of return $r$ from the global financial market it earns. We take this return as given parametrically. After some time has passed they will earn interest on the family savings which they will spend on joint family consumption $c^f_t$, investments $e^m_t$, $e^w_t$ on the education of the son and the daughter, respectively such that

$$c^f_t + e^m_t + e^w_t (1 + q) = (1 + r)s_t, \quad s_t = s^w_t + s^m_t. \quad (30)$$

The above equation presumes that there is an additional cost $q \geq 0$ related to educating the daughter, as mentioned in the empowerment and patriarchy case, just for the consistency of comparisons.

Family Utility  We assume that the utility of the family is determined in a unitary framework. At each date $t$, the family’s utility $U^f(t)$ depends on the family’s consumption $c^f_t$, the human capital $h^{m}_{t+1}$ and $h^{w}_{t+1}$ of the son and the daughter, respectively, such that

$$U^f(t) = \ln[c^f_t] + \gamma^m \ln[h^{m}_{t+1}] + \gamma^w \ln[h^{w}_{t+1}]. \quad (31)$$

The parents choose cooperatively $c^f_t$, $e^m_t$ and $e^w_t$ in order to maximise the family’s utility (31) subject to (30) and the constraint on the evolution of the son’s and daughter’s human capital, $h^{i}_{t+1}$, $i = m, w$ as per (7).
Value of the Family

Lemma 1: The value function of the family as a function of marital capital $s_t$ is given by:

$$V_\alpha(s_t) = \ln \left( \gamma^m \gamma^m (\gamma^w)^w \right) \left[ \frac{h_t^m (\delta^m_m + \delta^m_m (1 + q)) + h_t^w (\delta^w_l + \delta^w_l (1 + q)) + \tau(2 + q) + (1 + r)s_t}{1 + \gamma^m + \gamma^w} \right]^{(1 + \gamma^m + \gamma^w)}.$$  

(32)

Proof: Appendix.

3.2.3 Optimal Choices For The Modern Society

The optimal solution to the economic choices for men and women are as follows:

Private consumption (men):

$$c^m_t = z^m h^m_t + T \ast v(p),$$  

(33)

Private consumption (women):

$$c^w_t = z^w h^w_t - T \ast v(p)$$  

(34)

where, $T = 1$, for the married men and women along the Track 1 and $T = 0$, for the singles along the Track 2 and where $v(p)$ is given by,

$$v(p) = \beta \frac{z^w h^w_t}{1 + p} \left( 1 + \gamma^m + \gamma^w \right) \left( \frac{g(m, R, p)}{2(1 + p)} \right),$$  

(35)

where,

$$g(m, R, p) = mR(1 + p) - 1.$$
Gender Gaps, Gender Rents and the Violence Premium for Men  We may interpret \( g(m, R, p) \) as a gender specific rent for men that arises if and only if \( mR(1+p) > 1 \), where, \( p \) denotes the likelihood of VAW and \( m = z^m/z^w > 1 \) and \( R = h^m_t/h^w_t > 1 \), respectively, imply the gender gaps in earning per hour (or wages) and in human capital (or, capability) in a society. Clearly, \( v(m) \) denotes a zero-sum transfer of consumption from men to women or, a rent that men extracts from women as the violence premium which reduces to zero if and only if the gender rent due to gender gaps disappears from the society such that \( g(.) = 0 \).

The consumption pattern across marital status differs only by the violence premium \( v(p) \), given by (35). Presumably, men can extract a gender specific rent \( g(p, mR) \) in a society with VAW and gender gaps due to socio-economic structure of technology and parental bias in gender based preferences. However, they can only do so as married with children. It appears women can avoid paying these rents if they remain single without children.

Thus we get a counterintuitive result that in a society with a greater likelihood of VAW and larger gender gaps, women are better off staying as singles, because the rent men extract through marriage increases with the likelihood of VAW and gender gaps in the society.

**Labour Force Participation (men)**

\[
 n^m_t = 1 + \beta + T \left[ \beta (\gamma^m + \gamma^w) - (1 - \alpha) \beta \frac{(1 + \gamma^m + \gamma^w)(2 + g(m, R, p))}{2(1 + p)mR} \right] 
- \alpha \frac{h^m_t (\delta^m_m + \delta^w_m(1 + q)) + h^w_t (\delta^m_w + \delta^w_w(1 + q)) + \tau (2 + q))}{(1 + r)h^m_t z^m_t},
\]

and

**Labour Force Participation (women)**

\[
 n^w_t = \frac{1 + \beta + T \left[ \beta (\gamma^m + \gamma^w) \right]}{1 + p} - \frac{\alpha \beta (1 + \gamma^m + \gamma^w)(2 + g(m, R, p))}{2(1 + p)} 
- (1 - \alpha) \frac{h^m_t (\delta^m_m + \delta^w_m(1 + q)) + h^w_t (\delta^m_w + \delta^w_w(1 + q)) + \tau (2 + q))}{(1 + r)h^w_t z^w_t},
\]

where, \( T = 1 \), for the married men and women along the Track 1 and \( T = 0 \), for the singles along the Track 2.

**Saving**  \( T = 1 \), for the married men and women along the Track 1,

\[
s^m_t = \alpha s_t, \quad s^w_t = (1 - \alpha)
\]

28
where,
\[ s_t = \frac{\beta(1 + \gamma^m + \gamma^w)(mR(1 + p) + 1)}{2(1 + p)} z^w h_t^w \]  \hspace{1cm} (39)

\[ s_t = \frac{h_t^m(\delta^m_m + \delta^m_w(1 + q)) + h_t^w(\delta^w_m + \delta^w_w(1 + q)) + \tau(2 + q)}{(1 + r)} \]  \hspace{1cm} (40)

and

the parameter \( \alpha \) is either specified by the society parametrically or is determined by a Nash bargaining problem (see Appendix B) between the spouses as a part of writing the marriage contract such that

\[ \alpha = \alpha(p, q, m, R, \sigma). \]  \hspace{1cm} (42)

For the single men \((m)\) and women \((w)\) along the Track 2, saving \( s_t^i, i = m, w \):

\[ s_t^m = \beta z^m h_t^m, \quad s_t^w = \frac{\beta z^w h_t^w}{(1 + p)}. \]  \hspace{1cm} (43)

In response to a higher POV, women work less, plans to save less for the family and consume less. To offset the loss of capital for family, men work more but to compensate for the loss of utility consume more as well; yet, the overall effect on saving is negative.

We can see that both men and women’s consumption is affected by POV, but women’s consumption is affected by a greater degree. This shows that although men and women choose their consumption individually using their individual income, because of the notion of the pooled family savings, their individual consumption choices are affected by their partner’s income. Consumption increases with own income and decreases with partners income.

The number of hours men and women work is also affected by POV. Men have a positive effect on the number of hours they work when POV increases and women have a positive and a negative effect. For women we can see that the negative effect dominates. The reason why this positive effect exists is due to POV decreasing family savings, requiring each spouse to work more. We can also see that the positive effect from the POV term is multiplied by the opposite spouses contribution to family savings.

**Family Allocations:** The optimal choices of the family are as follows:

\[ c_t^f = \frac{\beta(1 + r)(z^w h_t^w + (1 + p)z^m h_t^m)}{2(1 + p)} \]  \hspace{1cm} (44)

\[ e_t^m = \frac{\beta \gamma^m (1 + r)(z^w h_t^w + (1 + p)z^m h_t^m)}{2(1 + p)} - (\delta_m h_t^m + \delta_w h_t^w + \tau) \]  \hspace{1cm} (45)
\[ e_t^w = \frac{\beta \gamma^w (1 + r)(z^w h_t^w + (1 + p)z^m h_t^m)}{2(1 + q)(1 + p)} - \left( \delta_m h_t^m + \delta_w h_t^w + \tau \right) \] (46)

We can see that due to POV women contribute less to family consumption as well as educational investment for the children.

