Evaluation of statistical properties of dune profiles

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ABSTRACT: A set experiments for dunes developing from a flattened sand bed was obtained in a narrow 0.44-m-wide and 12-m-long glass-sided open channel. The sand in use was a coarse uniform sand of D_{50}=0.85-mm and was exposed to a series of steady and uniform flow conditions. The chosen flow depths generated practically 2D dunes in flow direction over the length of the channel. Spatial sand-bed-elevation profiles were recorded on the centreline of the flume over a distance of 6-m, roughly every 23-sec over the time of development. The recorded dune geometries were evaluated with both the discrete and the continuous approach. For the discrete approach, discrete height and length values are obtained for dunes. For the continuous approach, the second-order distribution moments, standard deviations, were used to obtain characteristic height and length values of the dune field. It is shown that a lack of clear definitions for the discrete approach results in a wide range of averaged dune geometries, depending on how thresholds were set during the analysis process. The continuous approach provides more objective results, but interpreting the results of the analysis requires careful consideration. For this paper, the analytical results of applying both approaches are compared. A preliminary correlation between both approaches is discussed. The physical relationship between the flow field and characteristic height and length values, as obtained through the continuous approach, is not yet clear.

1 INTRODUCTIONS

1.1 *Background*

Dunes are sediment transport objects found commonly on river beds and in coastal areas (Bridge, 2003). The objective of this study is to evaluate in detail two different methods of obtaining statistical properties of dune profiles in laboratory channels. Determining the topographical characteristics of the bed surface is important for forecasting the growth of dunes under different flow conditions and modelling alluvial channel processes. As dunes strongly influence the hydraulic roughness and the sediment transport, a quantitative description of dunes is vital in providing relationships between the dune parameters and the hydraulic conditions.

1.2 *Dune bed forms*

Dunes, also called large-scale bed forms, have discrete lengths generally larger than 0.6-m and influence and interact with the water flow (Ashley, 1990). A significant portion of the flow depth is occupied by dunes. Therefore, dune topographies affect the flow with accelerations and decelerations of the flow field all the way to the free surface (Maddux, 2002). Dune wavelength l, and dune height h, are related to the flow depth H. In laboratory channels the discrete average length of developed dunes can be generally estimated as $6H$ (Yalin and da Silva, 2001). The averaged dune height can be estimated as $H/3$. The generated flow field over dunes is vastly different from the uniform flow field found over a flat bed (Best, 2005).

1.3 *Analytical methods*

Current methods for quantifying hydraulic roughness of river dunes generated in laboratory environments can be distinguished by two groups: a) discrete approach (also called direct approach); b) continuous approach (also called indirect or random-field approach). Commonly, the discrete approach is applied in determining geometric characteristics such as height and length of dune profiles. When applying the continuous approach, dune profiles are treated as a random field of sand-bed elevations.

1.4 *Discrete approach methodology*

Dunes are conventionally considered as periodic features with certain length, height and steepness (h/l) values. Average height and length parameters, obtained through discrete analysis, describe 2D dunes quite well. However, river dunes are highly irregular features, often not exposing such a periodic regularity in their natural environment. In reality the idealized triangular shapes of dunes (Figure 1), which form the basis of the discrete approach, do not exist (Figure 2). Additionally, the discrete approach has the drawback of being exposed to subjective identification (Van der Mark et al., 2005).

Commonly the height of a dune is the vertical distance between crest and trough. The first problem arises when dunes grow on cross-sets, where the vertical distance between crest and subsequent trough is different to the vertical distance of trough and subsequent crest. Furthermore the question arises, what constitutes a dune? Often sand sheets (Whiting et al., 1988; Venditti, 2003) migrate over the stoss side of dunes and can be identified in longitudinal records as individual dunes. If so, this so called superposition of dunes can reduce the statistical mean character of dune geometries significantly and averaged discrete height and length values will not represent the bed profile adequately.

Figure 1. Roughly 2D dunes as generated in a narrow 0.44-m wide flume. Flow is from left to right. D_{50}=0.85-mm.

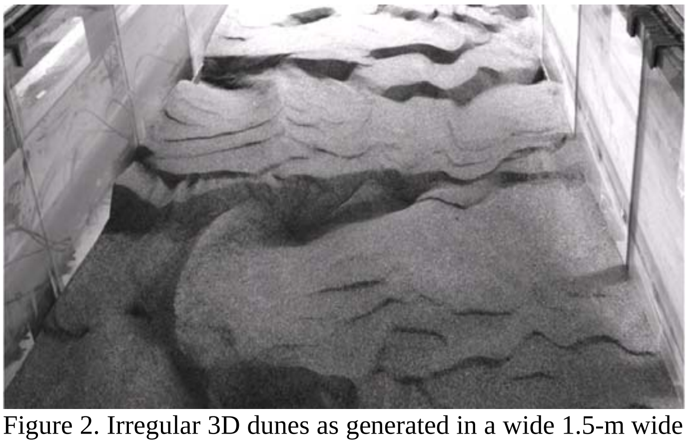

Figure 2. Irregular 3D dunes as generated in a wide 1.5-m wide flume. Flow is from top to bottom. Superposed sand sheets are clearly visible. D_{50}=0.85-mm.

