Evaluating the use of stereo-photogrammetry for gravel-bed roughness analysis

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Abstract- ***In this work, three different measurement technologies are employed to study the hydraulic roughness in a laboratory flume with a gravel-bed. The gravel-bed topography is measured with (i) an acoustic bed-profiler; (ii) a hand-held laser-scanner; and (iii) stereo-photogrammetry. Digital Elevation Models (DEMs) are obtained with each technique. Statistical roughness analyses of the DEMs are undertaken by estimating the vertical roughness length, represented by the standard deviation of bed elevations. In addition, Probability Distribution Functions (PDFs) and two-dimensional second-order structure functions of bed elevations are used for the analyses. All three measurement techniques were found precise enough to reach the grain-scale of laboratory gravel-beds. However, important disparities were found between the DEMs, which were not always visible statistically. The acoustic bed-profiler and the laser-scanner were found suitable for gravel-bed roughness characterisation. With stereo-photogrammetry, the complex operational processes must be improved to obtain realistic high-resolution DEMs and to derive faithful statistics.***

Keywords- **topography measurement; laser-scanner; stereo-photogrammetry; underwater; gravel-bed; roughness**

I. CONTEXT

In fluvial hydraulics, the bed roughness is a crucial parameter which intervenes in the determination of flow resistance and mean velocity in both natural and laboratory channels [1]. Traditionally, roughness coefficients were approximated by using long and tedious techniques, like size sieving and counting of the bed surface sediment particles. Advanced technologies now allow to obtain high-resolution point clouds of the in-situ river-bed topography and to represent the bed surface as 3D Digital Elevation Models (DEMs). Using these topography data sets, statistical analysis of the random field of surface elevation $z(x,y)$, where z is the surface height at coordinates x and y, has become fundamental to understand the morphology of water-worked beds and to quantify roughness parameters [2,3,4,5,6].

An infrared bed-profiler can be used to measure gravel-bed surfaces and sand bedforms in laboratory and field-based experiments, with 5 mm measurement resolution and a typical mean error in vertical elevation of 0.38 mm, [7]. Reference [8] studied submerged bedforms in a laboratory flume by using an acoustic bed-profiler with a resolution of 2.45 mm in traversing mode, various offset possibilities and a vertical measurement resolution of 0.37 mm.

Laser-scanning is another promising technology in measuring river-bed topography. In laboratory experiments, references [5,9,10] undertook bed-elevation measurements with a vertical resolution ranging from 0.5 µm to 0.1 mm by using Terrestrial Laser-Scanners (TLSs). This technology is actually considered as one of the most accurate large-scale measurement techniques. However, some TLSs may not be used outdoors and some others only allow measuring non-submerged beds because of a red beam.

Digital consumer cameras and photogrammetric software improved considerably over the last decade, allowing stereo-photogrammetry applications to be implemented in various fields. References [3,11] used stereo-photogrammetry to measure the bed morphology in a laboratory flume and study the hydraulic roughness. In [11], the precision of the system was estimated with control point coordinates, and a root mean square error in vertical elevation of 2 mm was found.

In this study, the topography of a water-worked gravel-bed is acquired in a laboratory flume with (i) an acoustic bed-profiler with a sound wave frequency of 2 MHz; (ii) an Inition MVT CLS60 hand-held laser-scanner; and (iii) a stereo-photogrammetric system composed of two Nikon D90 digital consumer cameras. Measurement techniques and experimental methods are presented in detail. Statistical analyses of the bed elevation focus on the determination of the bed roughness and the use of Probability Distribution Functions (PDFs) and 2D second-order structure functions. The results show the detrended DEMs and the statistics obtained with the three techniques. In the discussion, the suitability of the DEMs to make a statistical approach of the bed roughness in laboratory experiments is assessed. Finally, pros and cons of each technique are presented.

