Multi-Photon Blockade in Cavity QED

S.S. Shamailov, M.J. Collett, A.S. Parkins, H.J. Carmichael

Department of Physics, University of Auckland, Auckland, New Zealand.

Abstract

The driven Jaynes-Cummings (JC) model is studied in the strong-coupling, strong-drive regime by way of numerical simulations. The investigation is motivated by recent advances from Circuit QED which show conclusively that the model and parameter regimes considered are experimentally feasible. The focus is on the multi-photon transitions (MTPs) to higher excitation dressed-states, as observed by Bishop *et al*. [1]. Photon statistics of the light leaking out of the cavity are examined when the laser is tuned to such a multi-photon resonance. We find that with increasing drive strength, bunched, super-poissonian light is transformed into antibunched and sub-poissonian light. These results are interpreted and a simple model is constructed to explain our observations analytically. Moreover, the incoherent spectrum is computed, leading to observation of a saturation effect and multiple dynamic Stark splittings, resulting in a Mollow triplet. This is interpreted analogously to resonance fluorescence in the context of Floquet theory and the simplest example is treated analytically. In summary, we observe Multi-Photon Blockade.

1. Introduction

• Photon Blockade [2]: a system can only "hold" one photon of a given frequency at a time due to anharmonicity. Causes two-state behaviour:

→ Antibunched scattered light.
→ Saturation: strong coherent driving field induces semi-classical splitting and a Mollow Triplet.

• Photon Blockade occurs in strongly coupled Cavity QED due to the $\sqrt{n}$ non-linearity of the JC ladder. In this context:

→ First theoretical prediction in [3]; called "dressing of dressed states".
→ First experimental observation in [1]; called "supersplitting".

MPTs occur in the driven JC model when an integer multiple of the laser frequency is close to one of the dressed-state energies. This allows to extend the ideas of Photon-Blockade to *Multi*-Photon Blockade.

Fig. 1: The driven JC system: a 2-level atom confined to a mirrored cavity.

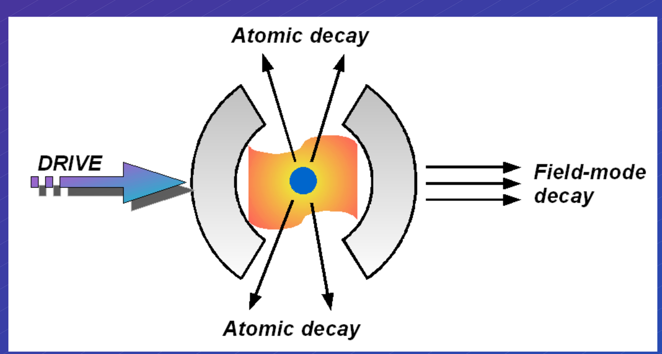
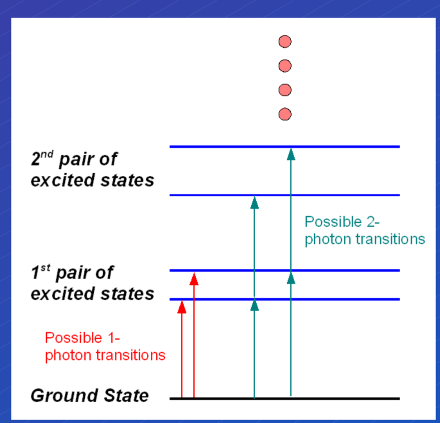


Fig. 2: JC ladder and possible 1- and 2-photon transitions.