When defining crest and trough elevations for dunes, one can determine local minima and maxima and use them for the growth analysis. One also can determine the top and bottom positions of the lee-side and use them for the growth analysis. Additionally, Van der Mark et al. (2005) point out that some researchers take into account smaller-scale dunes whilst some do not. For instance Allen (1982) defines dunes as bed forms longer than 600-mm and higher than 40-mm. The introduction of such threshold values can influence the histograms and the fitted Probability Density Functions (PDFs) considerably.

Similar to the lack of agreement about how to define bed-form height, there exists a lack of agreement about how to define bed-form length. Van der Mark et al. (2005) summarizes that bed-form length can be defined as the horizontal distance between two successive mean bed level upcrossings. A mean bed level upcrossing is defined as the point where the upward going bed elevation profile crosses the mean bed level. Again it is questionable if all mean bed level upcrossings of the bed elevation should be treated as distinguished individual dunes or if some dunes should be excluded. Dune length can also be defined as the horizontal distance between two successive mean bed level downcrossings or between two successive troughs or between two successive crests.

Another problem arises during bed-form initiation. Depth-sounding recording devices are not sensitive enough to let researchers analyse early discrete bed-form geometries when generated from a flat bed. Research showed (Coleman and Melville, 1996) that a wavelength does not start at zero; instead bed-form initiation growth depends on the grain size. Measurement device inaccuracies and different spatial or temporal resolutions of the device between subsequent data points can also result in different inaccuracies when determining bed-form length and height characteristics.

Generally, an experimental data set of developing bed forms is analysed with the same criteria for the whole bed-form development. A criteria adjustment, depending at which stage of growth development the bed-form analysis is carried out, could result in a more objective statistical evaluation of bed-form growth.

1.5 *Continuous approach methodology*

Compared to the empirical distribution functions, which are obtained through discrete analysis, the continuous approach focuses on the characterization of the stochastic process, which generates the bed forms (Moll et al., 1987). Nordin and Algert (1966), Hino (1968) and Jain and Kennedy (1974) treated bed-form profiles as continuous longitudinal profiles and applied spectral analysis. Nikora et al. (1997) treated sand-bed elevation fields as random fields rather than discrete sand waves, using longitudinal and transverse spectra, correlation and structure functions to describe statistical sand wave dynamics.

The continuous approach is widely accepted when studying turbulent structures in water flow and is an important tool to identify coherent structures in the turbulent flow field. Scaling relationships are used to facilitate understanding of the flow dynamics. Researchers re-introduced the continuous approach to bed-form studies during the last decade (Nikora and Hicks, 1997; Nikora and Goring, 2000; Nikora et al., 1997). The continuous approach is based on the premise that random fields can be described completely by the *n*-dimensional probability density when $n \rightarrow \infty$. A complete quantitative description using the *n*-dimensional probability functions is theoretically possible, but practically beyond reach (Nikora et al., 1997). Therefore selected applications of higher-order distribution moments (mean, standard deviation, skewness and kurtosis) and moment functions (correlations, spectra, structure functions, etc.) are evaluated, allowing for a quantitative evaluation of sand-bed elevations. In the following, the focus is on analysing the change in standard deviation of bottom elevations of developing dune profiles. The framework of the moving-bottom elevation field model (Nikora and Hicks, 1997) is described in more detail in Section 3.2.

1.6 *Outline*

For this paper, the direct and the continuous approaches as applied for this study are introduced. The experimental setup is described. The results are compared and the relationship between the two approaches is discussed.

2 EXPERIMENTAL SETUP

A new data set of developing dune bed forms from a flattened sand bed was obtained at the Fluid Mechanics Laboratory at the University of Auckland. The experiments were conducted in a 12-m-long, 0.38-m-deep and 0.44-m-wide glass-sided open-channel flume. Water is recirculated, as well as the sediment. The sediment in use is a coarse (D_{50}=0.85-mm) filter sand and was carefully flattened along the whole length of the flume. Once flattened, the flume was carefully filled with water, in order to not change the flat sand bed structure. At a given time $t=0$ the pump was turned on and the spatial recording of the sand bed proceeded simultaneously.