II. DATA ACQUISITION AND METHODS

A. Experimental setup

The following comparison of the topography measurement technologies uses experimental data sets obtained in the Fluid Mechanics Laboratory of The University of Auckland in 2010. Data collection was carried out using a 19 m long, 0.45 m wide and 0.5 m deep flume with a 1 m long vertically adjustable recess (Fig. 1). A relatively fine sediment mixture comprised of rounded and coloured gravel, with D_{50} = 4.5 mm

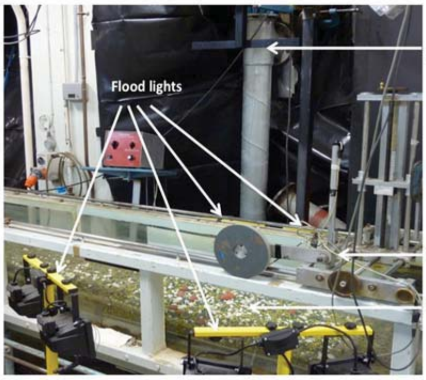

Figure 1. The flume setup

and a maximum gravel size of D_{98} = 50 mm, was placed randomly into the base of the adjustable recess and the surface was flattened to a thickness of 10 cm.

The laser-scanner could not be used directly at the flume and required the use of a metallic tray to move a sediment sample from the flume to the location where the laser is housed. The metallic tray was inserted into the flume‘s adjustable recess before the recess was filled with sediment, completely covered by the sediment layer. The 0.35 m long and 0.3 m wide metallic tray represented the measurement window used to compare the different techniques.

A constant flow rate of Q = 66 l/s monitored by a pressure gauge, and a constant water depth of 20 cm were applied to the test section over four hours. The upper sediment layer eroded downstream and armoured, with visible roughness structures. At the end of the experiment, the bed topography was measured with the acoustic bed-profiler and stereo-photogrammetry. Finally, the metallic tray was uncovered and carefully moved to the location of the laser-scanner, to undertake the data collection.

B. Acoustic bed-profiler

The gravel-bed topography was first measured with the acoustic bed-profiler while the measurement area was submerged. This device, mounted on a rack above the flume, comprises a sounding probe maintained 10 cm above the sediment bed to measure bed elevations, with digital potentiometers measuring the probe location in plan coordinates. The probe generates ultrasonic waves that reflect off the sediment bed and are received again by the probe. The time of passage of sound waves is measured electronically and allows determining the distance of the sediment bed from the probe. 2D longitudinal bed-elevation profiles were acquired while the probe was moved along lines in the flow direction, with automatic measurements every 2.45 mm downstream, and 2.54 mm spaces between the 2D profiles in the transverse direction. The vertical measurement resolution, Δz, was calculated from the frequency, f, of the sound wave using $\Delta z = \pm c/2f$. In this survey, a sound wave of frequency f = 2 MHz resulted in a depth resolution in water at 20°C of ± 0.37 mm. The 2D data sets were then opened with MATLAB® where a 3D DEM was obtained with a resolution of 2.45 mm downstream and 2.54 mm transverse, resulting in ~16,500 data points per measurement window. Data post-processing consisted in removing measurement spikes and interpolating gaps, using an automated procedure based on the mean of the surrounding points.

C. Laser-scanner

An Inition MVT CLS60 hand-held laser-scanner with a red beam, available in The University of Auckland Automated Systems Laboratory, was used for the project. The sample area height was quantified by 20,000 points per second with an accuracy of 0.050 µm (manufacturer specified) on a non-uniform spatial grid. Data were saved as an ASCII file and were directly read into MATLAB®. Data post-processing consisted of despiking the measured surface and transferring the non-uniform data cloud to a uniform grid with 1.45 mm resolution (resulting in ~50,000 data points per measurement window). Measurements took place in non-submerged conditions. However, preliminary tests showed that wet conditions, e.g. with wet particle surfaces, have a detrimental effect on the laser's accuracy (see Table 1).

D. Stereo-photogrammetry

Measurements of the exposed as well as submerged gravel-bed were carried out using a stereo-photogrammetric setup composed of two Nikon D90 digital consumer cameras, with an 18 mm lens and a 5.5 µm pixel pitch (12.3 megapixels). The two cameras were attached on a frame 1 m above the flume with a 28 cm baseline between the cameras using a mounting bar (Fig. 1). In accordance with [12], the setup was expected to provide a theoretical depth resolution of ~1 mm (see Fig. 2).

