Temporal Development of Sediment-wave Magnitudes

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ABSTRACT: A power-law expression can be used to describe the growth of bed forms (ripples or dunes) from flat-bed conditions to equilibrium magnitudes (lengths or heights) for a flow, where equilibrium magnitudes for the flow can be determined using established expressions from literature. In order to use this relation for prediction purposes, expressions are presented herein for the times required for ripples and dunes to achieve equilibrium magnitudes, and the respective growth exponents for the development of sand-wave heights and lengths. The proposed expressions are developed using the authors' experiments and published data, with the resulting description of sand-wave development provided by this approach found to be valid for ranges of flows and sediments.

1 BACKGROUND

1.1 *Introduction*

The presence of sand waves (ripples and dunes) on river beds greatly affects local flow conditions, influencing dispersion and mixing processes; biota habitats; riverbed resistance to flow (e.g. van Rijn, 1984) and flooding levels; and the passage of fluid, sediments and suspended or attached materials. The magnitudes of these sand-waves (heights and lengths) provide intuitively-sensible parameters to quantify their effects on the flow. Experimental investigations have produced relationships to estimate the average equilibrium magnitudes of ripples (e.g. Raudkivi, 1997) and dunes (e.g. van Rijn, 1984), for combinations of flow and sediment parameters. Such investigations have shown that a sand bed takes a significant period of time to reach its equilibrium configuration when developing from flat-bed conditions. Real-world flows are often constantly changing, leading to a need for prediction of magnitudes of non-equilibrium developing sand-waves.

1.2 *Investigations of developing sand-waves*

In line with earlier investigations, Baas (1993) suggested that the development of ripples from a flat bed could be described as an exponential process:

$$P = P_e - (P_e - P_i)e^{-C_p t} \qquad (1)$$

where P is the average value of a sand-wave parameter (length L or height H) after time t; P_i and P_e are the initial and equilibrium values of parameter P; and C_p is a growth parameter inversely proportional to the time to achieve equilibrium wave magnitudes. In contrast to the finding of Baas (1993), Nakagawa & Tsujimoto (1983) observe that the exponential process of (1) provides a poor description of bed-wave parameter development (Nikora & Hicks, 1997).

Nikora & Hicks (1997) alternatively propose a power function to describe sand-wave development, namely

$$\frac{P}{P_e} = \left(\frac{t}{t_e}\right)^{\alpha} \qquad (2)$$

where α is a growth exponent, and t_e is the time to achieve equilibrium magnitude P_e. The relationship of (2) is proposed to be valid for $0.01<t/t_e<1$.

Based on their own experimental data and the published data of Iseya (1984), the authors found α = 0.28 for both sand-wave lengths and heights, implicitly stating an invariability in sand-wave steepness during sand-wave development. The authors note, however, that the degree of fit of (2), with α = 0.28, to their data is a function of sediment size. They also comment that further work is required to enable α to be determined, and (2) applied, over ranges of sediments and flows, with α having been determined based on limited data.

In order to use (2) to predict bed-form development for a given flow, additional expressions are required to estimate P_e and t_e for the flow. Expressions for equilibrium sand-wave magnitudes P_e are available in published literature (e.g. Raudkivi, 1997; Yalin, 1992; van Rijn, 1984; and Julien & Klaassen, 1995). Raudkivi (1997) suggests that time to equilibrium for ripple development from a flat bed may be estimated by an equation of the form

$$t_e = a(\theta / \theta_c)^{-b} \tag{3}$$

where a and b are coefficients that require correlation from empirical data still to be obtained, θ is dimensionless shear stress, and θ_c is critical dimensionless shear stress for sediment entrainment. Raudkivi (1998) further notes that no systematic studies of the development time of dunes are known.

The present work is aimed at providing additional data and expressions for α and t_e to enable (2) to be used for the prediction of bed-form-magnitude growth with time from plane-bed conditions.