Bed profiles were measured with the help of a moving carriage. On the rails of the flume, the moving carriage traversed the flume back and forth. A depth sounding probe was installed on the carriage and recorded the sand-bed elevation field for every downstream and upstream traverse. The initial position of the moving carriage was located 4-m downstream of the inlet. The recordings covered a distance of 6-m, before the carriage stopped and returned to its initial position. A slight slippage of the chain-and-sprocket driven programmable-speed motorized carriage occurred, which was corrected for during the bed-profile analysis.

A depth-sounding system was used to record the profiles of the developing bed. The bed-profile measurement system is based on Coleman (1997), and has since been further improved. According to Coleman (1997), the accuracy of bed-elevation measurement is ±0.4-mm. The distance along the flume was measured using a potentiometer mounted on a 0.199-m diameter wheel running down the outside of the flume side-wall. The streamwise position was recorded at 2.45-mm distance increments, accuracy of measurement being ±1.23-mm.

Before each run, the slope of the flume (S_b) and water surface, as well as the temperature of the water were measured and recorded.

For each experiment, measurements were taken on the centreline of the flume. Therefore, a two-dimensional field of bottom elevations $z(x,t)$ was obtained for different flow fields. Bed profiles were recorded roughly every 23-sec for each experiment, lasting between 2.5 and 10 hours.

The mean depth-averaged flow velocity for each experiment was estimated by utilizing ADV flow measurements in the upper part of the water column, at the start of each experiment (with flat bed). The average flow velocity *U* was determined by fitting a logarithmic velocity profile through the data, and assuming average velocity at a height of 1/e of the water depth above the sand bed. Two shear velocities were calculated for each flow setting. $u_*^{(1)}$ was calculated as $\sqrt{gHS_b}$ (see explanation of symbols in Table 1), whereas $u_*^{(2)}$ was determined based on the ADV measurements.

All together a set of 24 experiments were recorded, of which 14 experiments started with a flattened bed and were exposed to steady uniform flow conditions. The remaining 10 experiments were part of a flood-wave research project and a PIV flow-field study and will not be discussed herein. The experimental conditions for the experiments discussed are given in Table 1.

3 ANALYSIS

3.1 *Discrete approach methodology*

The discrete approach, based on the presentation of sand-bed elevations as individual discrete bed forms, is commonly used to characterize dune geometry and that information can be used for incorporation in hydraulic formulas.

As introduced in Section 1.4, different algorithms can be used to determine discrete dune height *h*, dune length *l* and steepness of bed forms *h/l*. The applied algorithm for the recorded profiles is based on a routine which determines crest and trough positions based on identification of the lee face. This method requires a sufficiently small sampling resolution in the longitudinal direction. The following input threshold values are required:

- minimum dune height (default value: 0.005-m),
- minimum allowable horizontal separation distance between identified lee-face points (default value: 0.045-m),
- minimum number of points taken into account to constitute a lee face (default value: 6),
- minimum angle of the lee-face slope (default value: 12-degrees).

Figure 3 shows the results of applying the routine with different input parameters for each of the thresholds. Only one input threshold is changed in each case, the other three being set to the default values.

Table 1: Experimental Conditions

Flow Conditions	Experiment Names	Flow parameters B [m]	Q [l/s]	q [m^2/s]	H [m]	B/H [-]	S_b [-]	U [m/s]	Fr [-]	Re [-]	$u_*^{(1)}$ [m/s]	$u_*^{(2)}$ [m/s]
I	T3,T8	0.44	29.42	0.0669	0.1250	3.52	0.0010	0.53	0.48	66788	0.035	0.032
II	T6,T23	0.44	35.94	0.0817	0.1250	3.52	0.0015	0.62	0.56	77604	0.043	0.036
III	T4,T7,T11	0.44	38.56	0.0876	0.1250	3.52	0.0020	0.70	0.63	87633	0.050	0.041
IV	T5,T24	0.44	27.19	0.0618	0.1000	4.40	0.0015	0.58	0.58	57709	0.038	0.035
V	T13	0.44	20.46	0.0465	0.1000	4.40	0.0010	0.47	0.47	46508	0.031	0.028
VI	T14	0.44	31.47	0.0715	0.1125	3.91	0.0015	0.62	0.59	69975	0.041	0.037
VII	T15,T22	0.44	43.74	0.0994	0.1500	2.93	0.0015	0.65	0.54	97463	0.047	0.037

B = flume width, Q = discharge, q = specific discharge, H = water depth, S_b = initial water surface and bottom slope, U = average flow velocity, Fr = Froude number = U/sqrt(gH), Re = Reynolds number = UH/ν, Kinematic viscosity ν=0.000001-m^2/s, u_* = shear velocity, with $u_*^{(1)}$=sqrt(gHS_b), and $u_*^{(2)}$=based on ADV measurements

Note, for experiment T22,T23,T24 the downstream water depth was changed during the experiment, such that the water-surface slope was approximately equal to the bed-surface slope; for the other experiments the downstream water level was kept constant to the initial value