2. The Model

Hamiltonian of the driven JC model in the Schrödinger picture:

$$H = \hbar\omega_o\left(\sigma_+\sigma_- + a^\dagger a\right) + \hbar g\left(a^\dagger\sigma_- + \sigma_+ a\right) + \hbar\mathcal{E}\left(a^\dagger e^{-i\omega_L t} + a e^{i\omega_L t}\right)$$

Hamiltonian in the Dirac picture rotating with the drive:

$$H = \hbar\Delta\left(\sigma_+\sigma_- + a^\dagger a\right) + \hbar g\left(a^\dagger\sigma_- + \sigma_+ a\right) + \hbar\mathcal{E}\left(a^\dagger + a\right) \qquad \Delta = \omega_o - \omega_L$$

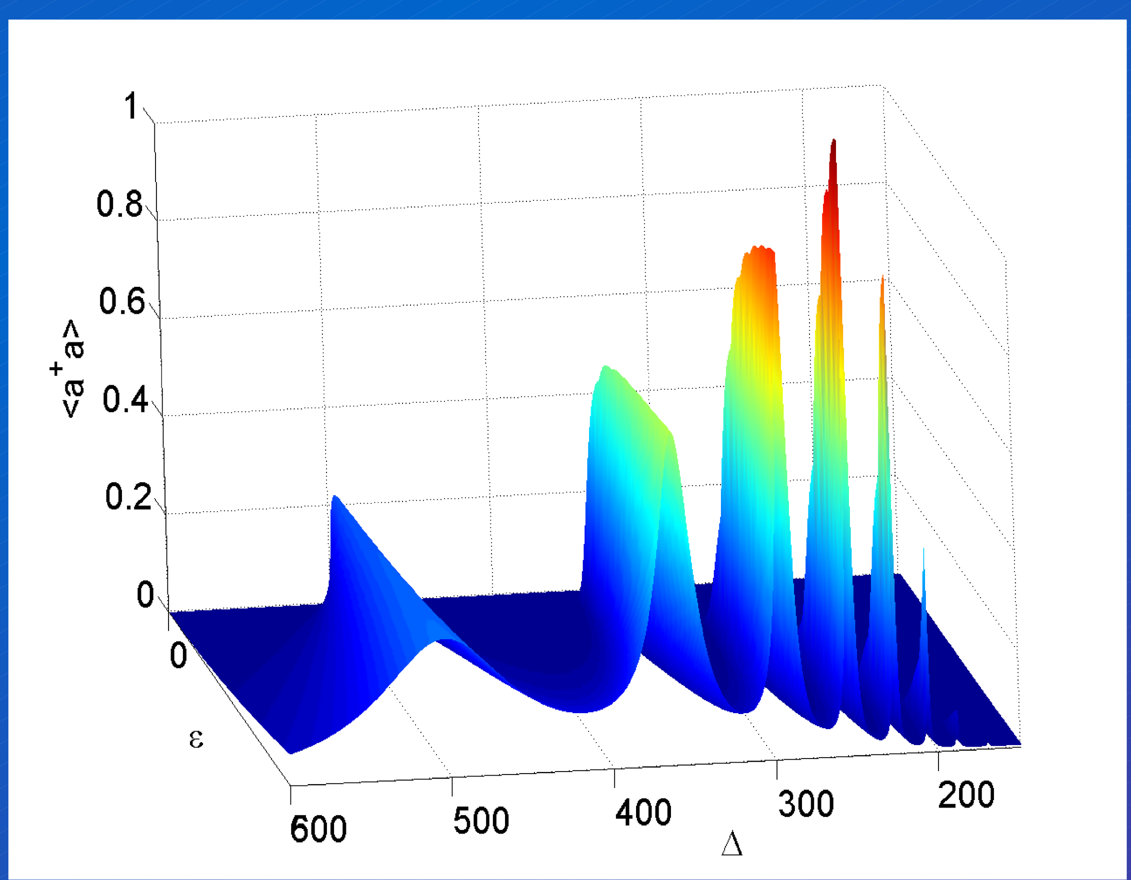
Energy is lost from of the cavity mode and atom at rates κ and γ respectively, so we have two collapse operators entering the Lindblad master equation for the reduced density matrix ρ: $C_a = \sqrt{\kappa}a$ $\quad C_{\sigma_-} = \sqrt{\gamma}\sigma_-$ $\quad \dot{\rho}(t) = \mathcal{L}\rho(t)$

with generalized Liouvillian

$$\mathcal{L} = \frac{1}{i\hbar}[H,\cdot] + \sum_{j=a,\sigma_-}\left(C_j \cdot C_j^\dagger - \frac{1}{2}C_j^\dagger C_j \cdot - \frac{1}{2}\cdot C_j^\dagger C_j\right)$$