2 EXPERIMENTAL PROGRAMME

2.1 *Experiments*

Two series of experiments focusing on bed development under the action of sub-critical water flow were carried out in glass-sided, tilting, recirculating flumes measuring 0.38m-deep, 0.44m-wide, 12m-long (Zhang, 1999) and 1.2m-deep, 1.5m-wide, 43m-long. The 15 experiments of sand-wave growth from plane-bed conditions involved varying flow conditions for each of four uniformly-graded sediments (Table 1). Velocity profiles (giving average flow velocity U and bed-shear velocity u_* for a run, Table 1) were measured with a laser-doppler velocimeter over equilibrium bed configurations for the first nine runs of Table 1. For the remaining runs, u_* and U were determined from equilibrium flume slope and PTV measurements of flow velocities. In regard to Table 1, b is flume width, y is average flow depth, S is equilibrium (uniform-flow) flume slope, τ is water temperature, d_{50} is median sediment size, s is sediment specific gravity, and u_{*c} is critical shear velocity for sediment entrainment.

As described by Zhang (1999), the bed configuration along the centreline of the flume was measured over a length of 4m (0.44m-wide flume) or 8m (1.5m-wide flume) at regular time intervals (e.g. 30s) for a run using an ultrasonic depth-sounder mounted on a carriage that was pushed along the flume. Bed profiles were measured from the initially flat bed to equilibrium sized ripples or dunes for the run. Measurements in a given bed profile, which is assumed to be recorded at an instant in time, were made at every 2.45mm along the flume, with bed-level measurement to an accuracy of +-0.4mm.

Table 1. Experimental parameters.

Run*	b (m)	y (m)	S ($x10^{-3}$)	U (m/s)	u_* (m/s)	τ (^{o}C)
FS	0.440	0.095	1.05	0.264	0.0380	17
FM	0.440	0.100	1.78	0.346	0.0497	17
FF	0.440	0.125	3.87	0.504	0.0657	18
MS	0.440	0.085	1.88	0.380	0.0296	21
MM	0.440	0.110	2.93	0.457	0.0457	20
MF	0.440	0.120	3.56	0.596	0.0607	21
C1S	0.440	0.100	1.26	0.432	0.0252	17
C1M	0.440	0.135	1.88	0.492	0.0488	17
C1F	0.440	0.145	3.77	0.728	0.0733	18
C2Sa	1.5	0.170	1.00	0.465	0.0373	20**
C2Ma	1.5	0.170	2.60	0.686	0.0590	20**
C2Fa	1.5	0.170	3.90	0.806	0.0725	20**
C2Sb	1.5	0.100	1.70	0.475	0.0373	20**
C2Mb	1.5	0.100	3.30	0.594	0.0522	20**
C2Fb	1.5	0.100	5.00	0.752	0.0641	20**

* The first letter indicates the sand type (fine, medium or coarse), and the second letter indicates relative flow strength (slow, medium or fast).
Fine sand: d_{50}=0.23mm, s=2.65, u_{*c}=0.0131 m/s
Medium sand: d_{50}=0.44mm, s=2.63, u_{*c}=0.0155 m/s
Coarse sand C1: d_{50}=0.74mm, s=2.62, u_{*c}=0.0197 m/s
Coarse sand C2: d_{50}=0.82mm, s=2.65, u_{*c}=0.0210 m/s
** Estimated temperature only.

2.2 *Experimental analyses*

For each recorded bed profile, sand waves were identified by the presence of a lee slope, as described by Coleman & Melville (1994). Identified waves were accepted if the wave height was greater than a minimum value of 2mm (fine sand) or 5mm (medium and coarse sands). From each profile, average sand-wave height and average wave length (number of sand-waves/length of measurement) were determined, along with the time to equilibrium bed conditions for the run. The time to equilibrium was defined as the time from when waves were first present on the bed to when average sand-wave heights remained essentially constant at their equilibrium level. Equilibrium bed-wave magnitudes, given in Table 2, are a representative average of sand-wave magnitudes recorded after this time (t_e). Growth exponents α_H and α_L for a run were evaluated from a best-fit of (2) to time series of H(t) and L(t), using the calculated values of t_e and H_e or L_e (Table 2).