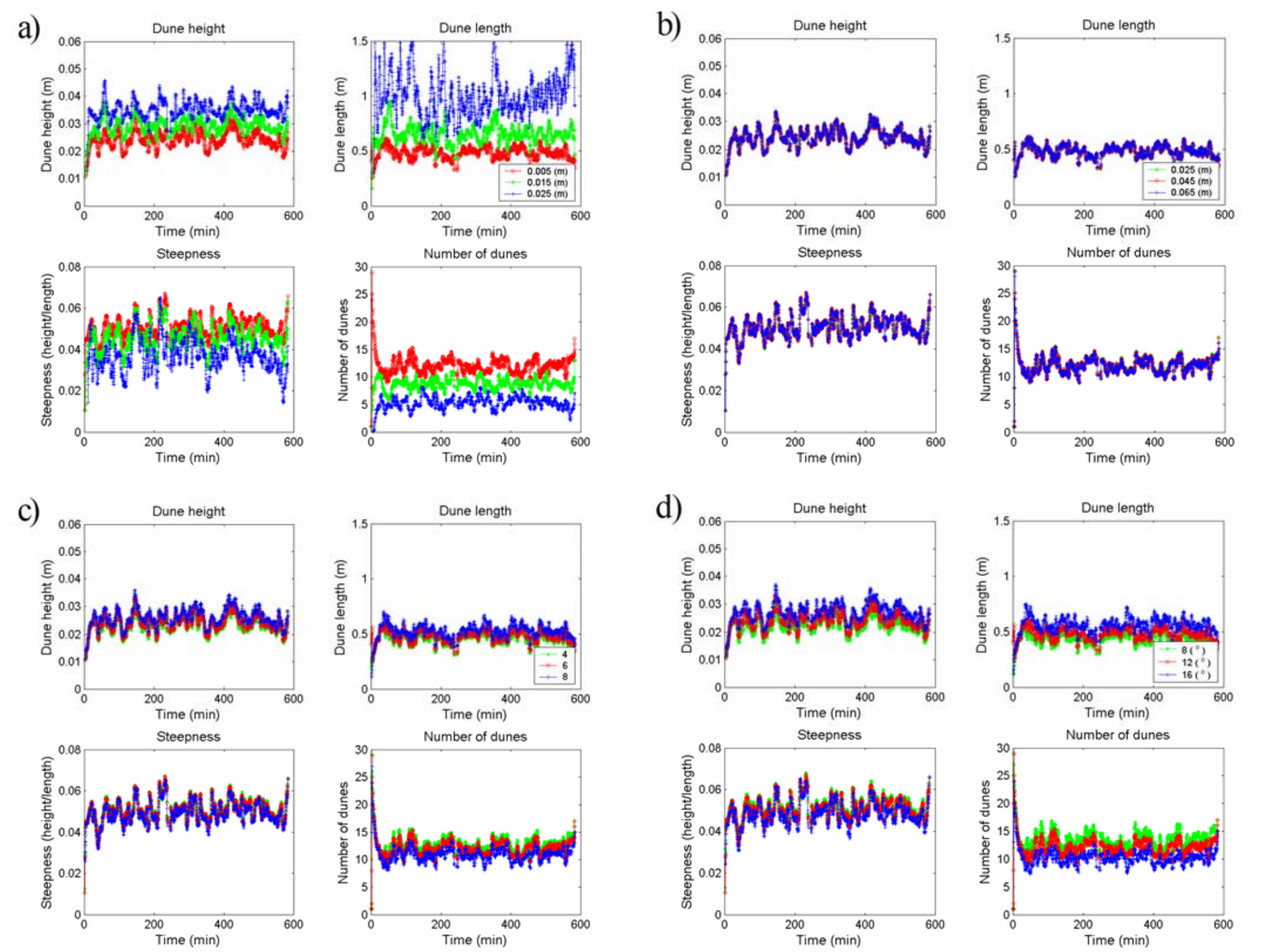

Figure 3. Discrete bed-form statistics of experiment T14. a) Change in threshold for dune height; b) change in threshold for allowable horizontal separation distance between identified lee-face points; c) change in threshold for number of sample points of lee face; d) change in threshold for angle of lee face.

Figure 3a shows that the threshold for the dune height can influence the discrete height and length values for dune geometries significantly. For instance, a minimum dune height threshold of 1.5-cm results in smaller averaged discrete dune heights than a minimum threshold of 2.5-cm. Consistently, the number of discrete dunes which are identified is much larger for a smaller threshold than for a larger threshold.

Figures 3b and 3c show the change of statistical dune geometry for changing input thresholds for allowable horizontal separation distance between identified lee-face points and for the number of points which must constitute a lee face, respectively. One can clearly see that the geometric values of the developing dune geometries do not vary significantly with changing input thresholds for these discrete dune characteristics.