Photogrammetric applications required special lighting conditions. A homogeneous light, with reduced reflection from the coloured stones, was achieved by using four flood lights, with two flood lights on either side of the flume and a light diffuser, which consisted of sheets of paper applied on the two transparent sides of the flume. The settings for both cameras were manually adjusted to be identical (shutter speed of 1.3 s, F/22 aperture, ISO 200) and manually focused on the gravel-bed. The images taken with the two cameras were overlapping at ~80% and 100% in the horizontal and vertical directions respectively, so that the Common Field of View (CFoV) of the two cameras was larger than the test section, and defined the area over which depth information was extracted. Before the gravel-bed was recorded, the cameras were calibrated with a 40 cm long and 20 cm wide checkerboard, consisting of a regular grid with 20 mm spacing. Images of the calibration checkerboard were taken in various positions; all degrees of freedom had to be used and the CFoV correctly covered by the checkerboard to ensure good calibration results.

The numerical treatment of the images involved several steps. Calibration of the cameras using the checkerboard's information and stereo-rectification were performed using MATLAB® and [13]'s camera calibration toolbox. From the calibration data and the rectified images of the gravel-bed, the Symmetric Dynamic Programming Stereo-photogrammetry (SDPS) algorithm allowed getting a depth map within the CFoV, as in [14]. With a distance of the cameras to the gravel-bed of 1 m, the mean ratio between the size of an image on a

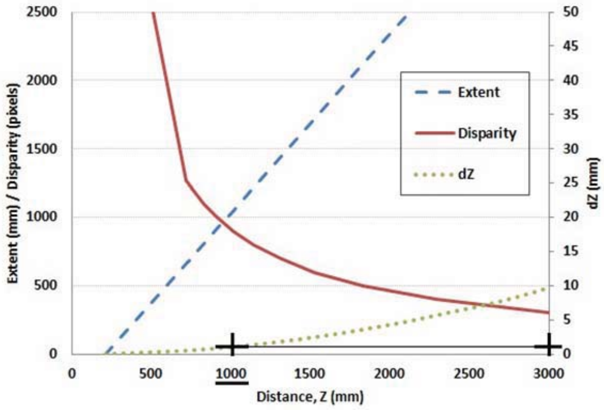

Figure 2. Characterization of the stereo-photogrammetric setup. The line in black shows the depth resolution of the setup used in this study, i.e., with a distance, z, from the cameras to the gravel-bed of 1 m. After [12].

camera's sensor and the size of the photographed area with one camera was around 1/45. This ratio also defined the resolution of the depth map, as being around 1/45 the resolution of the image on the sensor (5.5 µm pixel pitch), i.e., ~0.25 mm. The next stage was the extraction of the point cloud at the same resolution as the depth map, which equates ~1,680,000 data points on the measurement window. Finally, data post-processing was undertaken to despike the data and DEMs were interpolated with ~105,000 measured points "only", with a uniform grid resolution of 1 mm.

E. Statistical roughness analysis

Before analysing the obtained DEMs statistically, a detrending algorithm was applied on each DEM to remove larger scale trends than the grain-scale, such as bed slopes, which could obscure DEMs and bias statistics. In accordance with [15], a second-order biquadratic moving filter was applied to the original DEMs, reducing the variance of the original data series the most. Finally, all DEMs were normalised to have a mean bed-elevation equal to zero, and rotated to be aligned in the streamwise flow direction.

Initially, Probability Distribution Functions (PDFs) were obtained, representing the distribution of surface elevations. Parameters such as skewness, kurtosis and standard deviation of surface elevations were extracted from the detrended surfaces (Table 1). Similar to previous work by [6,9,10], the vertical roughness length was estimated using the standard deviation of the bed elevation σ_z. Further investigations on the morphology of gravel-bed surfaces were obtained by using generalised 2D second-order structure functions (also called semivariograms), defined by [6] as:

$$D_{G2}(\Delta x, \Delta y) = \frac{1}{(N-n)(M-m)} \sum_{i=0}^{N-n} \sum_{j=0}^{M-m} \{|z(x_i + n\delta x, y_j + m\delta y) - z(x_i, y_j)|\}^2 \quad (1)$$

where, $\Delta x = n\delta x$ and $\Delta y = m\delta y$; δx and δy are sampling intervals in the longitudinal and transverse directions respectively; $n=1,2,3,\ldots N$ and $m=1,2,3,\ldots M$, while N and M are the number of samples in the longitudinal and transverse directions, respectively.