If one drives at $\omega_L = \omega_o \pm g/\sqrt{n}$, the n-photon transition $|g,0\rangle \rightarrow |n\pm\rangle$ is excited as $n\omega_L = n\omega_o \pm \sqrt{n}g$ are exactly the dressed state energies.

Fig. 3: Resonances of the cavity-field intensity as a function of the drive strength and detuning. Parameters: g = 500, κ = γ = 1.



3. Characteristic Quantities

A) The intensity correlation function of the light leaking out of the cavity (assuming a stationary source) gives the dynamic probability of detecting two photons with a time gap of τ between them:

$$\left[g_{aa}^{(2)}(\tau)\right]_{ss} = \frac{\left\langle a^\dagger(0)a^\dagger(\tau)a(\tau)a(0)\right\rangle_{ss}}{\left\langle a^\dagger a\right\rangle_{ss}^2}$$

Antibunched & sub-poissonian light is a Quantum-Mechanical phenomenon.

B) The incoherent spectrum shows us the frequency components present in the light scattered by the system. The cavity mode spectrum, for example, is

$$S(\omega) = 2\int_0^\infty \frac{\left\langle a^\dagger(\tau)a(0)\right\rangle - \left\langle a^\dagger\right\rangle\left\langle a\right\rangle}{\left\langle a^\dagger a\right\rangle}\exp(-i\omega\tau)\,d\tau$$

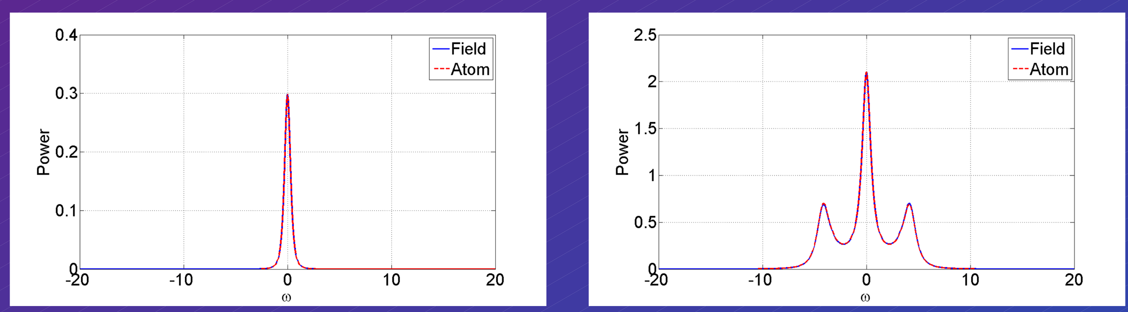
while that for the atom is obtained by swapping a for σ_-.

4. Results

We solve the steady-state Master equation, $\dot{\rho}_{ss} = \mathcal{L}\rho_{ss} = 0$, and use the formulas of section 3 to calculate the two-time correlation functions required. NB: with increasing drive strength, MPT resonances shift in frequency, and require "tracking".

4.1 Incoherent Spectrum

Fig. 4: Atomic and field spectra for 1-photon peak; Mollow triplet develops. g = 500, κ = γ = 1, Δ = 500, ε = 0.1 (left) and ε = 3 (right). Frequency axis is w.r.t. the drive.



• Saturation: coherent exchanges of excitation between the effective two-level system (ground and lower vacuum Rabi states) and the classical field dominate. Higher and higher orders (loops) become possible before an irreversible decay.