Figure 3d shows the influence of input thresholds for the minimum angle of the lee face. The statistical data here are more scattered than those in Figures 3b and 3c but still more uniform than the data displayed in Figure 3a.

In conclusion, it can be shown that the threshold for minimum dune heights as set during the analysis can skew the averaged discrete dune geometries significantly. A lower threshold results in depicting dune bed forms from early on during dune development. Additionally, sand sheets superposed on top of dune stoss sides, which develop later during an experiment, will be identified as independent bed forms and therefore reduce the average dune dimensions significantly. Alternatively, if a higher threshold for minimum dune heights is used for the analysis, dunes which develop early during bed-form development are not accounted for. Additionally, the averaged dune dimensions are substantially larger than for a smaller minimum dune height threshold.

3.2 *Continuous approach methodology*

The continuous approach, based on the presentation of sand-bed elevations as a 2D random field, is seen as an alternative to the conventional discrete approach. For the presented data set, the continuous approach considers the sand-bed surface as a random field $z(x)$ of bed elevations, where x is the longitudinal (the main flow direction) coordinate. Depending on the measurement frequency, z is also a function of time t. In order to compare statistical results relating to dune geometry, the detail of the continuous approach for this paper is restricted to the analysis of the change of the second-order distribution moments, i.e. standard deviations of bottom elevations, during dune development.

Nikora and Hicks (1997) introduce characteristic height and length values for sand-wave fields that are considered as a random field of bottom elevations. The root-mean-square deviation $\sigma_z(\delta x,t)$ over a certain distance δx (ranges from the smallest possible sub distance available for the spatial profile up to the whole length of the spatial profile) along the flume, at a certain time t during development, can be determined for every spatial profile during dune development starting from a flattened bed (Figure 4). When $\delta x \geq \delta x_0$ and $t \geq t_e$ (t_e is the development time necessary for establishment of equilibrium conditions), $\sigma_{z0}=\sigma_z(\delta x,t)$. When $\sigma_z(\delta x,t_e)$ becomes saturated for equilibrium conditions, $\delta x_0=\delta x$ (see Nikora and Hicks (1997) for more details).

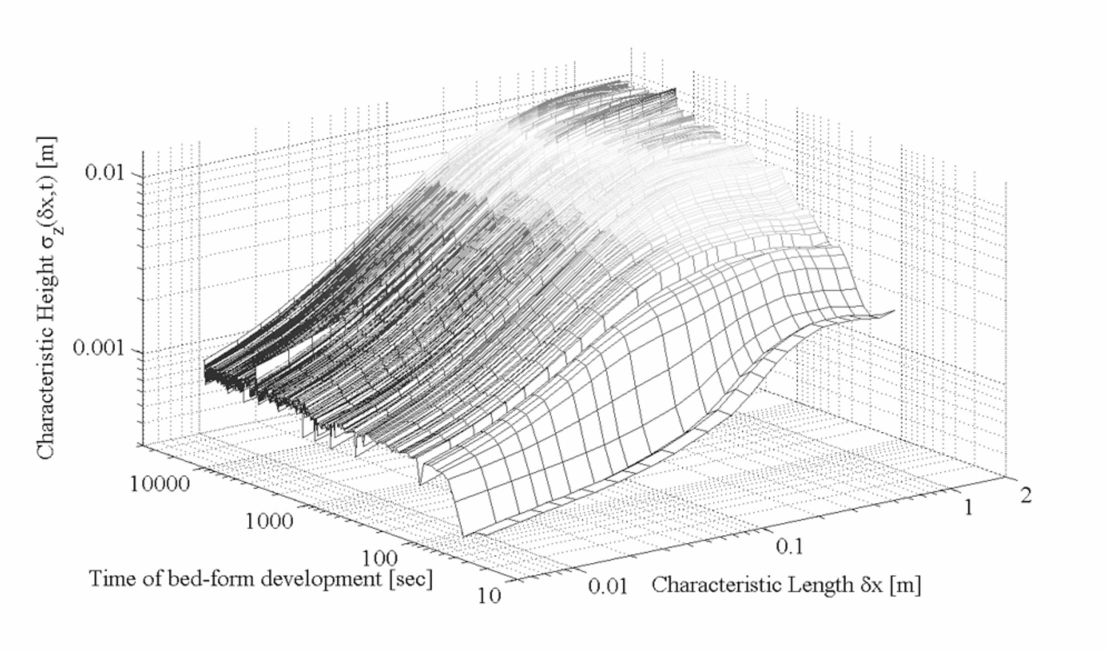

Figure 4. Development of characteristic height σ_z and characteristic length δx during dune development (here shown for experiment T13). It shows the initial fast development of the characteristic height σ_z , as well as the asymptotic approach of the characteristic length δx towards a saturated value during development of an equilibrium dune bed.