The relationship in (1) is used in this paper to represent contour lines of the generalised 2D structure functions. To enable a direct comparison between the various measurement techniques, the structure functions were normalised with the 'saturation level' $2\sigma_z{}^2$ and the contours of D_{G2} were plotted as proportions of the saturation level.

III. RESULTS

A. Digital Elevation Models (DEMs)

A visual examination of the DEMs in Fig. 3 shows that not all of the three measurement techniques were able to represent accurately the gravel-bed topography. With the acoustic bed-profiler, the gravel-bed surface appears rather smooth with little noise (Fig. 3a). The particles of larger size than the instrument resolution are correctly measured, and any visible errors are concentrated on the edges of larger particles and in the holes between grains, where major elevation changes occur rapidly. The laser-scanner's DEMs (Fig. 3b) are similar to those obtained with the acoustic bed-profiler. With a higher resolution, the laser-scanner enables a better representation of the grains' edges as well as the holes between particles. However, a detailed inspection shows erroneous points that remained after data post-processing, creating unrealistic narrow spikes above the gravel-bed surface. These errors are relatively small and do not alter significantly the quality of the DEMs. In this laboratory study, stereo-photogrammetry is the technique presenting the DEMs of least quality (Fig. 3c) despite the highest measurement grid resolution. The general aspect of the gravel-bed is not conserved since not all of the major gravel particles are well captured. The DEM also show major noise, even after despiking, a fact which considerably decreases the quality of the measurements.

B. Probability Distribution Functions (PDFs)

PDFs were generated to lead a finer comparison between DEMs, as well as to allow a simple statistical comparison of texture parameters for the various measurement techniques, such as skewness, kurtosis, and the vertical roughness length (fig. 4).

Whereas DEMs present obvious discrepancies, the PDFs obtained with the three measurement techniques are reasonably similar. Detailed inspections of Fig. 4a-b-c however reveal some differences in the distribution of surface elevation. A larger range of bed elevations is observed with the acoustic bed-profiler and more particles, proportionally, are lying beyond 15 mm above the mean bed level. It agrees well with the analysis made on the DEM, whereby the acoustic bed-profiler is said to correctly measure the major particles, but with a slight overestimation because of the issue of grain edges. With the laser-scanner (Fig. 4b), the distribution of surface elevation presents a higher and narrower peak than observed with the other instruments, traducing that more particles are reckoned to lie around the mean bed level. It could say that the laser-scanner correctly measures small variations in surface elevation around the mean bed level, features which require a good instrument resolution. Stereo-photogrammetry definitely misses some major gravel particles, resulting in fewer gravels lying beyond 15 mm above the mean bed level.

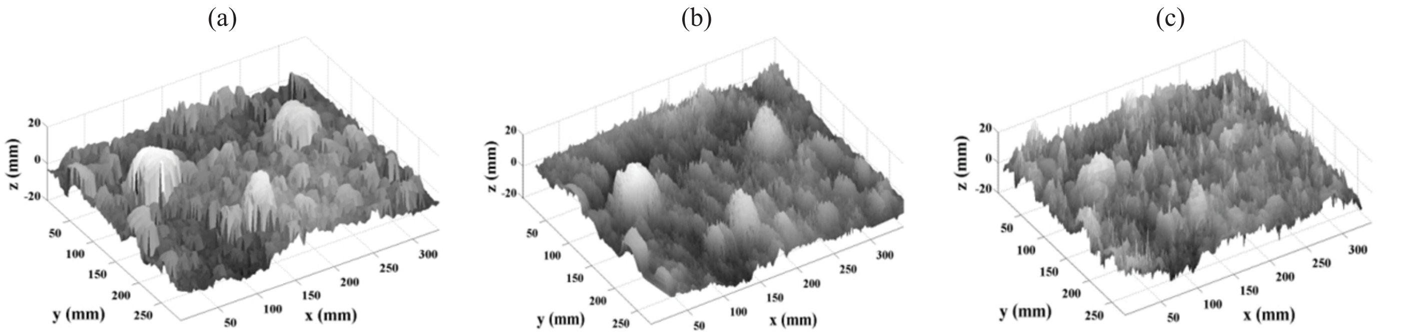

Figure 3. Detrended DEMs of the water-worked bed, focused on the region within the metallic tray, obtained with (a) the acoustic bed-profiler, (b) the laser-scanner (dry condition) and (c) stereo-photogrammetry (exposed condition)