• With drive, the system only has Floquet-type periodic solutions. The quasi-eigen-frequencies are found by diagonalizing the Hamiltonian in the interaction picture. Then the quasi-energies in the Schrödinger picture are found by adding those to each of the frequencies removed in transforming to the interaction picture.

Fig. 5: Dynamic Stark splitting for the 1-photon transition: quasi-eigen-frequencies and possible transitions.

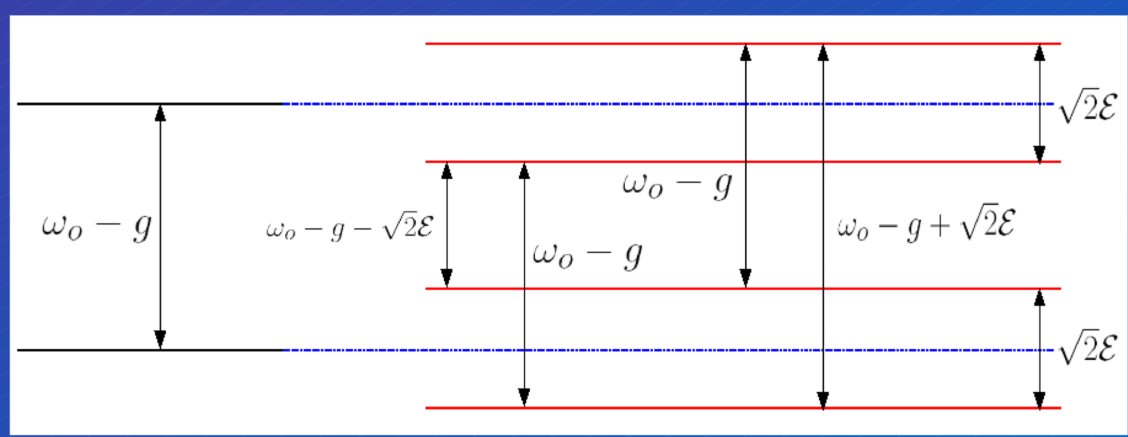
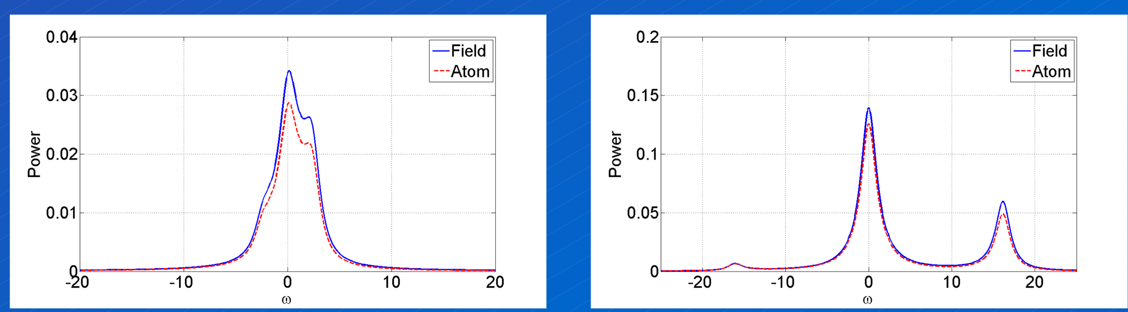


Fig. 6: Atomic and field spectra for 2-photon peak; Mollow triplet develops. g = 500, κ = γ = 1, Δ = 354.2, ε = 15 (left) and Δ = 358.1, ε = 40 (right). Frequency axis is w.r.t. the drive.