By analogy to height and length parameters being inter-changeable growth parameters when applying the power law (Nikora and Hicks, 1997) for discrete sand waves, there must exist a similarity relationship between $\sigma_z(\delta x)$ and δx for random fields. For self-similar features, which longitudinal spatial sand-wave profiles are considered to be, $\sigma_z(\delta x) \propto \delta x$ (Figure 5). Therefore δx is a characteristic scaling feature of the developing dune bed, similar to the wave length feature of discrete sand wave analysis. δx will grow similar to a discrete wave length during the development of the dune bed and will reach an equilibrium stage once saturation is reached.

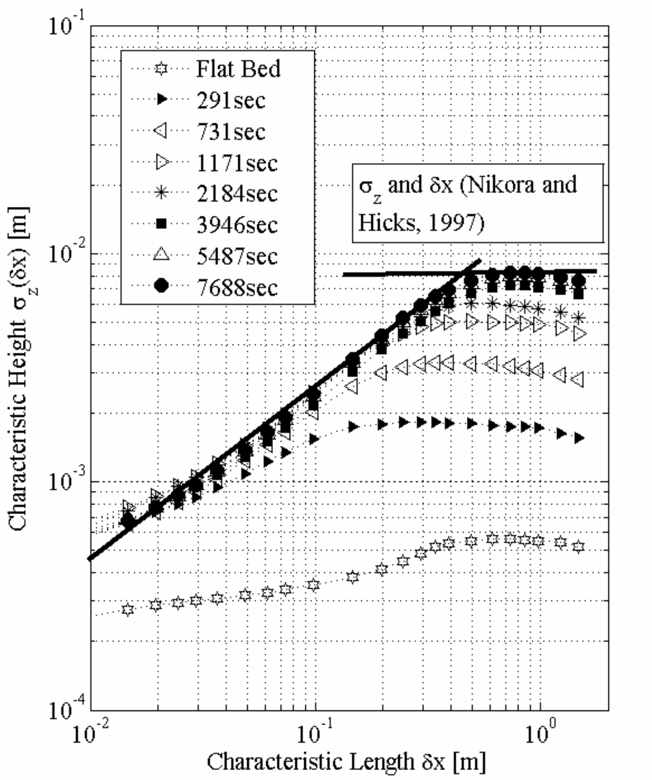

Figure 5. Exemplary functions $\sigma_z(\delta x,t)$ for experiment T3 during dune development starting from a flat bed. The self-similarity of dunes is characterized by the similar slopes in the scaling region after initial dune development.

4 RESULTS AND DISCUSSION

Dune development was recorded for seven different flow conditions. Table 2 shows the equilibrium values of characteristic height and length obtained through the continuous analysis as well the averaged dune height and length as obtained through the discrete analysis. Additionally bed-form geometry predictions after van Rijn (1984) and Julien and Klaassen (1995) are shown. The bed-form predictors generally overestimate the discrete dune dimensions, as they are based on mostly field data. For our laboratory data, the flow field was practically two-dimensional. The minimum width/ depth ratio is just under 3. Therefore side walls are expected to influence the dune formations in close proximity to the walls, but the centreline spatial dune profiles can be regarded as recording regular 2D dunes formed under interaction with 2D flow.

Figure 6 shows a typical comparison between dimensionless (in regards to the flow depth H) characteristic dune statistics and averaged discrete dune statistics, as obtained for experiment T3. The general trend as shown in Figure 6 is valid for all analysed experiments.

From Figure 6 it can be seen that averaged discrete height and length show a slightly jagged growth, compared to a statistically smoother growth for characteristic dune geometries. Besides the discussed variations of dune geometries (caused by applying different thresholds during the discrete analysis), other factors can be associated with the more jagged statistical growth behaviour of the discrete parameters, such as the limited spatial length of the profiles. The measurement section is only 6-m in length. This implies that average discrete dune dimensions are strongly influenced by dunes of different sizes migrating in and out of the recording length, resulting in the more jagged statistical growth.