**Human Capital Accumulation and Saving**  As mentioned above an increase the portion of human capital children receive from their parents and their inherent abilities will lower the need for educational investment from the parents prospective. This is because the children will be able to achieve the same level of human capital when they are older, despite the decrease in \( e_t^i, i = m, w \). We can see that, similar to the patriarchy and empowerment cases, the educational investment in the daughter is less than that of the son due to ACEW. The resulting human capital of the son and the daughter after the education investment has been taken into account is as following.

\[ h_{t+1}^m = \frac{\beta \gamma^m (1 + r)(z^w h_t^w + (1 + p)z^m h_t^m)}{2(1 + p)} \] (47)

\[ h_{t+1}^w = \frac{\beta \gamma^w (1 + r)(z^w h_t^w + (1 + p)z^m h_t^m)}{2(1 + p)(1 + q)} \] (48)

We observe that POV affects the human capital of both the son and daughter. This is because we assume that children receive a fraction of human capital from their mother and father and POV affects the human capital and productivity of the mother, thus affecting the children’s human capital. We also see that the human capital of the daughter is less than that of the son due to ACEW affecting the investment in education of the daughter. Also in societies where son bias is practiced the human capital of men will be higher than women, resulting in men earning greater income than women, increasing the gender gap.

The terms \( \delta_m h_t^m + \delta_w h_t^w + \tau, i = m, w \) expresses the reduction in educational investment for both the son and daughter. An increase in any of the terms above will result in parents investing less in education for the children. Therefore as ACEW increases parents will invest less on daughters’ education.

This will decrease total family expenditure as a consequence parents will save and work less. Both men and women have expected relationships with the real interest rate. As the real interest rate increase men and women will work and save more.

## 4  Growth Empirics

### 4.1  Endogenous Growth Rate

The balanced growth rate for the model economy satisfies:

\[ g = \beta(1 + r)(\gamma^m z^m(1 + p)(1 + q) + \gamma^w z^w)/2(1 + p)(1 + q) - 1 \] (49)
By taking logarithmic transformation of (49), by denoting male to female productivity ratio as $m = \frac{z^m}{z^w}$ and by plugging the equilibrium value of the male to female human capital ratio $R = \frac{(\gamma^m(1 + q))}{\gamma^w}$, where $\gamma^m$ and $\gamma^w$ denote are respectively denote the degree of parental care for the son and the daughter, it follows:

$$g = \ln\left[\beta(1 + r)\gamma^w z^w\right] + \ln(mR(1 + p)(1 + q)) - \ln(2(1 + p)(1 + q))$$  \hspace{1cm} (50)

The human capital ratio ($R$) is the same as in the previous cases. The growth rate of human capital is slightly different but similar relationships still hold. The growth rate decreases as POV and ACEW rises. This shows that men are also affected by POV and ACEW because it impacts the growth rate of human capital.

An increase in the productivity or the wage rate of men and women also improves the growth rate, as it creates an incentive for individuals to develop their human capital, in order to earn a higher wage. As expected, when parents place a larger weight on their children ($\gamma^m$ and $\gamma^w$) the growth rate of human capital increases. We extend this model to make the demand side play an important role as well by making the growth rate sensitive to the parameter $\beta$ which represents how we discount future consumption to determine our preference of consumption demand today over the next period. Therefore the formula for the growth rate changes to include $\beta$. Note as $\beta$ increases, i.e. we became more thrifty, the growth rate increases.

### 4.2 Regression results

There is a convenient development hypothesis that a country’s GDP increases, VAW declines, presumably because VAW is a problem for the poor and less educated population. With that hypothesis we expect a negative relationship between GDP and VAW. We also know from the standard neoclassical growth theory that the richer countries tend to have a lower rate of growth, because the scarcity of resources leads to diminishing returns to conventional investments in private goods, making it harder and harder to keep the rate of growth of GDP high. The implication of that theory for growth empirics is to expect a negative relationship between the level of GDP and the growth rate. If we can control for this negative effect of GDP then any additional effect of VAW on the growth rate would show up. By comparing errors of excluding a relevant variables or including irrelevant variables econometricians determine the significance of a variable’s explanatory factor. Based on the CAW data that we use to proxy of the POV variable of our model passes that test.
Growth Empirics

The cross country regression based on the equation (59) provide evidence for the existence of the above two channels. We run the following regression:

Dependent variable: the growth rate of real GDP per worker between 1985 and 2010.

Independent Variables: LNGDPW 1985, AYS15+ 1985, average investment to GDP ratio \((I/Y)\) for 1985-2010, average population growth rates \((n)\) between 1985-2010, and \(POV=\) the average values for the last 10 years of the percentage of all women in a country who experience some forms of violence as recorded by the UNDPâ€™s recently developed database on crime against women.

Violence against women and economic growth

\[ g = \ln \left[ \frac{\beta(1 + r)\gamma^w z^w}{2(1 + q)} \right] + \ln [mR(1 + p) + 1] - \ln [(1 + p)] \]

We use the growth rate of human capital equation above to find the relationship between violence against women and the growth rate of the economy. We assume the economy is on a balance growth path so that the growth rate of human capital is equal to, or an approximate of the growth rate of the economy. Note: we assume \(m = 1\) due to lack of available data.

Key variables

- \(g\) - the growth rate of GDP per capita
- \(p\) - the probability of violence against women in a society
- \(q\) - additional cost due to resistance against educating a girl child as a fraction of total costs for educating a child in a society
- \(m\) - male to female ratio of the average earning per hour
- \(R\) - male to female ratio of the average years of schooling
- \(\gamma^w\) - the average degree of parental care for a girl child
- \(z^w\) - the average female earning per hour
- \(\beta\) - a measure of thriftiness or how people discount future periods

The growth equation describes how the probability of violence against women may affect the long-run average growth rate of GDP per capita. Note in particular that the effect of \(p\) comes via two separate channels: one positive and one negative.

The positive effect shows that a higher likelihood of VAW encourages men and women to swap their working hours and that results in a higher GDP growth if there is a gender gap present.
The negative effect shows that the probability of violence lowers women’s working hours as well as negatively affecting their human capital. This lowers the growth rate of GDP per capita while raising the human capital gender gap \((R)\) in favour of men.

The data we have used in running the regressions below is as follows: The long run growth rate is found using RGDP per worker data from 1985 to 2010. The data for average years of schooling are also found by using the AYS and finding the average from 1985 to 2010. The similar technique is applied to average investment per income and average population growth. The POV data was sourced from the United Nations Development Programme which contains data showing the percentage of women that has been physically or sexually assaulted in a country. Due to incomplete data, we were only able to use 25 observations. The ratio of human capital between men and women is found by the ratio of average years of schooling between men and women. We were unable to collect data showing the ratio of the wage rate between men and women, the following regressions are run assuming \(m\) is equal to one. The regression above confirms the signs of our predicted growth rate model. The POV term \((\ln(mR(1 + p) + 1))\) has the predicted negative sign and the \(\ln[mR(1 + p) + 1]\) has the predicted positive sign. Initially, we included the control variables of average investment per income, average population growth, average years of schooling and real GDP in 1985 as a base. AYS controls for the spill over effect and GDP per worker controls for diminishing results. We find that the best model is predicted by Reg4. Although POV is not significant we can see that \(\ln[mR(1 + p) + 1]\) is. The regression above includes the same variables as