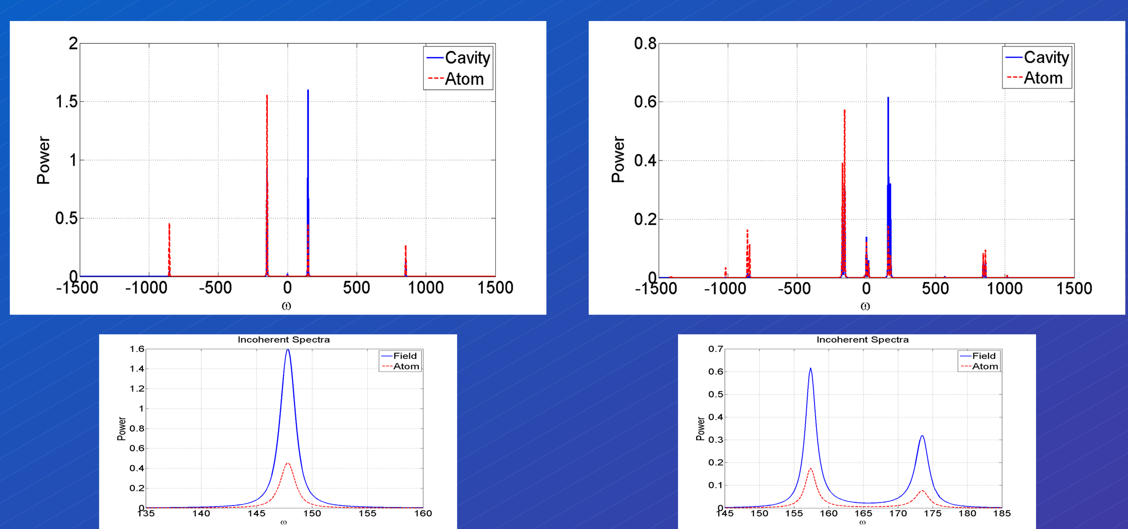


The asymmetry in the Mollow triplet is quite regular and is present for all orders of MPTs – see [4] for details.

• The Stark splitting frequency goes as an N^{th} order polynomial for the N^{th} MPT peak, as the transition which is being saturated requires N photons to occur.

• Other peaks in the spectrum, further removed from the origin, are also split into doublets or triplets after the onset of saturation. This happens at a lower drive, before the Mollow Triplet develops, and directly evidences the saturation.

Fig. 7: Atomic and field spectra for peak 2; Autler-Townes splitting. g = 500, κ = γ = 1, Δ = 353.8, ε = 10 (left) and Δ = 358.1, ε = 40 (right). Frequency axis is w.r.t. the drive. Subplots: magnified peak of the transition ground ↔ lower first excited state.