3 PUBLISHED DATA

In order to extend the results and implications of the present experiments, the results of previous published experiments were assessed and collated. The writers evaluated α_H and α_L from graphs of H(t) and L(t) provided by Baas (1993), Raichlen & Kennedy (1965) and Kühlborn (1993). The growth exponent of Nikora & Hicks (1997) was also included in analyses, along with t_e values from their experiments that they provided to the writers.

Table 2. Experimental results.

Run	t_e (s)	H_e (m)	L_e (m)	α_H	α_L
FS	18232	0.0113	0.1045	*	*
FM	10481	0.0120	0.1087	0.385	0.201
FF	855	0.0097	0.1386	0.380	0.431
MS	4509	0.0176	0.3321	0.386	0.592
MM	3833	0.0239	0.4543	0.453	0.255
MF	1812	0.0228	0.4668	0.297	0.470
CS	5450	0.0165	0.5000	*	*
CM	1757	0.0247	0.5584	0.522	0.469
CF	1204	0.0286	0.6904	0.482	0.555
C2Sa	*	0.0361	0.6162	*	*
C2Ma	3600/4800**	0.0398	0.5435	0.684	0.356
C2Fa	1290/2160**	0.0425	0.6068	0.522	0.316
C2Sb	4800**	0.0370	0.5330	0.490	0.29
C2Mb	1320**	0.0409	0.7176	0.280	0.293
C2Fb	630**	0.0302	0.5146	*	0.277

* The development of the bed for runs FS, CS and C2Sa was heavily influenced by the propagation of waves from upstream into the test reach rather than waves developing within the reach. Run C2Fb developed too rapidly for a reliable determination of α_H.
** Where two numbers are quoted, the first is in relation to bedform heights, the second bedform lengths. A single number is given in relation to bedform heights.

4 BED-FORM GROWTH

4.1 *Bed-form types*

Ripple dimensions are typically described as functions of sediment size, where-as dune dimensions scale with flow parameters. Plots of equilibrium bed-form magnitudes were then used to determine the respective natures of the bed forms obtained for the sediments of the present experiments (Table 2) and those of the earlier works of Raichlen & Kennedy (1965), Baas (1993), Kühlborn (1993), and Nikora & Hicks (1997).

4.2 *Growth exponents*

The growth exponents α_H and α_L of (2) were found to be insensitive to flow strength $\theta/\theta_c = (u_*/u_{*c})^2$ (Figure 1, with data differentiated as appropriate to ripples "r" and dunes "d"). The exponents were then plotted as a function of sediment (Figure 2), where $Re_{*c} = du_{*c}/\nu$ is the critical grain Reynolds number for sediment entrainment, d is representative sediment size, ν is fluid kinematic viscosity, and the critical shear velocity for sediment transport is taken as $u_{*c} = 0.0115 + 0.0125d^{1.4}$ (Melville & Coleman, 2000).

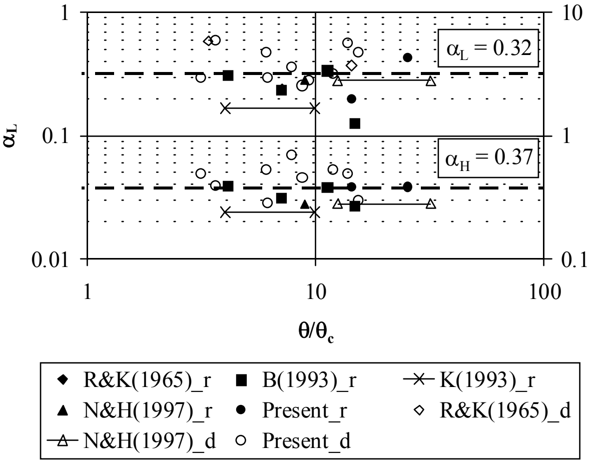

Figure 1. Growth exponents as functions of flow strength.

