1. INTRODUCTION

Consider the data in Table 1 taken from [1]. These data come from a randomized, double-blind clinical trial comparing an active hypnotic drug with placebo in patients with insomnia. There are many hypotheses available to analyze these data. For example, [1] proposed the log-linear model to characterize differential change among treatments, and [2] analyzed these data by using various marginal models.

Assume that $F_X + F_Y \neq 0$. Consider the submeasure defined by
\[
\Omega_{M1} = \left[ \frac{2 + \sqrt{2}}{2} \prod_{i=1}^{2} \prod_{k=1}^{R-1} \left( F_{(i)}^* - Q_{(i)}^* \right) \right]^{1/2}.
\]

Submeasure II: Let $S_N = 1 - F_X$, $S_Y = 1 - F_Y$, $\Delta_2 = \sum_{i=1}^{R-1} (S_N + S_Y)$, and let
\[
S_{1(i)} = \frac{S_N}{\Delta_2}, \quad S_{2(i)} = \frac{S_Y}{\Delta_2}, \quad Q_{2(i)} = \frac{1}{2} (S_{1(i)} + S_{2(i)}).
\]

Assuming that $S_N + S_Y \neq 0$, we shall define the submeasure $\Omega_{M2}$, which represents the degree of departure from MH, by $\Omega_{M2}$ with $(F_{1(i)}^*, F_{2(i)}^*)$ replaced by $(S_{1(i)}^*, S_{2(i)}^*)$ and $(Q_{2(i)}^*)$, respectively.

Measure for MH: Assume that $F_X + F_Y \neq 0$ and $S_N + S_Y \neq 0$. Consider a measure defined by
\[
\Omega_M = \frac{\Omega_{M1} + \Omega_{M2}}{2}.
\]

Properties of measure: (i) $0 \leq \Omega_M \leq 1$, (ii) $\Omega_M = 0$ if and only if there is a structure of MH, and (iii) $\Omega_M = 1$ if and only if the degree of departure from MH is the largest, in the sense that $F_X = 0$ (then $S_N = 1$ and $F_Y = 1$ (then $S_Y = 0$), or $F_X = 1$ (then $S_N = 0$) and $F_Y = 0$ (then $S_Y = 1$), for arbitrary cut point $i = 1, 2, \ldots, R - 1$.

3. TEST

Let $n_{ij}$ denote the observed frequency in the $(i, j)$th cell. The sample version of $\Omega_M$, i.e., $\hat{\Omega}_M$, is given by $\Omega_M$ with $(p_{ij})$ replaced by $(\hat{p}_{ij})$, where $\hat{p}_{ij} = n_{ij}/n$ and $n = \sum \sum n_{ij}$. Assuming that a multinomial distribution applies to the $R \times R$ table. Using the delta method, we obtain the following result.
\[
\sqrt{n}(\hat{\Omega}_M - \Omega_M) \text{ has asymptotically a normal distribution}
\]
\[
\text{with mean zero and variance } \sigma^2[\hat{\Omega}_M].
\]

For tables A and B (with sample sizes $n_A$ and $n_B$), denote $\Omega_M$ by $\Omega_M^{(A)}$ and $\Omega_M^{(B)}$, respectively. Then estimate the difference between $\Omega_M^{(A)}$ and $\Omega_M^{(B)}$ is given by the sample difference $\hat{\Omega}_M^{(A)} - \hat{\Omega}_M^{(B)}$. When $n_A$ and $n_B$ are large, this difference has approximately a normal distribution with standard error
\[
\left( \frac{\hat{\sigma}^2[\hat{\Omega}_M^{(A)}]}{n_A} + \frac{\hat{\sigma}^2[\hat{\Omega}_M^{(B)}]}{n_B} \right)^{1/2},
\]
where $\hat{\sigma}^2[\hat{\Omega}_M^{(A)}]$ and $\hat{\sigma}^2[\hat{\Omega}_M^{(B)}]$ are the estimated variances. For Table 1, the value of test statistic is 3.20. This is the significant at 5% level. So, we can infer that the active drug is more effective than the placebo.

REFERENCES