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Significance level of: 10% (*), 5% (**) and 1% (***)
<table>
<thead>
<tr>
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<th>Reg1</th>
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<th>Reg3</th>
<th>Reg4</th>
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<tr>
<td><strong>Intercept</strong></td>
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<td>-0.165*</td>
<td>-0.144*</td>
<td>-0.153***</td>
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<tr>
<td></td>
<td>(0.083)</td>
<td>(0.080)</td>
<td>(0.071)</td>
<td>(0.067)</td>
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<td><strong>Log of Average investment per income</strong></td>
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<td>0.018</td>
<td>0.012</td>
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<tr>
<td></td>
<td>(0.029)</td>
<td>(0.028)</td>
<td>(0.026)</td>
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<tr>
<td><strong>Log of Average population growth</strong></td>
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<td>-0.048</td>
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<td></td>
<td>(0.080)</td>
<td>(0.078)</td>
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<tr>
<td><strong>Log of Average years of schooling</strong></td>
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<td></td>
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<tr>
<td></td>
<td>(0.054)</td>
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<tr>
<td><strong>Log of RGDP(1985)</strong></td>
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<td>0.018</td>
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<td>(0.015)</td>
<td>(0.011)</td>
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<td><strong>POV</strong></td>
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<td>-0.026</td>
<td>-0.025</td>
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<tr>
<td></td>
<td>(0.031)</td>
<td>(0.027)</td>
<td>(0.026)</td>
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<tr>
<td><strong>ln(mR(1 + p) + 1)</strong></td>
<td>0.164*</td>
<td>0.155*</td>
<td>0.165*</td>
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<td><strong>Prob(F-statistic)</strong></td>
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<td>0.167</td>
<td>0.107</td>
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</table>

Significance level of: 10% (*), 5% (**) and 1% (***)
above except we have used the logs of the control variables. In this case we can see that RGDP is significant although AYS is not.

The regression above was run to ensure there is no endogeneity issue. We tried a large number of regressions and a wide variety of control variables used in the growth literature. We were very surprised by the robustness of the results and the consistency of the negative and positive signs of the above two key variables of the mode exactly as the theory predicts. We have only presented the simplest form of the regression.

**The effect of POV parameter \( p \) on the growth rate:**

We note from (59) two distinct channels through which the POV affects the growth rate. First, a negative channel which reflect adverse effect on female labour supply via (57) and an adverse effect on the investment on girl child via (52), which gets aggravated when the nuisance cost \( q \) of educating a girl child is high. Interestingly, however, there is a second channel through which POV can have a positive effect on growth in a perverse way. In particular, by (56), the POV encourages men to supply additional labour to offset the loss of savings for the family due to loss of income of his woman partner. The partial effect of that additional labour supply would be higher in a primitive society which uses a technology primarily with physical labour (or, equivalently discriminates women) to make the value of gender gap \( m \) in wages quite large. It would also be higher in a society where the gender gap in human capability \( R \) is quite large. In the model, however, \( R \) is endogenous; so strictly speaking, the partial positive effect of POV on the GDP growth rate would be higher if there is a stronger son bias \( \omega = \frac{\gamma m}{1-m} \) among the parents and the nuisance cost \( q \) of educating a girl child is larger.

5 Welfare Analysis

In this section we compare overall welfare of the society under different arrangement for decision making within a family. In particular, we compare and contrast the system under patriarchy in which men make all the decisions and its two alternatives discussed in the paper. In patriarchy, the social welfare reflects men’s value. Women indirectly influence men’s decisions by different degrees as per the degree men’s care for women within the underlying system of patriarchy. As per an alternative system, we consider a society which organises families that ensure decision making to follow women’s empowerment. We also consider a modern avenue, following the Basu (2006)’s idea of endogenous bargaining power, by allowing men and women to bargain cooperatively regarding the contribution of necessary marital capital for investing in marital goods such as family consumption and children’s education, before signing a marriage contract.

5.1 Patriarchy
The difference between the value functions of men and women under patriarchy with endogenous human capital is as follows.

\[
V_P^m - V_P^w = \frac{(1 - \sigma)(\ln(1/\sigma) + (1/\sigma)(1 + \gamma^m + \gamma^w + \sigma)}{\gamma^w z^w} - \frac{(1 - \sigma)(1 + p)(\gamma^m(1 + q)(\delta^m - \delta_w(1 + q) + z^m) + \gamma^w(\delta^m + \delta_w(1 + q))}{\gamma^w z^w}
\tag{51}
\]

The gap between the value function of men and women decreases with POV. This is because POV reduces the number of hours women work thus reducing the family income and the gap between the value functions.

We can also see a clear inverse relationship between the gap in the value functions and ACEW. This is due to men caring about their children’s human capital, which is negatively affected by ACEW.

The social welfare under patriarchy with endogenous human capital is satisfies,

\[
V_P = \ln \left[ \frac{h^w z^w}{\sigma(1 + p)} \right] + \sigma \ln \left[ \frac{h^w z^w}{(1 + p)} \right] + \gamma^m \ln \left[ \frac{\gamma h^w z^w}{\sigma(1 + p)} \right] + \gamma^w \ln \left[ \frac{\gamma^w h^w z^w}{\sigma(1 + p)(1 + q)} \right]
\tag{52}
\]

\[
- (1 + \gamma^m + \gamma^w + \sigma) + \sigma (1 + p) \frac{\gamma^m(1 + q)(\delta^m + \delta^w(1 + q) + z^m) + \gamma^w(\delta^m + \delta_w(1 + q))}{\gamma^w z^w}.
\tag{53}
\]

Note that under the patriarchy regime, the social welfare has a positive and a negative effect from POV. The positive effect occurs as a result of women working less due to a rise in POV, which decreases their disutility from working. The negative effect occurs because women work less as a result of a rise in POV, contributing less to family income which decreases family expenditure.

We can see that ACEW also has a positive and a negative effect on the family value function under the patriarchy regime. The negative effect is due to the human capital of the daughter decreasing as ACEW increases, which affects the utility and the value of men. The positive effect occurs through \( n^w \). As explained under equation (24) a rise in ACEW will decrease the number of hours women work which will increase the family value function under patriarchy, as women’s disutility from working decreases.

### 5.2 Empowerment

The social welfare under the women’s empowerment regime satisfies,

\[
V_E = (1 + \sigma) \ln \left[ \frac{h^w z^w}{(1 + p)} \right] + \gamma^m \ln \left[ \frac{\gamma h^w z^w}{(1 + p)(1 + \sigma)/2} \right] + \gamma^w \ln \left[ \frac{\gamma^w h^w z^w}{(1 + p)(1 + q)(1 + \sigma)/2} \right] - (1 + \gamma^m + \gamma^w + \sigma)
\tag{54}
\]

\[
+ \frac{(1 + \sigma)(1 + p) \gamma^m(1 + q)(\delta^m + \delta^w(1 + q) + z^m) + \gamma^w(\delta^m + \delta_w(1 + q))}{\gamma^w z^w}.
\tag{55}
\]
Similar to the patriarchy regime with endogenous human capital, $V_E$ rises and falls as POV and ACEW rises with the same explanations.

5.3 Patriarchy versus Empowerment: A Comparative Analysis

The social welfare difference between the regimes of empowerment and patriarchy is given by,

$$V^E - V^P = \ln[\sigma] + \ln\left[\frac{2\sigma}{1 + \sigma}\right] (\gamma^m + \gamma^w)$$

$$+ \frac{(1 - \sigma)(1 + p) \gamma^m (1 + q)(\delta^m_m + \delta^m_w (1 + q) + z^m) + \gamma^w (\delta^w_w + \delta^w_w (1 + q))}{\gamma w \omega}$$

We can clearly see that the social welfare difference between empowerment and patriarchy rises as POV and ACEW rises. This is primarily due to the weight allocated to women’s economic choices being greater under empowerment than in patriarchy. Therefore as POV and ACEW rise, $n^w_t$ decreases, as women attempt to reduce their disutility from work by working less. This causes the gap between the empowerment value function and the patriarchy to increase. A fall in $n^w_t$ also decreases family income and consumption, reducing men’s utility as well. Because consumption of men is higher in patriarchy than in empowerment $V^P$ will decreases more than $V^E$ causing the gap to increase further.