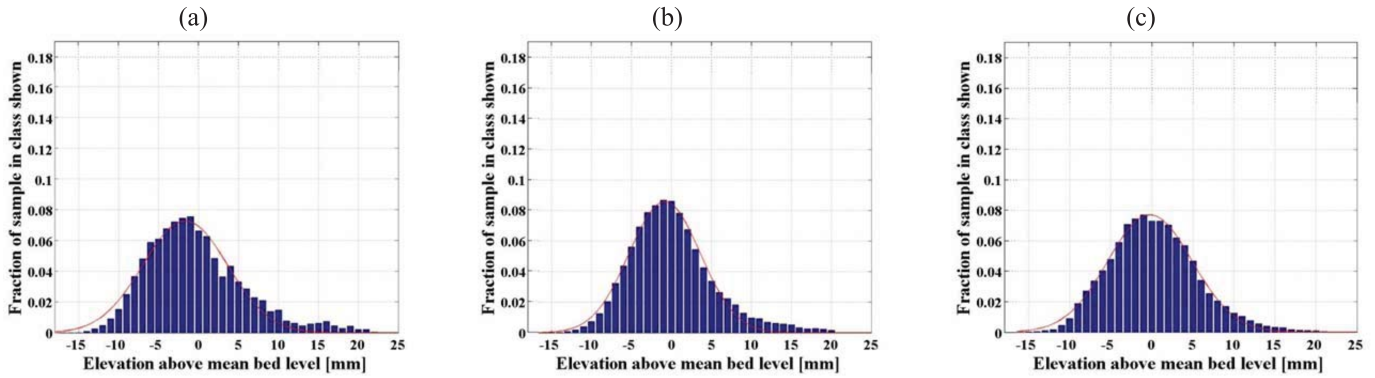

Figure 4. PDFs obtained with (a) the acoustic bed-profiler, (b) the laser-scanner (dry condition) and (c) stereo-photogrammetry (exposed condition)

Based on the detrended distributions of surface elevation, statistical parameters were extracted for the three techniques. As Table 1 shows, the values obtained for the standard deviation of the bed elevation, σ_z, vary, with the acoustic bed-profiler presenting the highest vertical roughness length of 5.76 mm, again certainly slightly overestimated. The laser-scanner measurements led to similar statistics for the dry and wet surfaces, indicating a good reproducibility of the results. With stereo-photogrammetry, serious problems in the gravel-bed surface representation were encountered, resulting in a reduced skewness and vertical roughness length. When stereo-photogrammetric measurements were taken over the submerged gravel-bed, the DEM's quality was the lowest, and resulted in a negative skewness for the statistical analysis.

Previous research showed that water-worked beds in both laboratory flumes and natural streams are positively skewed, e.g. [9,10]. Consequently, the measurements obtained with stereo-photogrammetry over the submerged bed are not realistic and should not be considered for statistical roughness analysis. Overall, considering all DEMs, no significant change in the kurtosis value is observed.

C. 2D second-order structure functions

Generalised structure functions (semivariograms) have become a familiar tool to study evolving processes. For instance, contour line representations of 2D second-order structure functions of bed elevations provide useful information on the history of surface forming mechanisms during the armour layer development, as presented by [4,9].

In this survey, plotting the 2D structure functions for data obtained with the three measurement techniques shows similar elliptical shapes and orientation of the main ellipse (Figure 5a-b-c). The water-worked gravel-beds are all found to be characterised at relatively small spatial lags by an elliptical shape. Geometrically, the elliptical shape of the contour lines reflects the general elliptical form of the dominant particles and an anisotropic surface structure of the bed [5,9,10]. In the experiment, visual observations of the gravel-bed evolution shows that most particles rotate to align their long axis across the flow direction, which agrees with previous studies, e.g. [5].

