Elastic scattering by planar fractures

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Outline

1. Introduction
2. Scattering by a plane crack
3. Laboratory experiments
4. Direct excitation
Faults and fractures

- Controls fluid flow: hydrocarbons, water, magma...
- Characterization of fracture properties with elastic waves
- Active or passive monitoring


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Theoretical expression and laboratory modeling

- Single fracture
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- Single fracture
- Linear-slip model: \( [u_i] = \eta_i T_r \), with \( [u] \) displacement discontinuity, \( \eta \) compliance \((1/\text{stiffness})\) and \( T \) traction
Theoretical expression and laboratory modeling

- Single fracture
- Linear-slip model: \([u_i] = \eta r T r\), with \([u]\) displacement discontinuity, \(\eta\) compliance (1/stiffness) and \(T\) traction
- Born approximation: scattered field small compared to incident field
Single fracture

Linear-slip model: \( [u_i] = \eta r T_r \), with \([u]\) displacement discontinuity, \(\eta\) compliance (1/stiffness) and \(T\) traction

Born approximation: scattered field small compared to incident field

Frequency domain: \( f(t) = \int F(\omega) e^{-i\omega t} d\omega \)
Single fracture

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Born approximation: scattered field small compared to incident field

Frequency domain: \(f(t) = \int F(\omega)e^{-i\omega t}d\omega\)

Previous work: small fractures \(\Rightarrow\) effective medium (Crampin, 1981; Hudson, 1981), or large fractures \(\Rightarrow\) reflection coefficients (Pyrak-Nolte et al., 1990; 1992)
Theoretical expression and laboratory modeling

- Fractured plastic samples
- Ultrasonic frequencies (100 kHz - 10 MHz) \( \Rightarrow \lambda \approx 10^{-4} - 10^{-2} \text{ m} \)
- Laser generation and detection of body waves

Units are cm
Ultrasonic laser receiver

- Wide bandwidth
- Absolute displacement
- Non-contact and small footprint compared to the wavelength
- No moving parts
- Scanning system
Decomposition of the compliance $\eta$:

$$\eta_{ij} = \eta_N f_i f_j + \eta_T (\delta_{ij} - f_i f_j)$$

- $\sigma$ stress
- $\omega$ angular frequency
- $\alpha$ P-wave velocity
- $\rho$ density of the material
- $k_\alpha$ wavenumber
- $R$ distance to the fracture
Decomposition of the compliance $\eta$:

$$
\eta_{ij} = \eta_N f_i f_j + \eta_T (\delta_{ij} - f_i f_j)
$$

Displacement as a function of the scattering amplitude:

$$
u_n^{(P)}(x) = f_{PP}(\eta) e^{i k_\alpha R} m_n$

- $\sigma$: stress
- $\omega$: angular frequency
- $\alpha$: P-wave velocity
- $\rho$: density of the material
- $k_\alpha$: wavenumber
- $R$: distance to the fracture
Scattering amplitude of a circular plane crack

For the experimental geometry:

\[
f_{P, \rho}(\psi, \theta) = \frac{\omega a}{2 \rho \alpha^3 (\sin \psi - \sin \theta)} J_1 \left( \frac{\omega a}{\alpha} (\sin \psi - \sin \theta) \right) \times \left[ \eta_N \left\{ (\lambda + \mu)^2 + (\cos 2\psi + \cos 2\theta)(\lambda + \mu)\mu + \mu^2 (\cos 2\psi \cos 2\theta) \right\} + \eta_T \mu^2 (\sin 2\psi \sin 2\theta) \right].
\]

⇒ term in \( \eta_N \), and term in \( \eta_T \) non-zero for \( \psi \neq 0 \)

\( \omega \) angular frequency
\( \alpha \) P-wave velocity
\( \rho \) density of the material
\( \lambda, \mu \) Lamé parameters
\( a \) fracture radius
Experimental setup

- Sample: PMMA cylinder (transparent plastic material), 150 mm high x 50.8 mm diameter
- Piezoelectric transducer source, 5 MHz, 400 V pulse
- Laser ultrasonic receiver: wide bandwidth (20 kHz – 20 MHz), absolute vertical displacement, small footprint, sensitivity in Å
- Fixed source-fracture angle $\psi$ and moving receiver ($\theta$ changes)
Experimental setup: geometry

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Non-fractured sample

- velocities $\alpha = 2600$ m/s and $\beta = 1400$ m/s
- $\rho = 1190$ kg/m$^3$ $\Rightarrow$ Lamé parameters $\lambda = 3.4$ GPa and $\mu = 2.3$ GPa
Non-fractured sample

- velocities $\alpha = 2600$ m/s and $\beta = 1400$ m/s
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- $f-k$ filter to remove surface waves
Fractured sample: data

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Fractured sample: data

Time (µs)

90 180 270 360

Displacement (nm)

-0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4

Scattered P

Top view

\psi = 0

PZT source

\psi = 50°

PZT source
Fractured sample: scattering amplitudes, $\psi = 0^\circ$

Influence of $\eta_N$: experimental amplitude
Fractured sample: scattering amplitudes, $\psi = 0^\circ$

Influence of $\eta_N$:

experimental amplitude

$\eta_N = 10^{-11}$ m/Pa
Fractured sample: scattering amplitudes, $\psi = 0^\circ$

Influence of $\eta_N$:
- Experimental amplitude

$\eta_N = 10^{-11} \text{ m/Pa}$

$\eta_N = 0.5 \times 10^{-11} \text{ m/Pa}$
Fractured sample: scattering amplitudes, $\psi = 0^\circ$

Influence of $\eta_N$:
- experimental amplitude
  - $\eta_N = 10^{-11}$ m/Pa
  - $\eta_N = 0.5 \times 10^{-11}$ m/Pa
  - $\eta_N = 2 \times 10^{-11}$ m/Pa
Fractured sample: scattering amplitudes, $\psi = 50^\circ$

Influence of $\eta_T$:
experimental amplitude

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Fractured sample: scattering amplitudes, $\psi = 50^\circ$

Influence of $\eta_T$:
experimental amplitude

$\eta_T = 10^{-12}$ m/Pa
Fractured sample: scattering amplitudes, $\psi = 50^\circ$

Influence of $\eta_T$:
experimental amplitude

$\eta_T = 10^{-12}$ m/Pa

$\eta_T = 10^{-13}$ m/Pa
Fractured sample: scattering amplitudes, $\psi = 50^\circ$

Influence of $\eta_T$:

- Experimental amplitude
  - $\eta_T = 10^{-12}$ m/Pa
  - $\eta_T = 10^{-13}$ m/Pa
  - $\eta_T = 10^{-11}$ m/Pa
Fracture scattering: summary

- Analytic expression for the scattering amplitude
- Good agreement between theory and laboratory data
- Estimation of the compliance $\eta_N \approx 10^{-11}$ m/Pa
- Same range of compliance as found in the literature (Pyrak-Nolte et al., 1990, Worthington, 2007)
- Low sensitivity to $\eta_T$
Direct excitation of a fracture
Experimental setup

- Same fractured sample
- Direct excitation by laser-induced thermal expansion
- Pulsed infrared laser source
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Results
Results

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Results

Surface excitation

Direct excitation

Laser noise

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Elastic scattering by planar fractures
Tip diffractions $\Rightarrow$ radius estimation $a = 3.5$ mm

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