Lemma 2: If men in a society are sufficiently enlightened to care about women’s welfare enough (i.e., $\sigma > \sigma_{min}$) then there exists $p^* < 1$ such that if the probability of VAW (POV) in the society exceeds that threshold of tolerance $p^*$ then men will prefer empowerment of women to continuing patriarchy; i.e., for all $p > p^*$, $V^E > V^P$.

Proof. Appendix C

One would be tempted to conclude that as a society perceives a greater likelihood of VAW, it would be motivated to switch toward women’s empowerment, to enjoy a higher level of welfare.

5.3.1 A Patriarchy Trap

However, there are a few subtle and intriguing observations that come as a surprise. It turns out that if $\sigma$ lies below a critical minimum $\sigma_{min} > 0$ threshold, then for all $0 < p < 1$, $V^E < V^P$. In other words, if the existing regime of patriarchy is an extreme one such that men care too little for their women partners, reflecting a rather primitive social attitude toward women, then irrespective of the likelihood of VAW, the society would always prefer patriarchy over women’s empowerment.

Fortunately, if the existing patriarchy is not too primitive, i.e., $\sigma > \sigma_{min}$, then there exists $p^* < 1$ such that if $p > p^*$, $V^E > V^P$. In other words, a sufficiently enlightened
patriarchy would tolerate the threat of VAW in the society only up to a limit. If the POV exceeds the critical maximum threshold, then men would prefer to empower women in order to raise overall social welfare. So, it is quite interesting to note that POV acts as a catalyst to promote women’s empowerment; thus this model helps us to discover a silver lining around the cloud of VAW but with a formidable caveat. There is no easy way to motivate men living in a primitive society of extreme patriarchy (i.e., $\sigma < \sigma_{\text{min}}$) to change their attitude, which is, however, the only path for bringing a change in those societies.

The optimal policy for societies in a patriarchal trap, therefore, would be the education of men before education of women until men’s care for women exceeds a critical minimum threshold. Afterwards, the patriarchy would reach a transitory level which would make them significantly more sensitive to economic losses due to the threat of VAW. Once the threat in the society exceeds a critical maximum transition to empowerment would take place.

5.3.2 Women’s Empowerment with Economic Development

We also discover that as the economy grows and women’s human capital grows with it, despite the continuation of any types of gender gaps that arise in our model, the maximum threshold of VAW that a patriarch society tolerates would decline, irrespective of its underlying primitiveness. Consequently, even for the societies under extreme patriarchy (i.e., $\sigma < \sigma_{\text{min}}$), economic development can lead to women’s empowerment. There is, however, one caveat. Economic development and the growth in women’s human capital may not be possible if the prevailing threat of VAW in a society is too high to choke off any growth or economic development.

6 Concluding Remarks

In conclusion we have shown that the perceived likelihood of violence against women (VAW) has a nontrivial impact on a country’s long-run economic growth. In particular, we show that the variation of the growth rate of GDP across countries is caused by the gender based inequality that VAW creates inefficiently between men and women who are otherwise born with equal talents and equal factor endowments. In our model, we show that men as well as women are affected by VAW, and that the gap between men and women’s value functions are reduced by the institution of marriage. A possible further extension of this model is to assign probabilities to show whether an individual prefers to remain single or form a family. The growth rate we derived from the model shows that there is a positive and a negative effect from the probability of violence. The positive effect is multiplied by the human capital ratio and the wage ratio between men and women, this shows that as POV rises women work less causing men to work more, and assuming gender inequalities exists the growth rate of the economy will increase. The negative effect shows the loss of skilled women in the workforce due to the probability of violence, decreases the growth rate of the economy as less skilled men are used to replace relatively higher skilled women. This effect is confirmed by the empirical results presented in this paper. Therefore we conclude that further analysing the roots of VAW and attempting to decrease or end it completely within a society or country
will result in a high level of growth and wellbeing for that economy. It matters little whether the religious, social and political leaders take responsibility for VAW or pass it conveniently on the "bad" people. What matters is the understanding of the reason behind the genesis of these "bad" people and the increasing phenomenon of VAW so that we can begin to work toward mitigating this bad outcome from the economy.

Most importantly, this paper reports that there exists a low welfare trap for a primitive society, operating under extreme patriarchy and with primitive technology, as a human evolution problem. It is hard to motivate men living in a primitive society governed under a regime of extreme patriarchy to change their attitude, which is, however, the only path for bringing a change in those societies. The possibility of such welfare trap makes a few noteworthy implications of for developmental policies. Potentially, much could be gained from public investment in men’s education regarding the potential gain in social welfare from a wider role of women in society than their traditionally limited role of "motherhood" and withholding investment in women’s education in a primitive society where the culture of VAW thrives with a high degree of tolerance. Also, a significant improvement of the law and order to ensure women’s right to education and work as well as autonomy in decision making and to lower the threat of VAW below a critical minimum threshold is a prerequisite to gain from investing in women’s education.

7 Appendix

Patriarchy versus Empowerment: A Comparison of the Value Functions

Proof of Lemma: that characterizes condition for $V_E > V_P$

$$V_E = (1 + \sigma)\ln \left[ \frac{h_t z^w}{(1 + p)} \right] + \gamma^m \ln \left[ \frac{2\gamma^m h_t z^w}{(1 + p)(1 + \sigma)} \right] + \gamma^w \ln \left[ \frac{2\gamma^w h_t z^w}{(1 + \sigma)(1 + q)(1 + p)} \right] + \frac{(1 + \sigma)(1 + p)}{2} \gamma^m (1 + q)(\delta^m + \delta^w(1 + q) + z^m) + \gamma^w (\delta^m + \delta^w(1 + q)) - (1 + \gamma^m + \gamma^w + \sigma) \tag{57}$$

$$V_P = \ln \left[ \frac{h_t z^w}{\sigma(1 + p)} \right] + \sigma \ln \left[ \frac{h_t z^w}{(1 + p)} \right] + \gamma^w \ln \left[ \frac{\gamma^m h_t z^w}{(1 + p)\sigma} \right] + \gamma^w \ln \left[ \frac{\gamma^w h_t z^w}{(1 + q)(1 + p)\sigma} \right] - (1 + \gamma^m + \gamma^w + \sigma) + \sigma(1 + p) \gamma^m (1 + q)(\delta^m + \delta^w(1 + q) + z^m) + \gamma^w (\delta^m + \delta^w(1 + q)) \tag{58}$$

$$V_E - V_P = \ln[\sigma] + \ln \left[ \frac{2\sigma}{(1 + \sigma)} \right] (\gamma^m + \gamma^w) + \frac{(1 + p)(1 - \sigma)}{2} \gamma^m (1 + q)(\delta^m + \delta^w(1 + q) + z^m) + \gamma^w (\delta^m + \delta^w(1 + q)) \tag{59}$$
Find a condition where \( V_E - V_P > 0 \)

\[
V_E - V_P = \ln[\sigma] + \ln\left(\frac{2\sigma}{(1 + \sigma)}\right) (\gamma^m + \gamma^w) + \left(\frac{(1 + p)(1 - \sigma)}{2}\right) \frac{\gamma^m(1 + q)(\delta^m + \delta^w(1 + q) + z^m) + \gamma^w(\delta^m + \delta^w(1 + q))}{\gamma^w z^w} > 0 \quad (60)
\]

\[
p > p^*,
\]

where, \( p^* =
\]

\[
-1 - \frac{2}{(1 - \sigma)} \left(\ln[\sigma] + \ln\left(\frac{2\sigma}{(1 + \sigma)}\right) (\gamma^m + \gamma^w)\right) \left(\frac{\gamma^m(1 + q)(\delta^m + \delta^w(1 + q) + z^m) + \gamma^w(\delta^m + \delta^w(1 + q))}{\gamma^w z^w}\right)
\]