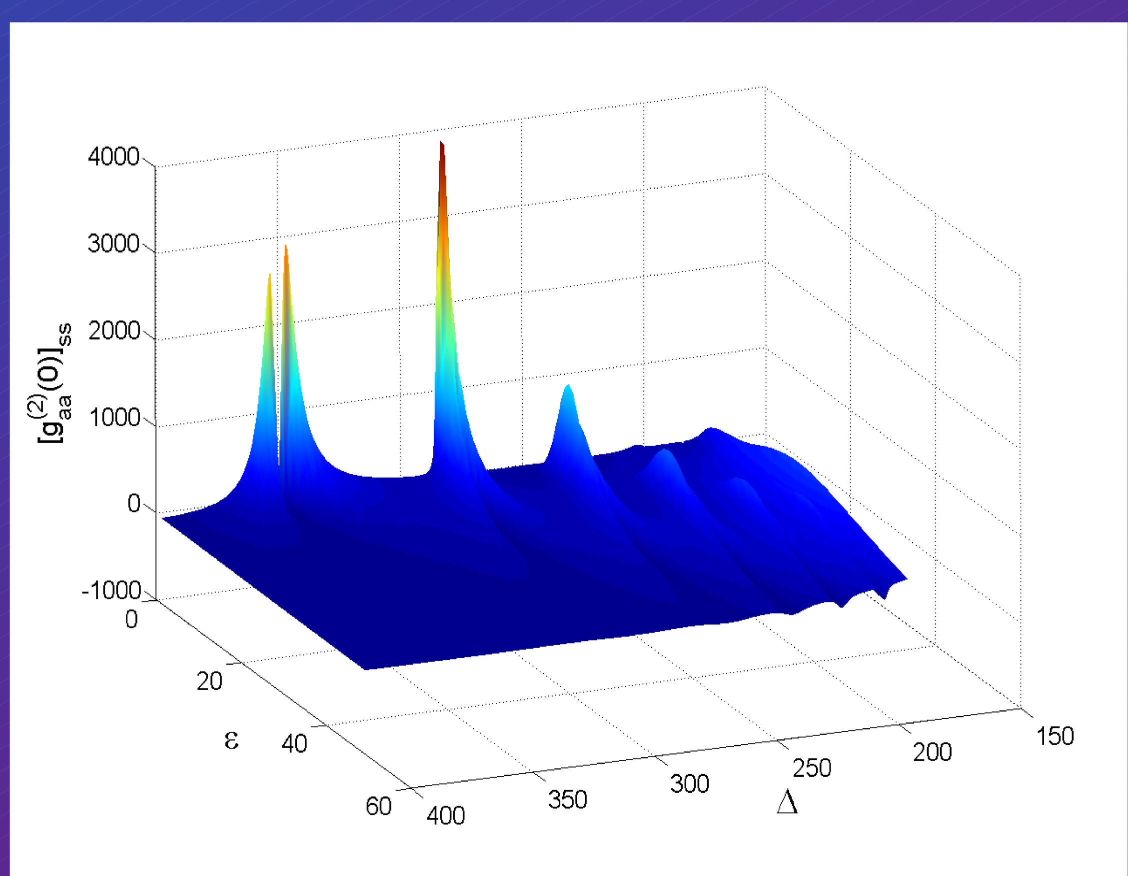


• This splitting of all peaks is understood with an analysis similar to that of the vacuum peak, i.e. periodic-solutions and their quasi-eigen-frequencies.

• Each eigenvalue of the undriven model is split into a band of quasi-frequencies, as many of them as there are basis states in the model. (Far removed states cause negligible splittings, though).

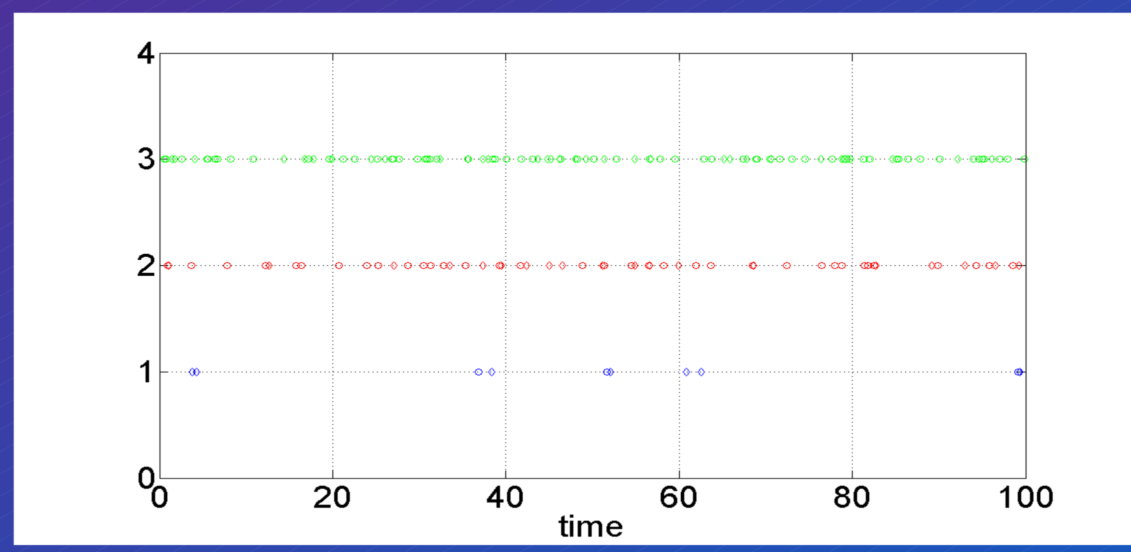
4.2 Intensity Correlation Function

Fig. 8: Intensity correlation at zero delay as a function of drive strength and detuning. Parameters: g = 500, κ = γ = 1.