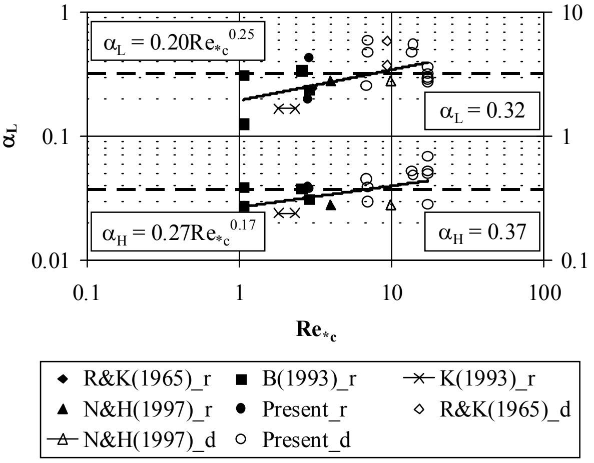

Figure 2. Growth exponents as functions of sediment.

Figure 2 shows a dependence of α on Re_{*c}, essentially sediment size, with trend fitting giving

$$\alpha_L = 0.20\,Re_{*c}^{\ 0.25} \tag{4}$$

$$\alpha_H = 0.27\,Re_{*c}^{\ 0.17} \tag{5}$$

for the growth of sand-wave (ripple and dune) lengths and heights respectively.

4.3 *Time to equilibrium*

Appropriate to sediment-transport phenomena, collated values of t_e were normalized by (u_*/d), with dimensional analysis giving $t_* = t_e u_*/d = f(\theta, Re_*, y/d, s)$, or for the current data pool of all quartz ($s \approx 2.65$) sediments, $t_* = f(\theta, Re_*, y/d)$.

Analogous to equilibrium ripple magnitudes (e.g. Raudkivi, 1997; and Baas, 1993), development time for ripples may be expected to be independent of flow depth and viscosity, giving $t_{*r} = f(\theta)$. With dune development typically assumed independent of viscosity effects, $t_{*d} = f(\theta, y/d)$ may be expected. Relationships for t_* were thereby sought in the form

$$t_*(d/y)^n = f(\theta/\theta_c) \tag{6}$$

where n was adjusted to optimize data collapse, with n=0 expected for ripple data.

The results of Figure 3 show that t_e can be estimated using

$$t_{*r} = (t_e u_*/d) = 2.08\times10^8(\theta/\theta_c)^{-2.42} \tag{7}$$

$$t_{*d} = (t_e u_*/d) = 2.05\times10^{-2}(d/y)^{-3.5}(\theta/\theta_c)^{-1.12} \tag{8}$$

where t_{*r} and t_{*d} are normalized equilibrium development times for ripples and dunes respectively. In regard to Figure 3, the postscripts "vf", "f", "m", and "c", refer to very fine, fine, medium and coarse sediments respectively, with dunes indicated by open symbols. Furthermore, "t_{eH}" and "t_{eL}" in Figure 3 refer to t_e for wave heights and lengths respectively.

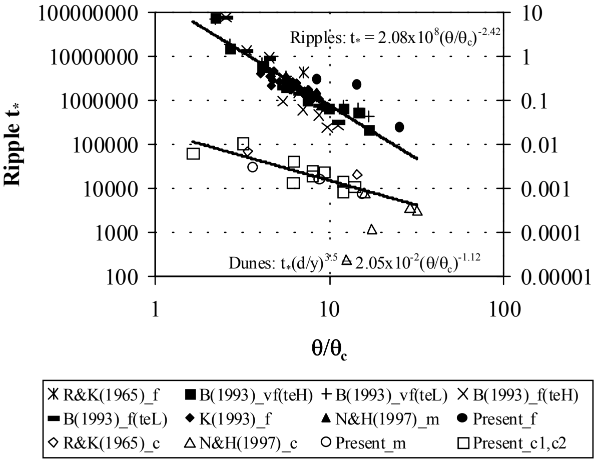

Figure 3. Equilibrium times for ripple and dune data.