Next, observe how \( p^* \) changes with \( \sigma \)

\[
p^* = -1 + f(\sigma)/A(\omega)
\]

\[
p^* \geq 0 \iff f(\sigma) \geq A(\omega)
\]

where,

\[
f(\sigma) \equiv \frac{2}{1 - \sigma} \left[ \ln\left(\frac{1}{\sigma}\right) + \ln\left(\frac{1 + \sigma}{2\sigma}\right) (\gamma^m + \gamma^w)\right], \text{ and}
\]

\[
A(\omega) = \omega(1 + q)(D^m + z^m) + D^w, \\
D^m = \delta^m + \delta^w(1 + q), \\
D^w = \delta^w + \delta^w(1 + q).
\]

Properties of \( f(\sigma) \):

1. [i.]
2. \( f(\sigma) > 0, \ 0 < \sigma < 1 \),
3. \( f(0) = \infty \),
4. \( \lim_{\sigma \to 1} f(\sigma) = 2 + \gamma \),
5. \( f(\sigma) < 0, \ 0 < \sigma < 1 \),
6. \( \frac{\partial f}{\partial \gamma} > 0 \),
7. \( \frac{\partial f}{\partial \gamma_w} = \frac{2}{1 - \sigma} \ln\left(\frac{1 + \sigma}{2\sigma}\right) > 0 \).
Proposition 1 There exists $1 > \hat{\sigma}_H > 0$ such that $p^{**} \geq 0 \iff 0 < \sigma < \hat{\sigma}_H < 1$.

Lemma A. If $\gamma_w$ increases, $\hat{\sigma}_H$ increases.
Proof. By (i)-(iii), $f(\sigma)$ decreases to zero from $\infty$ as $\sigma$ increases from $0$ to $1$. By (v), if $\gamma_w$ increases $f(\sigma)$ increases but $A(\omega)$ decreases. ■ ■

Lemma B. As $\gamma_w$ decreases, then $\hat{\sigma}_H$ decreases.
Proof. Same as Lemma A. ■ ■

Proposition 2 There exists $\hat{\sigma}_L < \hat{\sigma}_H$ such that $\forall \sigma \in (0,1)$, if $p^{**} \leq 1 \iff \sigma \geq \hat{\sigma}_L$

Corollary 3 1. If $\sigma < \sigma_L$, then $p^{**} > 1$ and $V_E < V_P$,

2. If $\sigma > \hat{\sigma}_H$, then $p^* < 0 \Rightarrow V_E > V_P$
Let $K_1 \equiv \frac{\gamma^w z^w (1 + \gamma^m + \gamma^w)}{\gamma^m (1 + q) (\delta^m_w + \delta^m_m (1 + q) + z^m) + \gamma^w (\delta^m_w + \delta^w_w (1 + q))}$

and $K_2 \equiv \frac{\gamma^m (1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m) + \gamma^w (\delta^m_w + \delta^w_w (1 + q))}{\gamma^m (1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m) + \gamma^w (\delta^m_w + \delta^w_w (1 + q))} - 1$

then by (23),

$$p^* = \frac{K_1}{\sigma} + K_2 \quad \text{(61)}$$

Lemma 4: If $w > 1$, then $K_2 < 0$ and $m > 1$

Proof.

$$K_2 = \frac{\gamma^w z^w}{\gamma^m (1 + q) (\delta^m_w + \delta^m_m (1 + q) + z^m) + \gamma^w (\delta^m_w + \delta^w_w (1 + q))} - 1$$

and so $K_2 < 0$ iff

$$\frac{\gamma^w z^w}{\gamma^m (1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m) + \gamma^w (\delta^m_w + \delta^w_w (1 + q))} < 1$$

iff

$$\gamma^w z^w < \gamma^m (1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m) + \gamma^w (\delta^m_w + \delta^w_w (1 + q))$$

iff

$$z^w < \frac{\gamma^m}{\gamma^w} (1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m) + (\delta^w_m + \delta^w_w (1 + q))$$

iff

$$\frac{z^w - (\delta^w_m + \delta^w_w (1 + q))}{\delta^m_m + \delta^m_w (1 + q) + z^m} < R$$

iff

$$\frac{z^w - (\delta^w_m + \delta^w_w (1 + q))}{\delta^m_m + \delta^m_w (1 + q) + z^m} < \omega(1 + q)$$

iff

$$\frac{z^w - (\delta^w_m + \delta^w_w (1 + q))}{(1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m)} < \omega$$

iff

$$\frac{z^w}{(1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m)} - \frac{\delta^w_m}{(1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m)} - \frac{\delta^w_w}{\delta^m_m + \delta^m_w (1 + q) + z^m} < \omega$$

now if $m > 1$ then

$$\frac{z^w}{(1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m)} < 1$$

also

$$- \frac{\delta^w_m}{(1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m)} < 0$$

and

$$- \frac{\delta^w_w}{\delta^m_m + \delta^m_w (1 + q) + z^m} < 0$$

$\blacksquare$
Lemma 5  By Lemma 1 and (23),

\[ p^* \leq 0 \Leftrightarrow \sigma \geq \frac{\gamma^w z^w (1 + \gamma^m + \gamma^w)}{\gamma^m (1 + q) (\delta_m^m + \delta_w^m (1 + q) + \gamma^w) + \gamma^w (\delta_m^m + \delta_w^m (1 + q)) - \gamma^w z^w} \equiv \sigma_H \] (62)

Proof. \( p^{**} \leq 0 \) iff

\[ \frac{K_1}{\sigma} + K_2 \leq 0 \]

iff

\[ \frac{K_1}{\sigma} \leq -K_2 \]

iff

\[ \sigma \geq -\frac{K_1}{K_2} \quad (K_1 > 0, \sigma > 0, -K_2 > 0) \]

\[ -\frac{K_1}{K_2} = -\frac{\gamma^w z^w (1 + \gamma^m + \gamma^w)}{\gamma^w z^w - (\gamma^m (1 + q) (\delta_m^m + \delta_w^m (1 + q) + \gamma^w) + \gamma^w (\delta_m^m + \delta_w^m (1 + q)))} \]

hence

\[ \frac{z^w}{(1 + q) (\delta_m^m + \delta_w^m (1 + q) + \gamma^w)} - \frac{\delta_m^w}{(1 + q) (\delta_m^m + \delta_w^m (1 + q) + \gamma^w)} - \frac{\delta_w^w}{\delta_m^m + \delta_w^m (1 + q) + \gamma^w} \overset{\omega}{\lessdot} 1 \]

now if \( w > 1 \) then

\[ \frac{z^w}{(1 + q) (\delta_m^m + \delta_w^m (1 + q) + \gamma^w)} - \frac{\delta_m^w}{(1 + q) (\delta_m^m + \delta_w^m (1 + q) + \gamma^w)} - \frac{\delta_w^w}{\delta_m^m + \delta_w^m (1 + q) + \gamma^w} \overset{\omega}{\lessdot} 1 \]

and so if \( m > 1 \) and \( w > 1 \), then \( K_2 < 0 \). \[ \blacksquare \]

Lemma 6  By Lemma 1 and (23),

\[ p^* \geq 1 \Leftrightarrow \sigma \leq \frac{\gamma^w z^w (1 + \gamma^m + \gamma^w)}{\gamma^m (1 + q) (\delta_m^m + \delta_w^m (1 + q) + \gamma^w) + \gamma^w (\delta_m^m + \delta_w^m (1 + q)) - \gamma^w z^w} \equiv \sigma_L \] (63)

Proof.

\[ p^{**} \geq 1 \Leftrightarrow \frac{K_1}{\sigma} + K_2 \geq 1 \]

\[ \Leftrightarrow \frac{K_1}{\sigma} \geq 1 - K_2 \]

\[ \Leftrightarrow \sigma \leq \frac{K_1}{1 - K_2} \]

\[ \blacksquare \]

Lemma 7  If \( w > 1 \) and \( m < 1 \) then \( \sigma_L > 0 \)
Proof. $\sigma_L > 0$ iff
\[
\frac{\gamma^w z^w (1 + \gamma^m + \gamma^w)}{z(\gamma^m (1 + q)(\delta^m + \delta^w (1 + q) + z^m) + \gamma^w (\delta^w + \delta^w (1 + q)) - \gamma^w z^w)} > 0
\]
iff
\[
z(\gamma^m (1 + q)(\delta^m + \delta^w (1 + q) + z^m) + \gamma^w (\delta^w + \delta^w (1 + q))) > \gamma^w z^w
\]
since $m > 1$ and $w > 1$ then this follows and so $\sigma_L > 0$.