TABLE I. TEXTURE COEFFICIENTS EXTRACTED FROM BED ELEVATION PDFs

Texture coefficients	Acoustic bed-profiler	Laser-Scanner		Stereo-Photogrammetry	
	Submerged	*Dry*	*Wet*	*Non-submerged*	*Submerged*
Skewness (-)	0.81	0.79	0.86	0.5	-0.12
Kurtosis (-)	3.8	4.07	4.26	3.58	4.79
σ_z (mm)	5.76	5.32	5.32	5.0	4.36

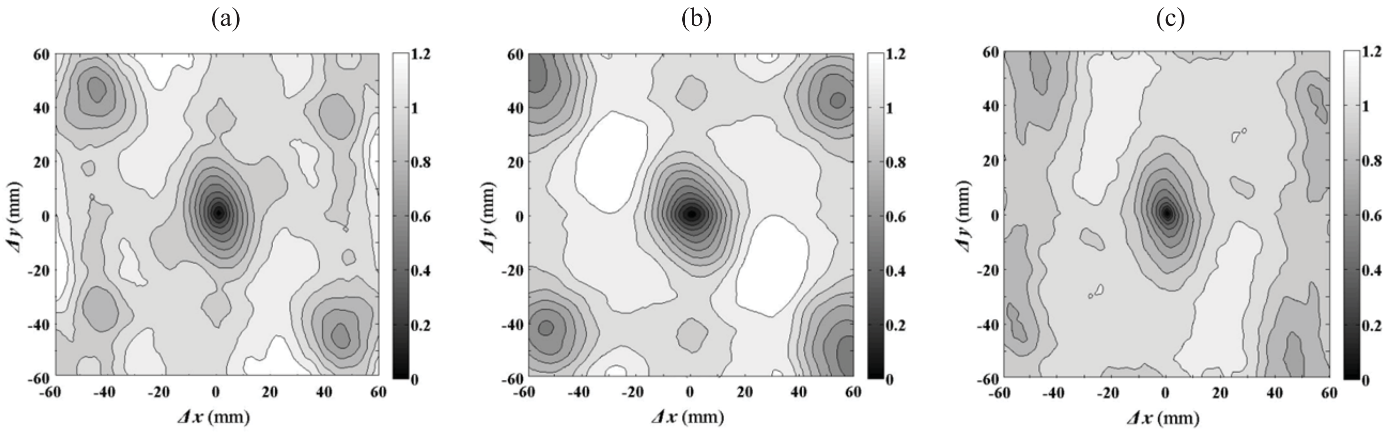

Figure 5. Contour lines of generalised 2D second-order structure functions obtained with (a) the acoustic bed-profiler, (b) the laser-scanner (dry condition) and (c) stereo-photogrammetry (exposed condition)

IV. DISCUSSION

In this study, 3D DEMs at the grain scale (estimated between 4,000 and 10,000 points/m^2) were obtained with the three studied measurement technologies. The acoustic bed-profiler allowed the topography of the 0.35 m long and 0.3 m wide measurement window to be attained with 16,500 data points (this equates ~157,000 points/m^2) and plotted on a 2.45 x 2.54 mm grid. Despite the least dense measurement grid resolution, the visual aspect of the post-processed DEM was representative of the real gravel-bed surface.

For the laser-scanner and stereo-photogrammetry, computer limitations required the downscale of the original data cloud for statistical analysis. The laser-scanner's DEMs were plotted with 50,000 data points (~500,000 points/m^2) on a uniform 1.45 mm grid. Theoretically, the laser-scanner was the most accurate measurement instrument and could have been considered beforehand as the reference instrument in this study. However, the laser-scanner was not able to scan the whole measurement window in one scan and thus required the merging of scanned areas. It resulted in small spikes, which remained in the DEMs despite systematic data post-processing but did not alter the overall DEMs quality.

The DEMs obtained with stereo-photogrammetry were plotted with 105,000 data points (~1,000,000 points/m^2) on a 1 mm grid. Despite the highest grid resolution, stereo-photogrammetry was the technique with the lowest depth resolution and results reflected this. The quality of the DEMs was significantly reduced due to important noise and the fact that large particles were not correctly captured. Compared to the other two measurement techniques, photogrammetry required a more complex operation process (calibration of cameras, creating depth map, extracting point cloud), which increased the error rate for the final DEMs. The relative influence of the different operations on the error rate is yet to be analysed. On the other hand, the use of painted gravel particles is assigned to cause severe inaccuracies in creating the depth map. Paint's reflection was still important despite the use of a dim and homogeneous light during photogrammetric measurements, and reflective painted particles were not correctly recorded due to a lack in texture. It is supposed that natural sediment beds are less reflective than painted particles, and thus photogrammetric results should be directly enhanced if natural sediment particles were used.