The vacuum Rabi peak is antibunched for all drive strengths. We thus witness the transformation of extreme bunching to antibunching when driving on a MPT.

Fig. 9: Examples of Quantum Trajectories for the 2-photon peak, showing emission times and channels (atom – diamonds, field – circles). Blue: ε = 5, Δ = 353.6, strongly bunched; Red: ε = 10, Δ = 353.8, slightly bunched; Green: ε = 15, Δ = 354.2, antibunched.



• With increasing drive, $g^{(2)}(0)$ decreases rapidly and bunching transforms to antibunching. This occurs because the excitation rate increases while decay rates are unchanged and so the intermediate states get populated. Any population in the vacuum Rabi states, for example, will contribute to the denominator but not numerator of $g^{(2)}(0)$.

Fig. 10: Field intensity correlation for peak 1; saturation causes slow oscillations at the sideband frequency in the spectrum. Parameters: g = 500, κ = γ = 1, Δ=500, ε = 0.1 (left) and ε = 3 (right).

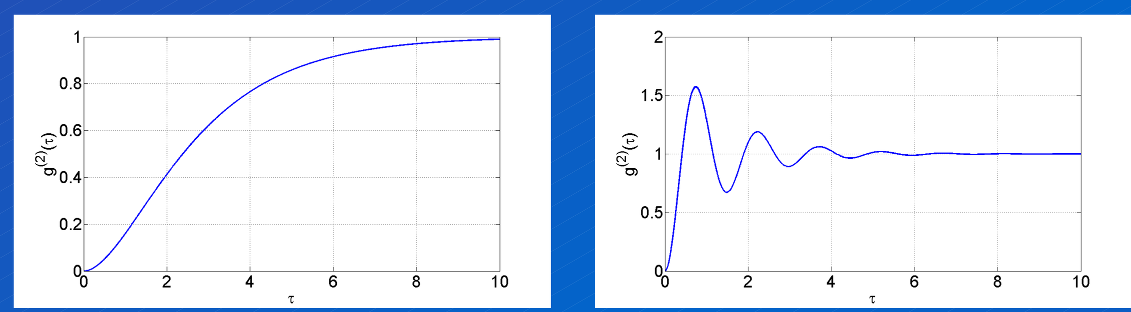
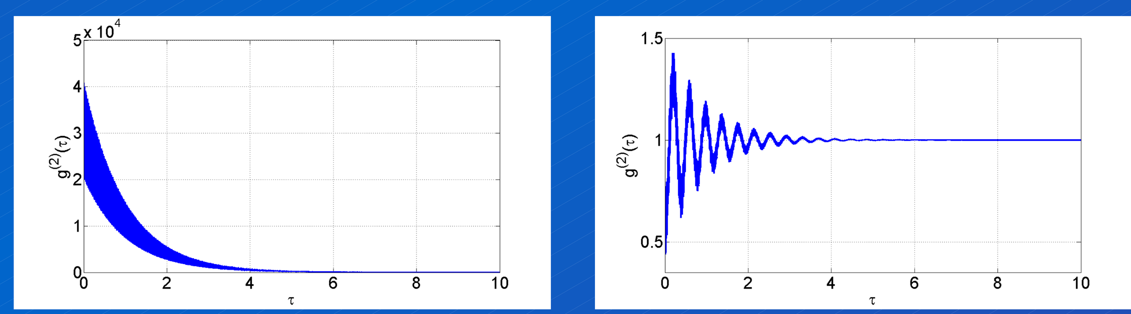


Fig. 11: Field intensity correlation for peak 2. Parameters: g = 500, κ = γ = 1, Δ = 353.6, ε = 0.5 (left) and Δ = 358.1, ε = 40 (right).