Lemma 8 $\sigma_L < \sigma_H$

Lemma 9 Let $D^m \equiv \delta^m + \delta^w (1 + q)$ and $D^w \equiv \delta^w + \delta^w (1 + q)$
\[
z^w (2 + \gamma^m + \gamma^w) \leq w(1 + q)[D^m + z^m] + D^w \Leftrightarrow \sigma_H \leq 1
\]
Proof. $\sigma_H \leq 1$ iff
\[
\frac{\gamma^w z^w (1 + \gamma^m + \gamma^w)}{\gamma^m (1 + q)(\delta^m + \delta^w (1 + q) + z^m) + \gamma^w (\delta^w + \delta^w (1 + q)) - \gamma^w z^w} \leq 1 \quad \text{(see Lemma 2)}
\]
iff
\[
\gamma^w z^w (1 + \gamma^m + \gamma^w) \leq \gamma^m (1 + q)[D^m + z^m] + \gamma^w D^w - \gamma^w z^w
\]
\[
\gamma^w z^w (2 + \gamma^m + \gamma^w) \leq \gamma^m (1 + q)[D^m + z^m] + \gamma^w D^w
\]
\[
z^w (2 + \gamma^m + \gamma^w) \leq \omega(1 + q)[D^m + z^m] + D^w
\]

Lemma 10 If $\sigma \leq \sigma_L$ then $n^w_t > 0 \ \forall p$

Lemma 11 If $\sigma \geq \sigma_H$ then $n^w_t = 0 \ \forall p$

Lemma 12 If $\sigma_L \leq \sigma \leq \sigma_H$ then $\exists \ p \in (0, 1) \ s.t \ n^w_t = 0$

7.1 Appendix on Women’s participation rate

Proposition 13 If
\[
C^m_t = \frac{h^w_t z^w}{\sigma (1 + p)}, \quad C^w_t = \frac{h^w_t z^w}{(1 + p)}
\]
\[
e^m_t = \frac{\gamma^m h^w_t z^w}{(1 + p)\sigma} - \delta^m h^m_t - \delta^w h^w_t - \tau,
\]
\[
e^w_t = \frac{\gamma^w h^w_t z^w}{(1 + q)(1 + p)\sigma} - \delta^w h^m_t - \delta^w h^w_t - \tau
\]
and $C^m_t + C^w_t + e^m_t + e^w_t (1 + q) = \gamma_t$ and $\gamma_t = z^m h^m_t + z^w h^w_t n^w_t$.

Then
\[
n^w_t = \frac{1 + \gamma^m + \gamma^w + \sigma}{(1 + p)\sigma} - (h^m_t (\delta^m + \delta^w (1 + q)) + h^w_t (\delta^w + \delta^w (1 + q))/h^w_t z^w - \frac{h^m_t}{h^w_t} \cdot \frac{z^m}{z^w}
\]

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to see this, firstly
\[ \gamma_t = z^m h_t^m + z^w h_t^w n_t^w \iff n_t^w = \frac{\gamma_t - z^m h_t^m}{z^w h_t^w} \]

and since \( \gamma_t = C_t^m + C_t^w + e_t^m + e_t^w (1 + q) \) then
\[ n_t^w = \frac{C_t^m + C_t^w + e_t^m + e_t^w (1 + q) - z^m h_t^m}{z^w h_t^w} \]

now by substitution
\[
n_t^w = \left( \frac{h_t^w z^w}{\sigma(1 + p)} + \frac{h_t^w z^w (1 + q)}{\sigma(1 + p)} - \frac{\gamma_m h_t^w z^w}{\sigma(1 + p)} \right) - \delta_m h_t^m - \delta_w h_t^w - \tau
\]
\[ + \left( \frac{\gamma_m h_t^w z^w}{\sigma(1 + p)(1 + q)} + \frac{\gamma_m h_t^w (1 + q)}{\sigma(1 + p)(1 + q)} - \delta_m h_t^m - \delta_w h_t^w - \tau \right) (1 + q) - z^m h_t^m) / z^w h_t^w
\]
\[ \iff \left( \frac{h_t^w z^w + \sigma h_t^w z^w + \gamma_m h_t^w z^w + \gamma_m z^w h_t^w}{(1 + p) \sigma z^w h_t^w} \right) - \delta_m h_t^m - \delta_w h_t^w (1 + q)
\]
\[ - \delta_m h_t^m - \delta_w h_t^w (1 + q) - (2 + q) \tau - z^w h_t^m) / z^w h_t^w
\]
\[ \iff \frac{1 + \sigma + \gamma_m + \gamma_w}{(1 + p) \sigma} - \left( h_t^m (\delta_m + \delta_w (1 + q)) + h_t^w (\delta_m + \delta_w (1 + q)) + (2 + q) \tau / z^w h_t^w - \frac{h_t^m z^m}{h_t^w z^w} \right)
\]

**Proposition 14** Whenever \( n_t^w \leq 0 \) then
\[ p \geq \frac{\gamma^w z^w (1 + \sigma + \gamma_m + \gamma_w)}{\gamma^m (1 + q)(\delta_m + \delta_w (1 + q) + z^m) + \gamma_w (\delta_m + \delta_w (1 + q) + (2 + q) \tau) \sigma} \]
provided \( n_t^w \) is as found previously.

Now if \( n_t^w \leq 0 \) then
\[
\frac{1 + \sigma + \gamma_m + \gamma_w}{(1 + p) \sigma} - \left( h_t^m (\delta_m + \delta_w (1 + q)) + h_t^w (\delta_m + \delta_w (1 + q)) + (2 + q) \tau / z^w h_t^w - \frac{h_t^m z^m}{h_t^w z^w} \right) \leq 0
\]
\[ \iff \frac{1 + \sigma + \gamma_m + \gamma_w}{(1 + p) \sigma} \leq \left( h_t^m (\delta_m + \delta_w (1 + q)) + h_t^w (\delta_m + \delta_w (1 + q)) + (2 + q) \tau / z^w h_t^w - \frac{h_t^m z^m}{h_t^w z^w} \right)
\]

Now from previously, \( h_t^m \leq \frac{\gamma^m (1 + q)}{\gamma^w} \) and so
\[
\frac{1 + \sigma + \gamma_m + \gamma_w}{(1 + p) \sigma} \leq \frac{\gamma^m (1 + q)(\delta_m + \delta_w (1 + q))}{\gamma^w z^w} + (\delta_m + \delta_w (1 + q))
\]
\[ + (2 + q) \tau / z^w + \frac{\gamma^m (1 + q) z^m}{\gamma^w z^w}
\]
\[ \iff \frac{1 + \sigma + \gamma_m + \gamma_w}{(1 + p) \sigma} \leq (1 + q)(\delta_m + \delta_w (1 + q))
\]
\[ + (2 + q) \tau = \gamma^w (\delta_m + \delta_w (1 + q) + (2 + q) \tau + z^m) / \gamma^w z^w
\]
\[ \iff \frac{1 + \sigma + \gamma_m + \gamma_w}{(1 + p) \sigma} \leq (\gamma^m (1 + q)(\delta_m + \delta_w (1 + q) + z^m)
\]
\[ + (2 + q) \tau / z^w)
\]