In light of the DEMs' analysis, it seems clear that the instruments' measurement resolution (horizontal resolution) is important to measure accurately small sediment particles, grain edges and holes between particles, but the higher resolution does not necessary result in visually better DEMs. Furthermore, the theoretical depth resolution estimated for each technique can be used beforehand as a marker of the reliability in vertical measurements, but does not correspond to the exact accuracy of the techniques. The use of control points would enable to estimate the accuracy (associated with the standard error) as well as the precision (associated with the standard deviation of error) of the three techniques, by comparing the control points vertical coordinates obtained with the studied techniques with the values obtained with a reference instrument. The standard errors in vertical measurements for the three techniques are certainly higher than the theoretical depth resolutions, especially with stereo-photogrammetry where the error rate is important.

With the visual study, the suitability of the DEMs to make a statistical approach of the bed roughness was assessed. It seemed clear that the DEMs obtained with the acoustic bed-profiler and the laser-scanner were well suited for statistical roughness analysis, whereas DEMs obtained with stereo-photogrammetry were not enough representative of the real gravel-bed surface to derive faithful characteristics. However, despite flagrant differences between the DEMs, the vertical roughness length, σ_z, was relatively similar for all DEMs (see Table 1). With the acoustic bed-profiler, statistics were hypothesised to be slightly overestimated because of the grains' edges measurement issues and the weak representation of holes between particles. The laser-scanner provided similar statistics for both the dry and wet gravel-bed surfaces. Hence, when comparing the bed elevation σ_z values with visual observations, it can be concluded that the best approximation of the bed roughness is based on the DEMs obtained with the laser-scanner. Further statistical analyses were conducted on the DEMs by using PDFs and contour lines of 2D second-order structure functions. As presented in Fig. 3 and Fig. 4, these analyses show in general a good agreement for the various DEMs. This indicates that although visible differences in the topography of the DEMs were observed, those relatively minor measurement differences are even less identifiable when DEMs are described statistically.

What criteria can be used for which measurement equipment is best suited for statistical roughness analysis? The acoustic bed-profiler has the advantage that surface measurement is simple, making this technology particularly efficient for laboratory experiments. However, the measuring time is long and because of a low measurement grid resolution, small topographic features were not recorded accurately. Moreover, the flow must be stopped to realise measurements, so that this technique is not really efficient to study evolving gravel-beds. The laser-scanner resulted in highly accurate DEMs, once problems with merging of scanned areas were addressed, which was time consuming. Further work is needed to reduce those merging problems, and once eliminated, the use of the laser-scanner could offer a high-speed and high-resolution measurement. On the downside, the measurement area is restricted in size and no submerged measurement is possible because of a red beam. With digital consumer cameras, stereo-photogrammetry is becoming more affordable and accessible. It has the advantage that it can be employed at various locations, as long as a frame holding the equipment can be positioned safely. The depth resolution is dependent on the cameras' specifications. Once the cameras are calibrated, this technology allows quick data acquisition. In a controlled environment, it is suited to study evolving processes, without interrupting the processes.

V. CONCLUSION

A Laboratory investigation to evaluate the most suitable measurement technique to study the hydraulic roughness for gravel-beds is presented. DEMs were obtained with an acoustic bed-profiler, a laser-scanner and stereo-photogrammetry. The random field of bed elevation was analysed statistically with techniques such as determination of the vertical roughness length, PDFs and 2D second-order structure functions. Pros and cons of the three studied measurement techniques are discussed in regards with a laboratory use, and the suitability of the techniques to characterize faithfully the gravel-bed roughness is assessed.

All three measurement techniques used for this study were able to examine the grain scale of laboratory gravel-beds. The vertical roughness length was approached with the three measurement techniques and similar results were obtained. Minor deviations can be explained with the differences identified in the visual study of the DEMs.

Although visual observation showed that the accuracy of the DEMs obtained with stereo-photogrammetry is inferior to the DEMs obtained with the other techniques, the promise of quick high-resolution data acquisitions, which can be employed at various locations, should warrant further research into this area. Additional work is needed to improve the quality of the complex operational processes for stereo-photogrammetry.

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