Table 2: Dune Parameters

				Equilibrium bed-form characteristics															
				Bed-form geometry (continuous)			Bed-form geometry (discrete)				Bed-form estimators (Van Rijn, 1984)			Bed-form estimators (Julien and Klaassen, 1995)			H/3	6H
Experiment Name	Flow conditions	Rec. Profiles	Development Time	Char. Height	Char. Length	Steepness	Height	Length	Steepness	m (Eq.1)	Height	Length	Steepness	Height	Length	Steepness	Height	Length
		[-]	[hrs]	[cm]	[cm]	[-]	[cm]	[cm]	[-]	[-]	[cm]	[cm]	[-]	[cm]	[cm]	[-]	[cm]	[cm]
T3	I	563	3.50	0.71	112	0.0064	2.5	67	0.038	3.5	4.17	91	0.046	6.87	78.54	0.087	4.2	75
T4	III	412	2.53	0.83	99	0.0084	2.9	54	0.054	3.5	5.55	91	0.061	6.87	78.54	0.087	4.2	75
T5	IV	391	2.44	0.53	90	0.0059	2.6	60	0.043	4.8	4.06	73	0.056	5.87	62.83	0.093	3.3	60
T6	II	482	3.03	0.90	130	0.0070	2.9	50	0.057	3.2	5.26	91	0.058	6.87	78.54	0.087	4.2	75
T7	III	325	3.02	0.64	100	0.0064	3.2	54	0.059	5.0	5.55	91	0.061	6.87	78.54	0.087	4.2	75
T8	I	524	3.60	0.79	111	0.0071	2.8	75	0.038	3.6	4.17	91	0.046	6.87	78.54	0.087	4.2	75
T11	III	458	2.92	0.91	159	0.0057	3.2	63	0.051	3.5	5.55	91	0.061	6.87	78.54	0.087	4.2	75
T13	V	1137	8.57	0.67	144	0.0046	2.3	65	0.036	3.5	2.85	73	0.039	5.87	62.83	0.093	3.3	60
T14	VI	1496	9.72	0.77	96	0.0080	2.6	58	0.044	3.4	4.69	82	0.057	6.38	70.69	0.090	3.7	67.5
T15	VII	676	4.39	1.12	158	0.0071	3.4	62	0.055	3.1	6.24	110	0.057	7.80	94.25	0.083	5.0	90
T22	VII	455	2.90	1.48	161	0.0092	4.1	67	0.061	2.8	6.24	110	0.057	7.80	94.25	0.083	5.0	90
T23	II	503	3.26	1.33	126	0.0106	3.6	69	0.052	2.7	5.26	91	0.058	6.87	78.54	0.087	4.2	75
T24	IV	575	3.74	1.07	149	0.0072	3.2	60	0.052	3.0	4.06	73	0.056	5.87	62.83	0.093	3.3	60