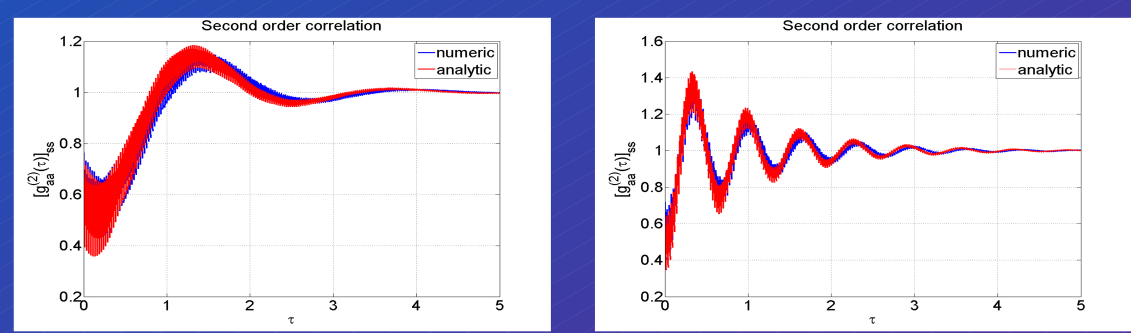


As before, a slow Rabi oscillation appears in $g^{(2)}(\tau)$ but this time there is a fast oscillation present as well. This is a so-called quantum beat: the "first" emitted photon sets up a superposition of the lower and upper dressed states with different energies, and the subsequent Schrödinger evolution causes the beat frequency (2g) to feature in the dynamics.

4.2 Approximate Analytical Model

In [5], we make an effective four-state model keeping only $|g,0\rangle,\ |1-\rangle,\ |1+\rangle,\ |2-\rangle$ and fully explain the observations for the two-photon transition. For example, Fig. 12 compares the analytic, approximate results with the full numerics.

Fig. 12: Intensity correlation functions: numeric and analytic, agreement is excellent. g = 500, κ = γ = 1, Δ = 354.2, ε = 15 (left) and Δ = 356.0, ε = 30 (right).



5. Summary

• We have looked at MPTs in the strongly coupled, strongly driven JC model. When the drive is tuned to such a MPT, the JC ladder is effectively truncated to a finite manifold due to the $\sqrt{n}$ nonlinearity of the JC spectrum. Consequently, two main results followed:

→ For small drive, the light is very strongly bunched, but with increasing drive, $g^{(2)}(0)$ decreases rapidly and we get antibunching.

→ The incoherent spectrum develops a clear Mollow triplet with increasing drive due to multiple dynamic Stark splittings of all energy levels nearby. Also, all peaks at higher frequencies are split into doublets (or triplets).

In conclusion, Multi-Photon Blockade is predicted to occur in our system. In our opinion, the most promising experimental platform for observing this effect is Circuit QED.

Acknowledgments

A huge thank-you to Simon Whalen for many helpful discussions and to Sze Tan for developing the software used. Thanks!

References

[1] Lev. S. Bishop *et al.*, Nonlinear response of the vacuum Rabi resonance, Nature Physics, **4**, 12 (2008).

[2] A. Imamoglu *et al.*, Strongly interacting photons in a nonlinear cavity, Phys. Rev. Lett. **79**, 8, 1467 (1979).

[3] L. Tian, H.J. Carmichael, Quantum trajectory simulations of the two-state behaviour of an optical cavity containing one atom, Phys. Rev. A, **46**, 11, R6801 (1992).

[4] Sophie S. Shamailov, Multi-photon blockade in Cavity QED, Honours Thesis, University of Auckland, 2009. Here you will find detailed workings, more results and plenty of relevant references. Downloadable at http://sites.google.com/site/sophieshamailov

[5] S.S. Shamailov *et al.*, Multi-photon blockade and dressing of dressed states, Optics Communications, **283**, 5, 766-772 (2010). The publication contains the simplified model analytical work not present in [4], achieving superb quantitative agreement with the numerics.