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Assumption, both sides positive and non-zero

\[ \begin{align*}
\Leftrightarrow & \quad \frac{(1 + p)\sigma}{(1 + \sigma + \gamma^m + \gamma^w)} \geq \frac{\gamma^w z^w}{(1 + \sigma + \gamma^m + \gamma^w)} \\
\Leftrightarrow & \quad p \geq \frac{\gamma^w z^w(1 + \sigma + \gamma^m + \gamma^w)}{(\gamma^m(1 + q)(\delta^m + \delta^w(1 + q) + z^m) + \gamma^w(\delta^m + \delta^w(1 + q) + (2 + q)\tau))} - 1
\end{align*} \]

Lemma 15 At the optimal levels of \( C_t^m, C_t^w, e_t^m \) and \( e_t^w \) along with the constraints, \( \gamma_t = z^m h_t^m + z^w h_t^w n_t^w \) and \( C_t^m + C_t^w + e_t^m + e_t^w (1 + q) = \gamma_t \). Then

\[ n_t^w = \frac{(1 + \sigma + \gamma^m + \gamma^w)}{(1 + p)\sigma} - \frac{h_t^m(\delta^m + \delta^w(1 + q))}{(1 + q)(1 + p)\sigma} - \frac{h_t^w(\delta^m + \delta^w(1 + q)) + \tau(2 + q)/h_t^w z^w - R_m}{} \]

where \( R = \frac{h_t^m}{h_t^w} \) and \( m = \frac{z^m}{z^w} \).

Proof. The optimal levels of \( C_t^m, C_t^w, e_t^m \) and \( e_t^w \) are,

\[ C_t^m = \frac{h_t^w z^w}{\sigma(1 + p)} \quad C_t^w = \frac{h_t^w z^w}{(1 + p)} \]

\[ e_t^m = \frac{\gamma^m h_t^w z^w}{(1 + p)\sigma} - \frac{\delta^m h_t^m - \delta^w h_t^w - \tau}{(1 + q)(1 + p)} \]

\[ e_t^w = \frac{\gamma^w h_t^w z^w}{(1 + q)(1 + p)\sigma} - \frac{\delta^m h_t^m - \delta^w h_t^w - \tau}{(1 + q)(1 + p)} \]

Now, since \( \gamma_t = z^m h_t^m + z^w h_t^w n_t^w \) and \( \gamma_t = C_t^m + C_t^w + e_t^m + e_t^w (1 + q) \) then

\[ n_t^w = \frac{C_t^m + C_t^w + e_t^m + e_t^w (1 + q) - z^m h_t^m}{z^w h_t^w} \]

provided \( z^w h_t^w \neq 0 \).

Now substituting in the optimal levels of \( C_t^m, C_t^w, e_t^m, e_t^w \) we obtain

\[ n_t^w = \left( \frac{h_t^w z^w}{\sigma(1 + p)} + \frac{h_t^w z^w}{(1 + p)} + \frac{\gamma^m h_t^w z^w}{\sigma(1 + p)} - \delta^m h_t^m - \delta^w h_t^w - \tau \right) \]

\[ + \left( \frac{\gamma^w h_t^w z^w}{(1 + q)(1 + p)} - \delta^m h_t^m - \delta^w h_t^w - \tau \right) \]

\[ \Leftrightarrow n_t^w = \left( \frac{h_t^w z^w + \sigma h_t^w z^w + \gamma^m h_t^w z^w + \gamma^w h_t^w z^w}{(1 + p)\sigma} - \delta^m h_t^m - \delta^m h_t^m(1 + q) \right) \]

\[ - \delta^m h_t^m(1 + q) - (2 + q)\tau - z^m h_t^m / z^w h_t^w \]

\[ \Leftrightarrow n_t^w = \left( \frac{(1 + \sigma + \gamma^m + \gamma^w)}{(1 + p)\sigma} - \frac{h_t^m(\delta^m + \delta^w(1 + q)) + h_t^w(\delta^m + \delta^w(1 + q))}{(1 + p)\sigma} \right) \]

\[ + (2 + q)\tau / z^w h_t^w - \frac{h_t^m}{h_t^w} \cdot \frac{z^m}{z^w} \]

\[ \blacksquare \]
Lemma 16 By (22) \( n^w_t \) is a function of \( D \). Let \( m \) denote that is \( n^w_t = \delta(p) \), \( n = n(p) \) and at the optimal levels of \( C^m_i, C^w_i, e^m_i, e^w_i \) and with the constraints

\[
\gamma_t = z^m h^m_t + z^w h^w_t n^w_t, \quad C^m_i + C^w_i + e^m_i + e^w_i (1 + q) = \gamma_t
\]

Then the value of \( p, p^* \), which gives \( n^w_t = 0 \) is

\[
p^* = \frac{\gamma^m z^w (1 + \sigma + \gamma^m + \gamma^w)}{(\gamma^m (1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m)) + \gamma^w (\delta^w_m + \delta^w_w (1 + q) + (2 + q) \tau)) \sigma} - 1
\]

Proof. Letting \( n^w_t = 0 \), and from Lemma 1.1, we must have

\[
0 = \frac{(1 + \sigma + \gamma^m + \gamma^w)}{(1 + p) \sigma} - (h^m_t (\delta^m_m + \delta^m_w (1 + q)) + h^w_t (\delta^w_m + \delta^w_w (1 + q)) + \tau(2 + q) / h^w_t \gamma^w) + \frac{h^m_t}{h_t^m} \cdot \frac{z^m}{\gamma^m z^w}
\]

\[
\Leftrightarrow \frac{(1 + \sigma + \gamma^m + \gamma^w)}{(1 + p) \sigma} = \frac{(h^m_t (\delta^m_m + \delta^m_w (1 + q)) + h^w_t (\delta^w_m + \delta^w_w (1 + q)) + \tau(2 + q)}{h^w_t \cdot \frac{z^m}{\gamma^w}}
\]

\[
\Leftrightarrow \frac{(1 + \sigma + \gamma^m + \gamma^w)}{(1 + p) \sigma} = \frac{h^m_t}{h_t^w} \cdot \frac{(\delta^m_m + \delta^m_w (1 + q)) + (\delta^w_w (1 + q) + \tau(2 + q))}{z^w} + \frac{\gamma^m (1 + q) \gamma^w}{\gamma^w z^w}
\]

Now by (16) pg 19

\[
\frac{h^m_t}{h_t^w} = \frac{\gamma^m (1 + q)}{\gamma^w}
\]

and by substitution

\[
\frac{(1 + \sigma + \gamma^m + \gamma^w)}{(1 + p) \sigma} = \frac{\gamma^m (1 + q) (\delta^m_m + \delta^m_w (1 + q)) + \delta^w_m + \delta^w_w (1 + q) + \tau(2 + q) + \gamma^m (1 + q) \gamma^w}{\gamma^w z^w}
\]

\[
\Leftrightarrow \frac{(1 + \sigma + \gamma^m + \gamma^w)}{(1 + p) \sigma} = \frac{\gamma^m (1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m) + \gamma^w (\delta^w_m + \delta^w_w (1 + q) + (2 + q) \tau)}{\gamma^w z^w}
\]

\[
\Leftrightarrow p = \frac{\gamma^w z^w (1 + \sigma + \gamma^m + \gamma^w)}{(\gamma^m (1 + q) (\delta^m_m + \delta^m_w (1 + q) + z^m) + \gamma^w (\delta^w_m + \delta^w_w (1 + q) + (2 + q) \tau)) \sigma} - 1
\]
7.2 Nash Bargaining Problem