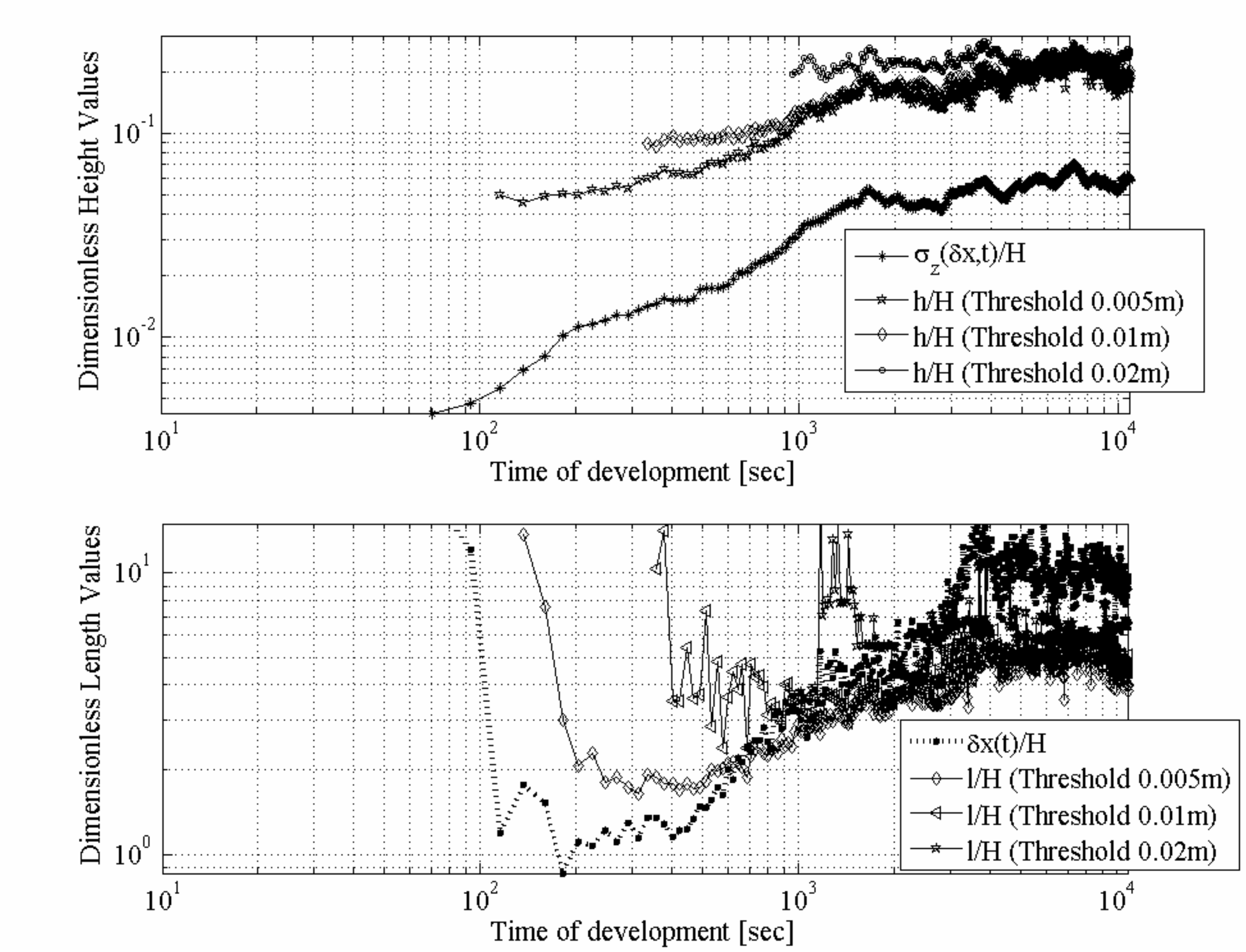

Figure 6. Relationship of averaged dimensionless discrete height *h/H* and length *l/H* values (different minimum dune height thresholds are applied) and dimensionless characteristic height $\sigma_z(\delta x,t)/H$ and length $\delta x(t)/H$ values during dune development. *m* (Eq. 1) does change significantly during dune development (see also Figure 7). The development is shown for the first 3-hrs for experiment T3.

The statistical growth as obtained with the help of the standard deviation of bottom elevations, however, results in a smoother growth, indicating that this analysis provides a more objective description of the actual averaged dune dimensions.

Both approaches, the direct and the continuous analysis, are related. Nikora et al. (1997) state that the average height *h* of bed forms relates to the standard deviation of bottom elevations σ_z in the following way:

$$h = m\sigma_z \qquad (1)$$

where *m* is in the range 1.7 to 2 for rivers. Table 2 shows values of *m* for our laboratory data for equilibrium conditions, which are all significantly larger than 2. Figure 7 shows the development of *m* during dune development for experiment T3. The general trend shown in Figure 7 is valid for all experiments, with *m* decreasing during dune development and fluctuating around a certain mean value (between 2.7 and 5; see Table 2) once the dune bed is in equilibrium. Most of the fluctuation is attributed to the migration of dunes in and out of the recording region, which influences the discrete averaged dune heights significantly.

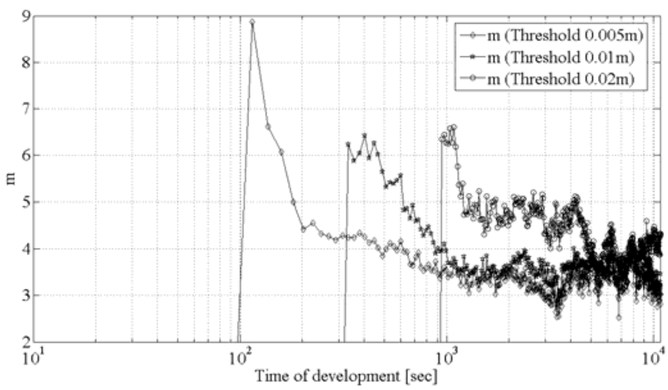

Figure 7. Development of *m* during dune development. The development is shown for experiment T3.

5 CONCLUSION

The present study was conducted in order to investigate two different deterministic and statistical analysis tools for dunes and to compare the results. A flattened sand bed of coarse sand was exposed to different flow strengths in a narrow 0.44-m-wide flume and the spatial development of dunes was recorded on the centreline of the flume.

Sand-bed elevation fields are treated in two different ways: as discrete bed forms and as a random field of bed-form elevations.

The discrete analysis utilizes a routine which can filter discrete dunes of less than a certain geometric threshold criterion, and exclude them from computing average dune properties. Slight variations of the input thresholds for detectable minimum dune height varies the statistical dune geometries significantly, although the general trend of exponential growth from initial bed features to equilibrium dunes is visible for all experiments.

In contrast, the continuous analysis results in unambiguous statistical characterisation of the growth of dunes during development from a flattened sand bed.

For total dune development, the value of $m=h/\sigma_z$ (Eq.1) is significantly larger than 2, the value associated with river data.

This paper shows that characteristic height and length values are independent of subjective thresholds during statistical analysis and describe the complete growth of average dune dimensions without discontinuity. This approach provides a substantial advantage compared to the traditional discrete approach.

More analysis and comparison of additional experimental data sets are required in order to improve the suggested relationship between these characteristic dune statistics and discrete dune geometries. Such a relationship can then be utilized for inclusion in flow resistance formulas and to help better understand the feedback mechanism between coherent flow structures and river morphology.

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