5 CONCLUSIONS

Bed-form (ripple and dune) growth from plane-bed conditions can be predicted using $P/P_e = (t/t_e)^\alpha$, where P is bed-form length or height; P_e is equilibrium bed-form length or height (estimated from existing equations); equilibrium time t_e for ripple development is given by $(t_e u_*/d) = 2.08\text{x}10^8(\theta/\theta_c)^{-2.42}$; t_e for dune development is given by $(t_e u_*/d) = 2.05\text{x}10^{-2}(d/y)^{-3.5}(\theta/\theta_c)^{-1.12}$; α for bed-form lengths is described by $\alpha_L = 0.20\text{Re}_{*c}^{0.25}$; and α for bed-form heights is described by $\alpha_H = 0.27\text{Re}_{*c}^{0.17}$. In terms of the earlier-identified knowledge gaps, this description of bed-form development (valid for ranges of flows and sediments) improves means of estimation of the growth exponent α, enables prediction of t_e for ripple and dune beds, and enables prediction of the growth with time of dunes (as well as ripples) from a plane bed to equilibrium magnitudes.

6 ACKNOWLEDGEMENTS

This research was partly funded by the Marsden Fund (UOA220) administered by the Royal Society of New Zealand.

REFERENCES

Baas, J. H. 1993. Dimensional analysis of current ripples in recent and ancient depositional environments. *Geologica Ultraiectina*, *Faculteit Aardwetenschappen der Rijksuniversiteit*, Utrecht, Netherlands, 106, 199pp.

Coleman, S. E., & Melville, B. W. 1994. Bed-form development. *J. Hyd. Engrg., ASCE,* 120(5), 544-560.

Iseya, F. 1984. An experimental study of dune development and its effect on sediment suspension. *Envir. Res. Ctr. Pap.*, 5, 1-56.

Julien, P. Y., & Klaassen, G. J. 1995. Sand-dune geometry of large rivers during floods. *J. Hyd. Engrg., ASCE,* 121(9), 657-663.

Kühlborn, J. M. 1993. Wachstum und Wanderung von Sedimentriffeln. *PhD thesis, Technischen Hochschule Darmstadt*, Darmstadt, Germany, 144pp.

Melville, B. W., & Coleman, S. E. 2000. *Bridge scour*. Water Resources Publications, Colorado, USA, 550pp.

Nakagawa, H., & Tsujimoto, T. 1983. Time-lag appearing in unsteady flow with sand waves. *J. Hydrosci. Hydr. Engrg.*, 1(1), 83-95.

Nikora, V. I., & Hicks, D. M. 1997. Scaling relationships for sand wave development in unidirectional flow. *J. Hydr. Engrg., ASCE,* 123(12), 1152-1156.

Raichlen, F., & Kennedy, J. F. 1965. The growth of sediment bed forms from an initially flattened bed. *Proc., 11th Cong. Int. Assn. for Hyd. Res.*, Leningrad, U.S.S.R., Vol. 3, Paper 3.7.

Raudkivi, A. J. 1997. Ripples on a stream bed. *J. Hyd. Engrg., ASCE,* 123(1), 58-64.

Raudkivi, A. J. 1998. *Loose boundary hydraulics*. 4th Ed., Pergamon Press, Inc., New York, New York, U.S.A., 496pp.

van Rijn, L. C. 1984. Sediment transport, Part III: Bed forms and alluvial roughness. *J. Hyd. Engrg., ASCE,* 110(12), 1733-1753.

Yalin, M. S. 1992. *River mechanics*. Pergamon Press, Inc., New York, N.Y., U.S.A.

Zhang, M. H. 1999. Sediment waves in subcritical water flows. ME thesis, The University of Auckland, Auckland, New Zealand, 264pp.