Nash Bargaining Problem

\[
\max_{s^m_t, s^w_t} = \ln[\beta(V(s_t) - \ln((1 + r)s^m_t))] + \sigma \ln \beta(V(s_t) - \ln((1 + r)s^w_t))
\]  

FOC's

\[
\frac{V'(s_t) - \frac{1}{s^m_t}}{V(s_t) - \ln((1 + r)s^m_t)} + \frac{\sigma V'(s_t)}{V(s_t) - \ln((1 + r)s^w_t)} = 0
\]  

\[
\frac{V'(s_t) - \frac{1}{s^w_t}}{V(s_t) - \ln((1 + r)s^m_t)} + \frac{\sigma V'(s_t)}{V(s_t) - \ln((1 + r)s^w_t)} = 0
\]

Divide above

\[
\frac{s^m_t}{s^w_t} = \frac{V(s_t) - \ln((1 + r)s^w_t)}{V(s_t) - \ln((1 + r)s^m_t)} \tag{67}
\]

\[
\frac{\sigma s^m_t}{\alpha} = \frac{V(s_t) - \ln((1 + r)s^w_t)}{V(s_t) - \ln((1 + r)s^m_t)} \tag{68}
\]

\[
\frac{\sigma s^w_t}{1 - \alpha} = \frac{V(s_t) - \ln((1 + r)s^m_t)}{V(s_t) - \ln((1 + r)s^w_t)} \tag{69}
\]

Define \(g(s_t)\)

\[
g(s_t) \equiv V(s_t) - \ln((1 + r)s_t) \tag{70}
\]

Then

\[
\frac{\sigma s^m_t}{\alpha} = \frac{g(s_t) - \ln(1 - \alpha))}{g(s_t) - \ln(\alpha)} \tag{71}
\]

So

\[
g(s_t) = h(\alpha, \sigma) \equiv \frac{\sigma \alpha \ln(\alpha) - (1 - \alpha) \ln(1 - \alpha)}{\alpha(1 + \sigma) - 1} \tag{72}
\]

Substitute in for \(s_t\)

\[
V(s_t) - \ln((1 + r)s_t) = h(\alpha, \sigma) \tag{73}
\]

7.3 Separation of variables

If we substitute in for the optimal \(s_t\) we find

\[
g(h^m_t, h^w_t, \delta^m, \delta^w, \gamma^m, \gamma^w, \tau, q) \tag{74}
\]

Note that the two terms here are the welfare gains respectively from having each child. We have separated the variables in the problem into those related to optimal saving on the LHS and those related to the division of saving on the RHS:

\[
g(h^m_t, h^w_t, \delta^m, \delta^w, \gamma^m, \gamma^w, \tau, q) = h(\alpha, \sigma) = \frac{\sigma \alpha \ln(\alpha) - (1 - \alpha) \ln(1 - \alpha)}{\alpha(1 + \sigma) - 1} \tag{75}
\]
7.4 Comparative statics of \( g() \)

\( g() \) is the gains from forming a relationship so we know \( g(s_i) \geq 0 \) otherwise no relationship would be formed.

Observe that \( g() \) is increasing in all human capital variables \( h^i \) and spillovers \( \delta^i_j \) and \( \tau \). We can also find conditions under which \( g() \) is increasing in \( q \):

\[
\frac{dg}{dq} = (\gamma^m + \gamma^w) \frac{h^m_t \delta^w_m + h^w_t \delta^m_w + \tau}{h^m_t (\delta^w_m (1 + q) + \delta^m_m) + h^w_t (\delta^w_w (1 + q) + \delta^m_w) + \tau (2 + q)} - \frac{\gamma^w}{1 + q} > 0 \tag{76}
\]

This holds iff

\[
\frac{\gamma^m}{\gamma^w} > \frac{h^m_t \delta^m_m + h^w_t \delta^m_w + \tau}{(h^m_t \delta^w_m + h^w_t \delta^w_w + \tau) (1 + q)} \tag{77}
\]

This holds if you assume that gender spillovers are symmetric \( (\delta^w_m = \delta^m_m, \delta^w_w = \delta^m_w) \) and boys are preferred to girls \( (\gamma^m > \gamma^w) \) since \( q > 0 \).

7.5 Existence of equilibrium

\( g() \) is the gains from forming a relationship so we know \( g(s_i) \geq 0 \) otherwise no relationship would be formed.

For a given \( \sigma \in [0, 1] \) we want a solution \( \alpha \in (0, 1) \) such that \( g() = h(\alpha, \sigma) \).

Consider a solution where both the numerator and the denominator of \( h(\alpha, \sigma) \) are positive. For the denominator this would require \( \alpha < \frac{1}{1 + \sigma} \) so \( \alpha \in (\alpha, 1] \subset [0.5, 1] \).

For this range of values the numerator is positive because:

- \( \alpha \in (0.5, 1] \) then \( \alpha > 1 - \alpha > 0 \) and \( 0 > \ln(\alpha) > \ln(1 - \alpha) \)
- \( \alpha \ln(\alpha) - (1 - \alpha) \ln(1 - \alpha) > 0 \)
- \( \sigma \alpha \ln(\alpha) - (1 - \alpha) \ln(1 - \alpha) > 0 \)

In this range \( h(\alpha, \sigma) \) is continuous, and monotone decreasing since \( \frac{dh(\alpha, \sigma)}{d\alpha} = \frac{\alpha (1 - \alpha) (\ln(\alpha) - \ln(1 - \alpha))}{(\alpha + 1 - \alpha - 1)^2} < 0 \)

So \( h(\alpha, \sigma) \to \infty \) as \( \alpha \downarrow \alpha \)

and \( h(\alpha, \sigma) \to 0 \) as \( \alpha \uparrow 1 \)

so for any \( g() \geq 0 \) there exists some \( \alpha \) such that

\[
g(h^m_t, h^w_t, \delta^m_m, \delta^w_m, \delta^m_w, \delta^w_w, \gamma^m, \gamma^w, \tau, q) = h(\alpha, \sigma) \]

This equilibrium is unique because it is not possible for there to exist an equilibrium \( g() = h(\alpha, \sigma) \geq 0 \) where both the numerator and denominator of \( h(\alpha, \sigma) \) are negative if \( \sigma < 1 \).

for the denominator to be negative we require \( \alpha < \frac{1}{1 + \sigma} \).

for the numerator to be negative we require \( \sigma < \frac{(1 - \alpha) \ln(1 - \alpha)}{\alpha \ln(\alpha)} \).

But this implies \( \sigma < \frac{(1 - \alpha) \ln(1 - \alpha)}{\alpha \ln(\alpha)} < \frac{\sigma \ln(\alpha) - \ln(1 + \sigma)}{\ln(1 + \sigma)} \) Which only holds if \( 0 < \ln(\sigma) \).
7.6 Compartive statics of h()

(This result really doesn’t make rsense, I need to think about this some more)
Differentiating implicitly

\[
\frac{d\alpha^*}{d\sigma} = \frac{\alpha(g(\cdot) - \ln(\alpha))}{\sigma \ln(\alpha) + \ln(1 - \alpha) + (1 + \sigma)(1 - g(\cdot))} < 0
\] (78)

Large gains from marriage (If \(g(\cdot) > 1\)) are sufficient to guarantee this fraction is negative and so the share the man pays (\(\alpha\)) will decrease as the woman’s bargaining power (\(\sigma\)) increases.

Also we know that from our comparative statics of \(g(\cdot)\) that if \(h^i, \delta^j, \tau\) or \(q\) increase, \(g(\cdot) = h(\alpha, \sigma)\) will increase and so \(\alpha^*\) will decrease

8 References


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